VALUATION, DEPRECIATION AND THE RATE-BASE

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PREFACE

A thorough discussion of the principles which control the engineer, the economist, the assessor and the business man in making valuations will not be attempted in these pages. This book is the result of personal contact with the valuation problem. No apology need be made for the fact that, in presenting an original analysis of the problem, the arrangement of the material is not always as logical as might have been expected if the present state of the art only had been brought under review. Special consideration has been given to a discussion of the non-agreement of the actual life of articles which have a limited period of usefulness with their probable or normal life. The effect of this non-conformity has been studied by the author and the results have been thought worthy of a special chapter. These results show that there is great advantage in adopting, instead of "present value," a rate-base without deduction of depreciation, which will include but little, if anything, other than legitimate and properly estimated cost as the starting point when rates are to be fixed. He therefore recommends for the thoughtful consideration of the student, the method of procedure which he has named the Unlimited Life Method and which is herein fully explained. The author feels, too, that the owner of a public utility is, generally, entitled to larger earnings than will yield a bare interest return on the invested capital. Volume of business transacted and also the unearned increment should, sometimes, be taken into account when estimates are made of the earnings that may with propriety be allowed. Such matters as these have received attention and it is hoped that some of the ideas that are herein advanced may stimulate further thought along similar lines and may, here and there, prove helpful to those who have appraisals to make or

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rates for an output or for a service to establish. Careful attention has been given to the fundamental principles which should control when appraisals of public utilities are made for rate-fixing purposes and their practical application is explained.

The tables which are presented in this volume are intended to meet the needs of valuation engineers, but will also be found useful by any one having problems of finance and bonding to solve. They are based throughout on original calculations. Those relating to replacement requirements and to expectancy are of a novel type not to be found elsewhere. Special care has been taken to secure accuracy and convenient arrangement.

Acknowledgment is due to the office staff of the American Engineering Corporation the members of which have assisted in the preparation of the tables.

C. E. GRUNSKY.

San Francisco, Cal., February 1st, 1916.

ABBREVIATIONS AND NOTATION

Amort. = Amortization.

Ann. = Annual.

Beg. = Beginning

Eq. = Equal.

Payt. = Payment.

- A = the annual replacement requirement for each dollar of capital invested annually in a growing plant.
- A_m = the accrued amortization in m years when the annual amortization installment is a and the interest rate is i
- A' = the amount of \$1.00 at compound interest at the end of the nth year at the interest rate i.
- A'' = the amount of an annuity of \$1 00 paid at the end of each year at compound interest at the interest rate i.
- $a_n =$ the amortization installment which must be invested annually, in order to amount at compound interest to \$100 in n years.
- a_n' = the annual installment which at compound interest at the rate i will amount to \$1 00 in n years
- a_n'' = the annuity receivable at the end of each year which \$1.00 will buy for n years.
 - a_m = the current amortization in the mth year, i.e., the amortization increment a plus interest on the amortization fund already accumulated. It is the amortization installment which in the remaining years of life will retire the remaining capital.
 - $C = \cos t$ of replacing a group of articles.
 - c= the annual renewal requirement for a group of articles whose cost of replacement is C.
 - e = expectation, that is the probable remaining years of usefulness of any article whose probable life new was n years.
 - e' = relative expectancy of an article whose probable life new is 10 years when compared with an article m years old whose probable life new is n years.
 - g = the average annual investment in additions to a plant.
 - i= the rate of interest per year expressed fractionally thus for 6 per cent, i= 0.06.
 - m = a number of years.
 - m' = relative age of an article whose probable life new is 10 years, when compared with an article m years old, whose probable life new is n years.

- n = the probable life term in years of any article, or the term of amortization.
- P = the present value of \$1.00 due at the end of n years.
- P'= the present value of an annuity of \$1.00 receivable at the end of each year during n years.
- R = amount in the replacement fund expressed in percentage of the original investment.
- S= the sum of all annual replacement requirements estimated for a number of articles of various ages
- S_n = the sum of all annual replacement requirements during n years.

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CHAPTER I

INTRODUCTION AND GENERAL NOTES

Conflicting Views Relating to Procedure. — The need of harmonizing the conflicting views relating to the best methods of dealing with the establishment of rates that are to be charged by public service corporations for the service which they render or for the commodity which they supply is recognized by economists and engineers. This need is the more pressing in view of the fact that the public, in exercising control over the operation of the utility through properly constituted authority, may and no doubt frequently has established onerous and burdensome regulations, resulting at times in the confiscation of property. At any rate the limitations relating to permissible charges have frequently been such that the question of the reasonableness and sufficiency of these charges has been taken into court and the courts have reached certain conclusions from which there is apparently no appeal but concerning some of which the wisdom of permanent enforcement may well be called in question.

Rapid progress is being made in recognizing the fundamental principles which should control when an appraisal of public service properties is to be made as a basis for the establishment of rates for service rendered. This subject is, however, today still in a controversial stage. The assembling of facts relating to such fundamental principles as will be of service to the appraisers of operating plants of any character, whether used

in the public service or not, is the task which was set in undertaking the preparation of this volume. It is hoped that the same will fill the gap for a time until wider experience of the valuation experts shall have eliminated the involved and ofttimes useless procedures which are still in practice and explanations of which have, therefore, been included in the text.

The Public and the Owner. — The question may well be asked whether the reformer, who has been seeking protection against the demands of the trusts and corporations which have frequently been guilty of over-capitalizing, has not in his turn gone too far in the endeavor to reduce the earnings of the corporation-owned public utilities. The public has demanded and, in some measure, has obtained, from various rate-fixing bodies, and from the courts, the introduction of methods of procedure that are not always equitable, but the baleful effects of which are slow in becoming manifest. There has, for example, been no little confusion in the matter of defining depreciation and not a few valuation experts have had difficulty in grasping the fact, which when once recognized seems fundamental, that the repayment of capital (amortization), is one thing and that the making of provisions for the replacement of worn-out or discarded parts at the end of their term is another thing.

It is fundamental that the owner of a public utility should be allowed:

- a. To obtain, sooner or later, a reasonable return on his investment.
- b. To recover at some time the capital invested for the public good.
- c. To reap a suitable reward for establishing and managing the enterprise.

It is the knowledge that these principles have obtained universal recognition which prompts the capitalist to embark upon such ventures as those known as public utilities.

Control of Public Utilities a Recent Innovation. — Until within the last few decades the control by the public of the operations of the public utility concern has generally been so loosely ex-

ercised that the margin for a possible profit was large; regulation was more or less perfunctory and, very frequently, due to the imperfect knowledge of the problem, neither the owner of the utility nor the rate-payer knew whether the earnings were deficient, reasonable, or excessive. The recent tendency of the public to insist upon its right to control and the attempt to make a close study of what would in each case be reasonable earnings has stimulated the work of the economist and the valuation engineer and has led to a consideration of many new phases of an interesting subject.

Prof. John H. Gray in an address before the Economic Club of San Francisco on July 28, 1914, in presenting a historical survey of public utilities said:

"We have very recently entered upon the experiment of administrative control through central commissions. But we have failed entirely to grasp the significance of the problem, or the steps necessary to solve it, if we fail for a moment to remember that until the establishment of the Board of Gas Commissioners in Massachusetts, in 1885, no industries, save that of transportation, were classed as utilities, and that the other states have been very slow to follow the lead of Massachusetts in this matter. In fact, it is only within the last decade that the problem may be said to have been taken up seriously by the country as a whole. That is altogether too short a time to adjust the law, the theories, or the practices to the economic needs and conditions of the situation. Much more is it too brief a period in which to have modified to any considerable degree the pioneer philosophy that still dominates the private owners of these industries.

"The scientific theory is that the utilities should render adequate, safe, and universal service, at just, reasonable, and fair prices to all, and that the sovereignty shall be the final judge in every case of these matters. This statement implies, of course, that the total gains or rewards of the owners shall be reasonable under all circumstances, including that of virtually guaranteed monopoly, and that they shall have just compensation in case of expropriation."

Fundamental Principles Apply to Industrial Establishments.

— The work which has been done in studying the fundamental

principles looking to the fair treatment of both the rate-payer and the owner has also been of benefit in a large way to the factory, and to all the great industrial establishments of the country, because it points the way to a closer approximation of the actual cost of production, thereby establishing a better basis for fixing the sale price of the output.

The discussions in the following pages are necessarily directed mainly to the valuation of public service properties and to a discussion of the procedure to be followed in determining what are reasonable earnings, but the general fundamental principles are equally applicable to the private concern.

Value must be ascertained when the ownership of property is to be transferred from one person to another as in the case of a sale; it must be ascertained when the property is capitalized as a basis for the issuance of securities, and, too, for taxation purposes. When rates or prices to be charged for service or commodity output are to be fixed, it is necessary to know the amount of capital which is properly invested — to know, in other words, what the amount is which should be taken into account as a "rate-base."

Depreciation and Withdrawal of Capital. — The practice is altogether too common of treating every earned depreciation allowance as equivalent to a withdrawal of invested capital and of assuming that any of the various methods of estimating present value are equitable, regardless of the past history of the plant under investigation. So, too, there is sometimes a too rigid construction of the generally recognized requirement that only those parts of a property which are actually in use or which are useful elements are to be considered in valuing the property for rate-fixing purposes. This has occasionally worked a hardship upon the public service corporation and may therefore be regarded as one of the factors which have caused a demand for large allowances for all kinds of intangible and more or less indefinite elements of value to offset losses and unproductive investments.

Every private industrial establishment, too, has its depreciation

problem to deal with. Depreciation enters into the cost of production and must be covered in the sale price if there is to be any profit. The cost of the perishable parts of the plant which are consumed, just as fuel is consumed though at a slower rate, must be recovered sooner or later.

Definition of Terms. — The definitions of terms, herein presented, relate to the sense in which these terms are used in this volume. There is not yet any perfect agreement among engineers and accountants relating to all of these terms and, as time goes on, some modification of terms or of the definitions of terms is to be expected. These definitions are therefore offered with due appreciation of the fact that some of them may not survive and that others remain subject to modification.

The Agency Theory.— Every owner of a public service property is in a certain sense the agent of the rate-payer. If regarded as such agent he is under obligation to render the service which he has engaged to perform in a satisfactory manner; he is entitled to protection against unreasonable competition, and he is entitled to protection of his investment, provided, of course, that the investment is a proper and reasonable one. There may be some cases, however, where unwise investments have been made, and others where part compensation for making the investment has been found in the increased value of real estate or other property, perhaps not connected with the utility but owned by the same individuals who own the utility, and in such cases the value of the service rendered to the public may bear an unusual relation to the investment.

Thus an irrigation canal, which was constructed to develop a region of little or no value without water, when considered by itself, apart from the land which it irrigates, may have cost so much that the service charges cannot be made high enough to yield a revenue on its cost. And yet the canal was justified, though perhaps not as a separate venture. In such cases a transfer of the canal to the land owner would be appropriate in order that the question of a reasonable return on invested capital might be eliminated, or as an alternative a bonus charge

against the land whether under this name or under the name of "water right," or otherwise, might be agreed to, such that the capital remaining in the canal enterprise, as a proposition apart from the land, would be reduced to a reasonable amount. If neither of these alternatives are adopted the canal venture may be permanently unprofitable, from the standpoint of present owners. This will be the case when the rates that would yield interest on the investment are in excess of the value of the service rendered and cannot be supplemented by a participation of the canal owner in the unearned increment which the construction of the canal has brought to the land owner.

As another illustration a railroad may be considered which was primarily constructed for the development of a tract of land or for the development of mineral or timber resources. There can be no question that when considered from the broad standpoint of the advantage to society the construction of a railroad under such circumstances may be justified. When this railroad begins to serve the public and the freight tariff and passenger rates are to be fixed, these may have to be established on the basis of the fair value of the service rendered without any attempt to make the earnings yield interest on the capital invested in the road, or suitable consideration may have to be given to the fact that a part of the investment was made for the benefit of some special enterprise such as a rock quarry or a lumber business and should not have been charged to the road at all.

These limitations upon the use of an appraisal as an element for consideration in fixing rates are fully recognized, but these and other limitations are not such as to detract from the desirability of referring to and using appraisals under ordinary circumstances.

Basis for Sale Price — Business Enterprises. — Everyone who engages in a business enterprise must know:

The cost of the article in which he deals at the time the article is sold.

The cost of the plant such as buildings, roads and equipment

which are necessary to manufacture or to care for the article pending the time of sale.

The operating costs of the business.

The depreciation of the plant or the replacement requirements thereof.

The profit which it is desired to make.

When these elements are known and a proper analysis of operating cost has been made the prudent business man will distribute his fixed elements of cost, such as management, depreciation and the like with a view to making the price of the article or the charge for the service which he renders, attractive to his customers.

In a large machine works where the charges for repair work appeared unusually high, it was found upon investigation that overhead expenses were unequally distributed to new and to the repair work. This was done in order that new work which had to be obtained in competition with other concerns could be taken at lower figures. While this is only an isolated example, it shows the importance of being in a position to analyze costs in order that charges may be fixed low enough to secure business and yet high enough to yield an adequate profit.

Reference to the Tables in this Volume.—As an aid to the appraisers of industrial and public utility properties, this volume would fail in its purpose without tables containing information relating to the probable term of service and the expectancy of the articles in common use in such establishments or plants and also interest and annuity, amortization and depreciation tables.

The amortization and depreciation table not only covers a range from 2-year life to 75-year life, but it has been so arranged that the remaining years of service, instead of age, can be made the starting point when it is desired to enter the table for present value and the so-called current rate of depreciation.

While such tables and explanations as are furnished in this volume are generally applicable they have been prepared, as stated, with special reference to the requirements of appraisers and of the public service commissions and of other rate-fixing

bodies. They will meet, too, the requirements of the owners of industrial and other properties, who owing to lack of such information occasionally allow over-capitalization and excessive dividends, coupled with inadequate earnings, to cripple their enterprises.

The Panama Canal and Fundamental Principles. — The necessity for a clear understanding of the fundamental principles to be followed when rates are to be fixed may be illustrated by reference to the case of the Panama Canal. Here is a great work representing an investment by the United States of about \$375,000,000. It is a type of canal which involves large operating expense. In connection with this canal the question arises: What should the earnings be?

The problem of the canal tolls may be broadly considered along the following lines:

First: Who is ultimately to pay for the canal? Is it to be the user of the canal?

Second: Has the canal been built by the United States as a revenue-producing investment or is the canal to be regarded as worth what it has cost for military purposes and as an instrument of general benefit?

Third: What shall be included in the operating expenses?

The ordinary citizen is under the impression that the \$375,-000,000 which the canal has cost is an investment similar to those which the United States has made in constructing the Eads jetties at the mouth of the Mississippi river, in constructing the Ambrose channel for access to the New York harbor, and in building the San Pedro breakwater for the protection of the harbor of Los Angeles and the Columbia and Eureka bar jetties for the improvement of other Pacific Coast harbors of more or less local importance. If this is the fact, the investment in the canal has been made as a non-revenue-producing investment for all time and there will be no need of recovering the cost of the canal from those who use it nor even interest on this cost. If the United States plans to collect the cost of the canal in installments from the users of the canal, no matter how small

these installments may be, the canal will ultimately be paid for, in part at least, by foreign people. And if interest only is covered by the toll charges, then this Nation will stand in the position of having made a loan to the commercial interests of the world of \$375,000,000 for the construction of the canal.

Once the fundamental principle has been adopted relating to the extent to which commerce shall recoup the United States for its outlay and the question has been settled as to whether the users of the canal shall pay interest on the investment, the question of operating expenses will have to be taken up. Salaries, wages, materials and supplies are readily determinable. All expenditures for repairs and upkeep will be included with perhaps no difficulty until the question of depreciation is to be considered.

Shall each element or class of elements be considered separately, according to probable useful life, and some sum be set aside which together with interest thereon will replace the wornout parts, or shall the amount to be set apart for replacements be determined directly from the relation of probable life to the cost of replacement of each perishable part by setting apart annually such a fraction of the cost that these annual increments, without interest, added together will be equal to the cost, or finally shall provision be made from year to year for the replacements that will probably have to be made in each year? It is hoped that what is presented in this volume may prove of some assistance in the solution of problems of this character.

Unprofitable Expenditures and Early Losses. — Unprofitable and unproductive expenditures which may be the results of mistakes or of accidents and early losses in the business are elements which should not be regarded as adding value to a property. They belong in some measure to the hazards of the business. They are, nevertheless, to be taken into account when rates are to be fixed because they usually represent legitimate expenditures. They represent an outlay which in nearly every case would have been incurred under consent of the rate-payer, if the owner had been acting as the agent for the rate-

payer and with his approval. Consequently the rates should be fixed with a view to amortize sooner or later such expenditures.

In thus making provision for unproductive expenditures determined, perhaps, from actual expenditures legitimately incurred in excess of cost of works in use, care must be taken not to go too far. Wasteful expenditure is not to be sanctioned and wise and prudent management is entitled to reward. Experience alone can determine what allowance should be made for hazards of the business and for the cost of establishing the business and bringing it up to a paying basis. If it be found. for example, that a suburban electric road will not be on a paying basis for a number of years, the losses (or deficient earnings) during these years may be added to the cost of the road not as elements of value but as a part of the investment on which an interest return is to be allowed or as the preferable alternative. net earnings in excess of interest on the cost of the road can be allowed in such amount that within a reasonable time the early losses will be amortized. Thereafter a continuation of some excess of earnings above the returns from ordinary safe investments will be the owner's reward for having engaged in the enterprise.

Franchise and Water-right Values. — When the public through properly constituted authority grants a franchise or confers a privilege to enter upon a business which is in the nature of a public service, as, when it grants the use of water for power, for irrigation or for other purposes, the franchise or water-right is valuable only to the extent that the public provides a market for the service rendered or commodity supplied. When the rates to be charged are subject to regulation and are not fixed in the franchise or water-right grant, no basis exists (except in the cases of strategic value) for determining franchise or water-right value. This value depends, as will be explained, upon earnings in excess of a fair return on the investment and if earnings are not protected by franchise terms, it will lie in the power of the rate-fixing body to eliminate franchise and water-right value altogether. This is as it should

be. At the same time with due regard to the share in the general prosperity to which the owner of the utility is entitled, he should be allowed to earn more than ordinary interest on his investment and a capitalization of any such excess earnings when these are fairly assured will furnish him with a basis for approximating the sum of all intangible values of whatever nature.

Amortization and Depreciation. — A clear distinction should be made between the amortization of capital that is invested in any enterprise and provision for depreciation or the replacement of perishable elements. The sinking fund for the retirement of bonds, or other debt, should not, in other words, be confounded with the fund usually referred to as the "depreciation" fund which is intended to meet the replacement requirements as they arise.

Valuation for Purchase or Sale. — In appraising for a sale, various matters are to be taken into account which are of minor importance when rates are to be fixed. If the accrued depreciation has actually been allowed in the earnings and has been permitted to accumulate in a fund, and if the accumulated fund properly represents the accrued depreciation, and is a part of the property to be transferred, then the appraisal including this fund should be in substantial agreement with the legitimate investment. If, on the other hand, there is no depreciation fund, if the earned and collected adequate depreciation allowance in excess of the expenditures therefrom for replacements has been absorbed for other purposes by the owner, then a purchaser would make the depreciated value or the legitimate investment less depreciation his starting point in determining the amount which a plant is worth, because in purchasing the property he assumes an obligation to replace perishable parts as they go out of use. If the plan is followed of allowing earnings which will meet all replacement requirements together with interest on the full investment, without deduction for depreciation, then a purchaser in view of the assured return will make the legitimately invested capital without deduction for depreciation his starting point in determining what he can afford to pay.

This last case is in conformity with the requirements herein explained, that the investment in a public utility should remain, at all times, unimpaired. It is at variance with the plan advocated by many and approved in various court decisions, that the appraisal for rate-fixing purposes shall fluctuate from year to year with the age and remaining service value of the perishable elements that go to make up a public service plant.

Present Value as a Starting Point. — Despite prevailing sentiment and rulings of the public service commissions and decisions of the courts in favor of making "present value" the starting point when rates are to be regulated, it will be apparent to the student that what is estimated as "depreciation" which may never have been earned and collected has not necessarily been a repayment of capital and that therefore "present value" is not always the best starting point.

The wisdom of starting with "present value" or with "value" in any form may well be questioned because value is a result of assured earnings and such values as are not covered by the investment do not exist until the earnings create them. In appraising for rate-fixing purposes the aim should be to make the legitimate investment and not the depreciated value the rate-base.

It is sometimes difficult to modify the "fair value" basis of rates as heretofore insisted upon by the courts so that it will fit local conditions as in the following illustrations:

r. Adjacent to the built-up section of a community which is being supplied with water at satisfactory and in every way equitable rates, a large subdivision of acre property is being made. Streets have been laid out and paved, sidewalks have been constructed, an adequate sewer system has been provided and a system of water mains has been constructed. These water mains conform in every respect to the requirements of the owner of the water-works to whom this system of mains is now offered on the sole condition that he supply prospective customers with water.

Acceptance of this system of mains may bring no immediate increase of revenue. The development of the newly subdivided area may be slow. Some years may elapse before the sales of water in the district will be equal to the added burden of maintenance, collection of rates, and depreciation.

Shall part of the system thus acquired be included in the valuation on which the owner is to receive a return? Or shall such inclusion be deferred? And if deferred for how long a time?

The inclusion of such property merely because it has "value" can perhaps be justified on the plea that it makes no difference what a plant has cost the owner and that according to the decision of the courts value alone should be considered. It will be held that it represents a necessary extension of the distributing system that could best be made at the same time that other street improvements were made, and that it would be impracticable to attempt in the case of this considerable addition to the system to fix the time when the extent of the use of such new pipe and the addition of revenue resulting from such use would no longer add to the burden of the remaining, or rather, older rate-payers. But the fact remains that when considered as the agent of the public the water-works owner has not made the investment. The investment has been temporarily made by the owner of the subdivided tract. He will be reimbursed by the purchaser of lots, for the outlay for this and all other improvements made on the tract will appear in some form in the selling price of the lots.

If the municipality owned the water-works, in what situation would it now find itself? The same treatment should be accorded the owner as would be accorded to an agent of the municipality. Viewed from this standpoint, the owner, when he accepts the addition to his distributing pipe system as a gift, assumes at once also the obligation to keep this part of the system in repair and to replace the same when worn out or discarded for other reasons. The owner is therefore entitled (necessity for the extension of the system being admitted) to the

necessary allowance for operating expenses including a proper allowance for anticipated future replacements but should not always immediately be allowed interest on the full cost of this extension which, as already stated, together with other improvements such as street grading, paving, sidewalk and sewer construction will be included in the price charged by the owner for the lots and will therefore represent a contribution by this new section of the municipality to the cost of the privately owned water-works

In some measure a donated extension to a water-works system, as here used for the purpose of illustration, bears a similar relation to the plant, as does property held for future use in excess of the capacity of the plant, with the difference that in the one case the public donates the property to the owner, in the other, the owner secured the same by investing his own capital. In the fixing of rates the increase of capital due to the receipt of gifts and donations should generally be considered separately from the actually invested capital. Individual cases will arise requiring special treatment. The owner in most cases will not be entitled to as large a return on this donated capital as on his actual investment.

2. The above case is not materially different from that of the municipality which needs water-works but cannot afford to make the necessary expenditures. To induce the investment of private capital in a water-works enterprise, the offer of a bonus is made. To avoid legal restrictions this has sometimes been done through the medium of extra large annual payments extending through a number of years for the water required for street sprinkling, for sewer flushing, for extinguishing fires and other municipal uses. Whenever a direct or an indirect contribution is thus made, the owner's invested capital — the ratebase — is not necessarily represented by the cost of the reproduction of the entire water-works.

Fair Value as a Rate-Base. — In concluding this introductory chapter attention may be called to the accepted view of the courts in the matter of valuations for rate-fixing purposes. It

appears from the various decisions of the courts which have been cited and from many others that might be referred to, that the majority of the courts are definitely committed to the idea of making the "fair" value of the property used for the convenience of the public, that is to say the "present value" of this property the basis of all calculation as to the reasonableness of the rates to be charged by a public service corporation.

If the general analysis of the fundamental principles, which must control when rates are to be fixed, as presented in this volume, contributes in any degree to the modification of this attitude of the courts, the author will feel repaid for having attempted the analysis. It seems to him illogical to make the earnings dependable on value which itself is the direct result of the earning capacity of the property for whose output rates are to be fixed. The real starting point is the properly invested capital and the volume of the business which is transacted. What consideration should be given to these elements and to other factors affecting a legitimate return to the owner of the public service property will be further discussed.

CHAPTER II

DEFINITIONS

Value. — Value is the worth of anything measured by any standard of purchasing power. It is the exchange power which one commodity or service has in relation to another. The word value is used in this sense throughout this volume. The other meaning of the word relating to adaptability for a certain purpose has not been brought under discussion. It is only value in exchange and not utility which is of interest to the appraiser.

A careful distinction must be drawn between the cost of an article and its value. The cost frequently determines the price at which an article is offered and taken in exchange and there may therefore be at times a close relation between cost and value. But the words are in no sense synonymous and cannot be used interchangeably.

Value in the sense of worth estimated by any standard of purchasing power, is, in the case of such properties as public utilities, a result of the earning capacity. In the case of certain properties such as highly improved residence property a determination of value from earning capacity may not be immediately apparent but the rental value is nevertheless generally there and can be determined. In the case of such properties as works of art, however, which are not ordinarily revenue producing, but which are desired for the pleasure which they give and for educational purposes, the value in exchange cannot thus be measured, and consideration must be given, to the more abstract question of supply and demand. The discussion in these pages will be restricted to the valuation of properties which have an earning capacity.

Market Value. — In the sense in which used in this volume "value" is synonymous with market value.

Fair Value. — The term "fair value" is used so frequently in the valuations of public service properties for rate-fixing purposes, that it deserves more than passing notice. As a basis for the fixing of rates it is difficult to define. Apparently it is usually intended to mean market value, — the value which would be ascertained by a prudent purchaser making thorough inquiry relating to all circumstances affecting value. But in reality the value is a result and not a basis of earnings. It should not be made the "rate-base." The term "fair value" has been used in a measure, no doubt, to rule out "book value" or cost which may include items and amounts of doubtful propriety and to rule out estimates of value based on stock and bond issues and the market value of such securities.

The term fair value as used by the courts has not yet been satisfactorily interpreted and no attempt will here be made to reconcile divergent views in relation thereto. But attention may be called to the difficulty which has been experienced by all who have attempted to make appraisals for rate-fixing purposes, in reconciling the value to a purchaser with the "fair value" which the courts wish to have considered when fixing rates. Why should there be one valuation for purchase and another for rate-fixing? The answer that has been given by the courts is practically to the effect that there should be no such difference, and experts have found difficulty in so appraising values that any such difference shall disappear.

The value to an investor is unhesitatingly determined from the net earnings, with due regard to hazards of the business. The value for rate-fixing purposes as the courts say is to be that value on which, with the same regard for the hazards of the business, the owner is to be allowed to earn a fair interest return. Value should be the same whether determined by a rate-fixing body or whether determined for a purchaser.

When the acquisition of the properties of the Maine Water Company by the Kennebec Water District was under consideration, the Supreme Judicial Court of Maine included in its instructions to the appraisers (Dec. 27, 1902) the following:

"The capitalization of income, even at reasonable rates, cannot be adopted as a sufficient or satisfactory test of present value. But while not a test, present and probable future earnings at reasonable rates are properly to be considered in determining the present value of the system."

In this case, "Kennebec Water District vs. City of Waterville et al" (97 Maine 185; 54 Atlantic 6), the district was authorized by law to acquire the entire plant property and franchise, rights and privileges of the Maine Water Co. and the instructions referred to were issued by the court upon a joint request.

According to the decisions of the courts, as the matter stands today, that which is to be ascertained and made the starting point when rates are to be fixed, is the present value of the property devoted to the public use.

In this connection the Supreme Court of the United States says in the Minnesota Rate Cases (230 U.S. 352): "The basis of calculation is the fair value of the property used for the convenience of the public." In San Diego Land and Town Co. vs. National City the Court says, "What the company is entitled to demand, in order that it may have just compensation, is a fair return upon the reasonable value of the property at the time it is being used for the public." In the Minnesota Rate Cases the Court also quotes with its approval from Smyth vs. Ames (169 U.S. 466).

"In order to ascertain that value, the original cost of construction, the amount expended in permanent improvements, the amount and market value of its bonds and stock, the present as compared with the original cost of construction, the probable earning capacity of the property under particular rates prescribed by statutes, and the sum required to meet operating expenses, are all matters for consideration and are to be given such weight as may be just and right in each case. We do not say that there may not be other matters to be regarded in estimating the value of the property. What the company is entitled to ask is a fair return upon the value of that which it employs for the public convenience. On the other hand, what the public is entitled to demand is, that no more be

extracted from it for the use of a public highway than the services rendered by it are reasonably worth."

It is to be noted, too, that the U.S. Supreme Court holds to the view that the value as a basis for establishing rates is to be the present value. The Court uses the following language in Willcox vs. Consolidated Gas Co. of N. Y. (212 U.S. 19, 52).

"And we concur with the court below in holding that the value of a property is to be determined as of the time when the inquiry is made regarding the rates. If the property, which legally enters into the consideration of the question of rates, has increased in value since it was acquired, the company is entitled to the benefit of such increase. This is at any rate the general rule. We do not say there may not possibly be an exception to it, where the property may have increased so enormously in value as to render a rate permitting a reasonable return upon such increased value unjust to the public. How such facts should be treated is not a question now before us, as this case does not present it. We refer to the matter only for the purpose of stating that the decision herein does not prevent an inquiry into the question when, if ever, it should be necessarily presented."

The Court has apparently taken the view that the unearned increment may generally be allowed to go to the owner of the utility either as an offset in whole or in part to early losses or as a reward for having undertaken the enterprise. The fact should, however, not be lost sight of that any such increase in value is, strictly speaking, a part of the aggregate earnings and due consideration to this fact should be given in making appraisals.

While these rulings of the highest tribunal in the country seem to compel the valuing of a public utility property according to accepted standards at present worth, when value is to be ascertained and made the starting point for fixing rates, we may nevertheless be permitted, and we consider it a duty, to point out the limitations of these rulings and the desirability of securing some modification thereof. This matter will be further discussed in the chapter on depreciation.

Remaining Value. — The term "remaining value" is equivalent in its meaning to present value when applied to an article whose value lessens with age. It is generally dependent upon and to be computed from three elements; (a) the probable useful life term of the article new; (b) its expectancy or probable remaining term of usefulness; and (c) the cost of replacing it with some article of equivalent usefulness, which, when prices are not subject to change, is the original cost less the residual value of the article when it goes out of use.

Residual Value. — The "residual value" is the value which remains in any article after the same has ceased to be useful as an integral part of a public service or other property. The residual value may be only scrap value, or it may be the price at which it can be disposed of for use in connection with some other property. It is usually estimated as the value to an outside purchaser, less the cost of delivery to such purchaser.

Condition Per Cent. — The existing condition of any article can be expressed by comparison with the condition of the same article new. If the article when new is taken at 100 per cent and present condition in comparison therewith is noted in per cent this is called the condition per cent. It is the present or remaining value in comparison with value new, based upon the condition of the article, expressed in percentage.

Accrued Depreciation. — The accrued depreciation is the difference expressed in money between the original cost of an article and its remaining value.

Probable Life.—The probable life term of an article is the time, usually expressed in years, during which it may reasonably be expected to render useful service. The probable life term of any article is to be determined by the experience, the world over, with other articles of the same kind. It depends not alone upon the time required for an article to become valueless by use, by ordinary wear and tear, but also upon the time when by reason of accidental destruction, inadequacy or obsolescence, the article must be replaced by a new one better adapted to fulfill its purpose. If the life of a large number of articles of

the same kind, as for example telegraph poles, be recorded it will be found that a few fail almost immediately, being destroyed by storms, by fire or by accidents, that a larger number will fail within a few years due to inherent defects in the timber and other causes in addition to the causes of failure already named, that failures will multiply at some period such as 10 to 15 years according to the character of the timber of which the poles are constructed and that some will survive many years beyond the average life term of all. From such records the probable life term of a new pole is ascertained; but while this probable life, here synonymous with average life, is known or can be ascertained as explained, the actual life of individual poles will depart therefrom more or less. The total number of service years of the poles which fail early will fall short of the expected total number by as many service years as those of the poles which survive the probable term will exceed this number.

Expectancy or Remaining Life. — The expectancy of any article is the probable time, usually expressed in years, during which it may reasonably be expected to render efficient service. For a new article the expectancy is its probable life. When the life of a candle which will burn for a number of hours is under consideration, the expectancy is ascertained by subtracting age from probable life new. But in all ordinary cases the expectancy cannot be ascertained by any such simple mathematical process. Every article which has been in service for some years, and has escaped the accidents which might have put it out of business in its early life, stands a better chance of being among those which will outlive the probable life term fixed for it when it was new, than it had when new to outlive this term. Consequently the expectancy is not to be determined by subtracting age from probable life. It is to be determined from the actual condition of the article and all local circumstances which may affect its continued usefulness.

A high-duty pump, which originally had a probable life of 25 years, when it reaches the end of this term, may be in a condition almost as good as new. It has escaped the possible accidents of the early years of its life and by careful attention and replacements of its wearing parts is still rendering first-class service. The value of this pump is not to be written off the books, neither should it be regarded as good as new. Its value is ascertained by determining its probable additional years of usefulness and the probable cost of replacing it at the end of this term.

An irrigation canal usually improves with age. So far as wear and tear is concerned, it has unlimited life. But under the development of extensive areas, the small original canal may in the course of time be superseded. The probable life of this canal, and therefore, too, the annual replacement increment, is estimated on some assumption relating to the rate of this development. Finally the time comes when the project for a comprehensive canal system has taken shape and it may reasonably be assumed that within a definite period, five or ten, or some other number of years, the original canal will be superseded; its diverting dam and its headworks, and perhaps the canal itself, will then be abandoned. The remaining life or expectancy of the canal is at that time only five or ten or some other number of years, as the case may be, and within this time the remaining investment in the canal is the amount under consideration for replacement.

Composite Life. — The composite life of a complex plant is that term of years within which the accruing depreciation of all items of which the plant is composed, on the assumption of no replacements, would amount to the cost of the plant. As in the case of articles in classes according to their probable life, so in the case of entire plants, the life thus determined for various types of plants is made an aid in estimating the current depreciation. Without recourse to the more laborious method of dealing separately with each item of which the plant is made up, it becomes possible if composite life is known to approximate the current depreciation or replacement requirement.

Composite Age. — The composite age of a complex plant is that age which it would have to acquire, if treated as a depreciating unit, to make its accrued depreciation equal to the

aggregate accrued depreciation of the individual items of which the plant is made up.

"Amortization." — In the sense used in this paper the term amortization applies to the retirement of capital. When an article is to be permanently retired from connection with a business and not to be replaced, the annual depreciation increment or the annual replacement increment allowed in the earnings becomes an amortization increment.

Ordinarily a discarded article is replaced by a new one which will render equivalent or better service. The amount to be provided during the life of the article and made available at the time it goes out of use is, for all practical purposes, its replacement cost, being the cost of installing an equivalent new article, less the residual value of the original article. The amount thus provided may be treated as amortization, but there is no need of doing so as will be fully explained.

Depreciation. — Depreciation is the lessening in worth of any perishable article which takes place from use and advancing age. It is not solely due to inherent deterioration or to wear and tear but results, too, from the fact that owing to progress in the arts and sciences and methods of manufacture, an article may become obsolete and that due to growth of communities or other causes, appliances or structures may become inadequate and have to be replaced in the course of time by new ones better adapted to the requirements. Every circumstance tending to limit the term of usefulness of an article should be taken into account when its probable life or expectation and consequent rate of depreciation are to be determined.

Annual Depreciation. — The annual depreciation or the current depreciation is the annual theoretical lessening in worth, expressed in money.

Appreciation. — Appreciation is the increase in worth expressed in terms of money. It applies not alone to real estate but to any article, structure or thing which increases in value due to general prosperity or to the more obvious cause of higher wages and greater cost of materials.

The Current or Annual Replacement Requirement. — The current or annual replacement requirement is that amount which should annually be covered by the earnings, to meet the renewals which must be made from time to time. The amount necessary to accomplish this may be estimated from the known character, number and cost of the articles which must annually be replaced or in the case of large units, from the rate of depreciation.

The Obligation to Replace. — The obligation which every owner of a public utility has assumed to continue in business for either a definite or an indefinite time period carries with it an obligation to replace the articles which are essential for the proper conduct of the business with equivalent new articles whenever the original articles cease to be useful. This obligation to replace grows with the age of the article in service. When expressed in money, it is equivalent to the accrued theoretical depreciation. This obligation to replace, together with deferred maintenance, must be taken into account by the appraiser of property which is to be purchased.

Deferred Maintenance.—"Deferred Maintenance" is the neglect, expressed in dollars, which has resulted from failure to keep an article in good condition and repair. It is the sum which should at once be expended to restore the article to ordinary good service condition, and to protect it against causes of rapid destruction so that its deterioration will not be unduly rapid. Ordinarily there should be no "deferred maintenance." By proper attention to maintenance and repairs the service rendered should be kept at the standard which is expected of the plant. The plant in other words should, practically at all times, be at 100 per cent efficiency.

Wear and Tear.—"Wear and tear" is the term applied to the deterioration of an article from use. The article is kept in serviceable condition by the renewal of its worn-out parts and by suitable attention to its wearing parts. Maintenance and repair are to be included in operating expenses. The expenditures for repairs and maintenance are intended to keep the article at or near 100 per cent efficiency.

It is sometimes difficult to draw a close distinction between repairs and replacements and between deferred maintenance and accrued depreciation. It is generally assumed that deferred maintenance results from neglect to make the lesser repairs and renewals such as are current from year to year and that depreciation comes into consideration when dealing with the more important items which when new have a probable life of at least a number of years.

Replacement Cost. — The replacement cost is the cost of a new article with which a worn-out article is replaced, less the residual value of the original article. It is the cost of effecting a change from the worn-out part of the property to a new part of equivalent service value.

Invested Capital or Investment.— The investment or the capital invested hardly needs a definition. It is the aggregate of the expenditures which have been made and which remain in the business. It is the summation of the cost of the various items which make up the property in question and render the same efficient for the purpose for which it is intended, less such sums as may have been applied out of earnings for the retirement of the capital. In connection with the valuation of public utility properties it may be regarded as the aggregate of the reasonable and proper expenditures which have been incurred to make these properties serviceable. It may and generally does include such items as the cost of establishing the business.

Wearing or Service Value. — When the fact is taken into account that an article after it has ceased to be useful in connection with one enterprise or property may still have value in connection with another, and usually does have some scrap value, it will be plain that the lessening of worth is not to be estimated from the original full cost of the article. It is the difference between the first cost and the residual value which is being consumed during the useful life of the article. This difference is frequently referred to as the wearing or service value of the article.

Remaining Service Value. — The service value of an article which is no longer new is its remaining service value. It is determined from the original service value by deducting the accrued depreciation.

Intangible Values. — The term intangible value is applied to any element of value other than elements of a physical nature. The value of a franchise is an intangible value, so too the "good-will" of a business; "going value;" "rights of access;" "water-rights," "power-rights," and the like. The cost of organizing and establishing a business is an element for consideration in making an appraisal of value just as legitimate as the cost of the physical elements of a property.

Cost of Business Development or the Cost of Establishing the Business. — The cost of development or the cost of establishing the business is a summation of the expenses not chargeable to construction which have been incurred in building up the business and bringing it upon a paying basis. Preliminary expenses, advertising, commissions and losses due to inadequate revenue in the early years may all be brought under review in making up the cost of development. It is not always a simple matter to know where to stop in the matter of charging up expenses to the cost of development and to the capital account. To a certain extent the losses due to deficient business in early years may be regarded as part of the capital reasonably invested. The European practice is to do this to a far greater extent than is customary in the United States. When feasible, the better practice would be to recognize the early losses as a necessary incident to the business and to make provision for their amortization as circumstances may justify. In general it may be said that any legitimate expense which it may be difficult to assign to construction or operating accounts can be regarded as development expense.

Franchise. — A franchise is a privilege to do business granted by the public. A franchise may be limited to a specified period of time, it may be indeterminate or it may be perpetual. It may be, but is not necessarily, an exclusive privilege. It may

be coupled with all sorts of conditions such as division of profits; the fulfilment of all kinds of obligations; the limitation of charges for the service rendered or the commodity furnished. It may be nothing more than a privilege to occupy the public highways with rails for transportation purposes or with pole or pipe lines for the transmission or delivery of electricity, water, oil or gas. It may be a grant without cost or may have to be paid for at a fixed price or subject to competitive bidding. It is one of the intangible elements which may represent value and frequently appears among the items of cost.

Going Concern. — The term "going concern" is applied to such business enterprises as are in actual operation. If in successful operation, the value of the "going concern" should be somewhat greater than the sum of the value of the physical elements of which the property is made up.

Going Value. — The increment of value which is due to the fact that a business has been established and brought upon a paying basis is its "going value." In some degree the cost of developing the business, such as the early losses, can be made the measure of going value. This can be only to the extent that the going concern has an advantage over a similar enterprise subject to ordinary reasonable business development expenditures. If the early losses have been treated in the accounts as operating loss and not as invested capital and if subsequent years of operation have wiped out the early losses, it may be found difficult to justify the inclusion of any cost of developing the business among the intangible elements of value. It may in other words be difficult to determine how much of the aggregate of the intangible value should be classed as "going value."

Good-Will. — The term "good-will" is closely related to "going value." It represents the intangible element of value in an ordinary business and is applied generally in those cases where there is competition with other like business ventures. The courts hold that where there is no competition there can be no good-will.

Overhead Expenses. — Certain expenditures in connection with the development and operation of any revenue-producing property are usually classed as overhead expenses. Economists and engineers are not all agreed as to just what part of the cost of construction or operation should be classed as overhead. Ordinarily the cost of management is treated as the overhead. When work is done by contract, the amount paid to the contractor is the cost. An analysis of cost to the contractor when compared with what he has received may show a profit but this is no part of the overhead expense. The compensation of the general manager, on the other hand, and of the engineer and legal staff and the expense of the general office should be treated as overhead - so, too, interest during construction, taxes and insurance. Whether or not they are taken into account in making valuations for the purpose of establishing a "rate-base" (it being no concern of the public how the funds for establishing an enterprise are raised), the commissions to brokers who sell bonds are a real item of expense to the owner and will be classed by him either among the overhead expense or as a promotion expense.

Promotion Expense. — The term "promotion expense" is used to designate certain expenditures, which cannot well be included in the "overhead" nor yet in the legitimate cost of establishing the business. To this class of expenditures belong a variety of expenditures usually incurred prior to the permanent organization of the business enterprise and may include commissions to real-estate agents, and the various expenditures that have been incurred apart from actual cost in acquiring the properties forming the nucleus of the enterprise. Advertising and discounts and commissions to bond brokers may also be at times properly classed as promotion expense. The cost of bringing together the various fundamental properties into one holding, representing the result of the promoter's activities, except to the extent that the cost of promotion work appears in enhanced value, is generally accepted as being covered by the item "promotion expense."

The amount of the promotion expense may vary within wide limits according to the character of the works, the difficulties to be overcome and the thoroughness with which preliminary investigations are made. Such expenditures are generally much less in connection with the additions to an established enterprise than they would be in connection with an original plant of the same character and magnitude.

Reproduction Cost New. — The ascertainment of what it would cost to construct an exactly equivalent property, identical with that to be valued, is frequently an acceptable aid in determining value. Any article which forms a part of a revenue-producing property has a value in the service which may reasonably be measured by the cost of replacing Its value in the service may be estimated from the cost of installing a new article at the end of the expectancy term. Generally the value at some particular time is under consideration and it would be theoretically correct to apply the prices of material and labor which are current at that time in making the estimate of reproduction cost. But construction was a process requiring time and reconstruction would also require time. Furthermore the appraisal may have to serve for some time, perhaps for a term of years. It is reasonable and logical, therefore, to depart from the strictly theoretical requirement and to adopt, in making such an estimate, unit prices which represent average conditions, preferably for a period of about 5 years. The instructions to appraisers by the Supreme Judicial Court of Maine, in the Kennebec Water District Case (1902), (97 Maine 185; 54 Atlantic 6), when the properties of the Maine Water Co. were to be valued contained the following reference to reproduction cost:

"The appraisers may properly consider what the existing system can be reproduced for. But the cost of reproduction will not be conclusive. It will be evidence having some tendency to prove present value. The inquiry along the line of reproduction should be limited to the replacing of the present system by one substantially like it."

Public Utility. — A public utility is an enterprise which renders a service or which supplies a commodity of general necessity or convenience to the public. There may be public ownership of the utility, or the privilege of serving the public may be delegated to a private person. When the private person is a corporation, we have the public service corporation.

Rate-base. — The "rate-base" is the starting point, the basis of the calculation when earnings are to be determined to which the owner of a public utility is entitled. The owner is entitled to a fair return on the legitimate investment which he has made for the benefit of the public. He may also be entitled to more than what would be considered a reasonable return on ordinary investments but this fact is to be determined by the circumstances in each particular case. To know the legitimate investment, whether ascertained from book records or by estimating the cost of reproduction or otherwise, is fundamental.

The rate-base which is the most satisfactory is the actual reasonable and proper investment or original cost new, including a proper allowance for the cost of establishing and developing the business, undiminished by depreciation, though in some cases, perhaps, diminished by the bonus which may have been contributed by the public or by the rate-payer.

The endeavor is made in these pages to show that the value of the service rendered and the earnings are independent of accrued depreciation and that for this reason the present or depreciated value of the physical elements is not the best criterion for a determination of the legitimate amount of the earnings. But the sacrifice which the owner has made, the amount of money which is in the business legitimately as of the date of construction, the original cost, in other words, less the capital which has been contributed to the work as a bonus and not as amortization, is the essential element for consideration and should be made the starting point when rates are to be regulated. It is to be understood, as a matter of course, that special consideration may have to be given to the

overbuilt plant and to property held for future use. If the owner has gone too far in the matter of providing capacity or in the matter of securing property not immediately required, the cost of such property may have to be omitted from the rate-base or the rate of return on the rate-base should be lower than would otherwise be allowed.

CHAPTER III

FUNDAMENTAL PRINCIPLES WHICH CONTROL WHEN APPRAISALS OF PUBLIC SERVICE PROPERTIES ARE TO SERVE AS A BASIS FOR FIXING RATES

- 1. The earnings of the public service property should be such that, within the life of the property, there will be returned to the owner, the capital which he has properly invested in it, and in addition thereto, interest at a reasonable rate upon such amount of capital as from time to time actually and properly remains as an investment in the property.
- 2. The reasonable cost of each item which goes to make up a property may be returned to the owner during the probable or during the actual life of that item or it may be returned to him in a lump sum when that item ceases to be useful. In the case of items which are of sufficient importance to be individualized the amortization of cost may be progressive during a fixed term which is generally determined by the probable life of the item. In the case of numerous articles the amortization, with equal propriety, may be deferred to the end of the term of usefulness of each article.
- 3. During the early years of most public utilities the operating costs will exceed the revenue. The business is conducted at a loss. When such losses are legitimate, they may be regarded as the cost of establishing the business and may be added to the invested capital. They may, as an alternative, be regarded as temporary advances which are then to be amortized within a reasonable time.
- 4. The owner of the public service property is entitled to proper compensation for assuming the hazards of the business and for establishing and operating the utility.

- 5. The owner of a public utility is entitled to a reasonable share in the general prosperity of the community which he serves. For this reason he should be allowed a share in the increase of value which results from population growth. If present value is made the rate-base, he will get the appreciation of real estate (if such increase be not excessive). If the public utility has but little or no property which appreciates then some other factor may be brought into consideration in determining the fair return. It would be sound doctrine to exclude appreciation from the rate-base thus avoiding uncertain and accidental reward and to allow the unearned increment to appear in the net earnings.
- 6. Where the invested capital is small and the volume of business large, the owner of the utility is entitled to have the volume of business considered as well as the invested capital when rates are to be fixed. It would generally be good practice to take the volume of business into account as well as the rate-base when rates are to be regulated, but this is not yet an established practice.
- 7. The depreciation of an item used in the public service does not reduce the value of the service or commodity which the public utility supplies to the consumer. The value of the service is ordinarily unaffected by depreciation. Depreciation is a lessening of worth which may result from any cause. Wear and tear suggest themselves as the prime cause of the depreciation of articles in use, but it is sometimes difficult to establish a connection between the rate at which an article is consumed in service by wear and tear and its lessening worth. The accrued depreciation of any article is ascertainable from the probable life new of the article, the cost of replacing it at the end of its life, its probable remaining term of usefulness, and its original cost. In practice the accrued depreciation is usually computed from the cost of the article or rather from its wearing value, its probable life new and its expectancy. The cost affords a convenient basis for approximation. (This does not apply to certain articles which are readily replaced such as automobiles when

considered apart from a complex property. Treated by itself the automobile shows a marked drop in value as soon as it goes into service and thereafter may be considered as decreasing at a fairly uniform rate per year.)

- 8. Whenever the rates to be charged for the service rendered by a public utility are to be fixed the current depreciation or the current replacement requirement must be estimated. This is usually done by compound interest annuity methods or by the so-called Straight Line Method. According to the procedure under which the necessary amount of earnings are determined, either the current depreciation or the current replacement requirement are then made a part of the necessary earnings.
- 9. According to the procedure which is adopted for treating the replacement requirement, depreciation and amortization in the accounting system, the computation of earnings that are necessary from year to year will vary. Any one of a number of methods of procedure may be adopted with due regard to the following:
- a. Is it desirable to keep the required earnings in the early years relatively low?
- b. Is any portion of the invested capital to be amortized or is there to be any amortization of expenditures for unproductive work?
 - c. Has there been any amortization of capital in the past?
 - d. What has been the relation of earnings to operating expenses?
- e. What amount of prospective business is to be taken into account?
- ro. An earned depreciation allowance may be regarded as an amortization increment. The depreciation allowance is then for all practical purposes a repayment to the owner of a part of his invested capital. It will, in that event, make no difference in the ultimate result, at what rate the repayment of capital is made. Ordinarily capital is retired as follows:
- a. In equal annual amounts which without interest will in the life of the article whose cost is to be amortized amount to that cost. (The Straight Line Method.)

- b. In amounts which increase from year to year, figured on the compound interest basis. (Equal Annual Payment Method.)
- c. In equal annual amounts which together with compound interest thereon will in the life of an article amount to its cost. (Sinking Fund Method.)
- 11. A public service property may be regarded as having an unlimited life. Taken as a whole when rates are to be fixed the accrued depreciation may then be disregarded; but provision must be made for the renewal of those parts which for any reason become useless. In this event there need be no repayment of capital, except only those portions thereof which may be regarded as temporarily invested.
- 12. In so far as this may be practicable, the earnings of the public utilities should be adjusted to the ability of the rate-payer to pay. Ordinarily, therefore, the aggregate earnings should be kept relatively low in the early years when the number of rate-payers is small
- 13. The owner of a public utility has the right to do what he pleases with any portion of his capital which is paid back to him, which is, in other words, eliminated from the rate-base.
- 14. The owner of a public utility should be held accountable for all sums collected from the rate-payers for the specific purpose of making repairs and renewals. A diversion to other uses of any fund intended for this purpose is equivalent to a repayment of capital.
- 15. The basis of calculation, the starting point when rates are to be fixed for a public utility, is the amount of capital whose investment was necessary to build the plant and develop its business and on which there should be a proper interest return to the owner. This may be either the legitimate original cost new, or it may be the properly invested capital reduced by the accomplished amortization. This basic amount is for convenience called the "rate-base."
- 16. As a general proposition the earnings present and prospective should be such that they will give the property a value in excess of the capital actually (and properly) invested the

excess of value being the amount of the reward (including perhaps some unearned increment) to which the owner is entitled for having established and for managing the business.

- 17. Strategic value, resulting from a combination of circumstances such as low operating cost and ample market for the output at prices fixed by or for competing concerns which have not the same advantage of low operating cost, is a part of the reward to which the owner of a public utility may be legitimately entitled.
- 18. The various procedures for estimating the required earnings of public service properties are usually based on the assumption that the same procedure has been used from the beginning of operations. A procedure which may be correct if consistently and continuously applied, may be unfair to either the owner or the rate-payer, if introduced at a later period. Consequently it should be regarded as imperative that consideration be given to past history when the rates of an operating concern are to be fixed.
- 19. Franchises and water-rights and in general the privilege to do business are not to be made a part of the rate-base except in certain cases in which such rights and privileges have definitely ascertainable strategic value or have had to be purchased as, for example, when a city sells the right to construct and maintain a street car system or when the acquisition of a water-right involves the purchase of adverse riparian or other rights.
- 20. The net earnings of a public service property should in some measure exceed the return from ordinary safe investments.
- 21. The prospective business must be taken into account when the rates are to be fixed particularly in the case of a new enterprise, where the rate-payers are too few at the outset to produce, at a reasonable charge for the service, the revenue which would yield a proper current return on the investment.
- 22. The replacement requirement is to be determined from the expectancy (remaining years of usefulness), the probable life when new of similar articles and the estimated cost of effecting the replacement. It is not dependent on original cost nor age,

although original cost and age may frequently be valuable aids in making an estimate thereof.

23. Whenever the current replacement requirement or depreciation are to be estimated, due consideration must be given to the fact that the actual period of usefulness of perishable articles rarely coincides with their probable term of life.

CHAPTER IV

ESSENTIALS OF VALUE

Cost of Physical Elements

Supply and Demand. — The value of any property is usually determined by the law of supply and demand. This is perhaps best illustrated by reference to perishable property such as articles of food which can be preserved in marketable condition for a limited time only. According to information that may be available relating to the world's supply of wheat and of coffee the value of the wheat or coffee will be high or low. The harvest prospect in the case of such perishable elements has a direct effect upon the market value. Or as an additional illustration take the limited output of a great sculptor or artist. The demand, particularly after the sculptor or artist has ceased to produce, may greatly exceed the supply and in such case the market value may rise far in excess of what would otherwise be the value.

The popular actor, or vocalist, the skillful musician, the poet and the author have something to offer in the world's market which is unique and may be in great demand, and therefore may command a return out of all proportion to the compensation for ordinary individual effort.

Cost of Physical Elements in Relation to Value. — Productions of this type which have value are not here under consideration. It is the revenue producing property which is to be discussed and to which the chapters of this volume are addressed. Nearly every such property is made up of both physical and intangible elements. Taken in the aggregate this combination of physical and intangible elements has "value" based upon earning capacity. This is a fundamental fact which if lost sight of may

lead into confusion when the term "value" is used. Both the physical and the intangible elements which make up the entire property may have been acquired by the outlay of money or in exchange for other valuable property. Consequently the question of cost deserves consideration and may at times be found a fair guide in determining value. Usually the physical elements of any property in successful use are worth at least as much as it would cost to reproduce them in the condition in which they are found at the time of the valuation. At any rate a prudent purchaser would desire to know what this cost would be and he would be very apt to make it his starting point in determining what he could afford to pay for the property.

The first step therefore in practically every valuation is the determination of the cost of the physical elements which go to make up the property.

In making the valuation of the physical properties account must be taken:

- 1. Of the cost of replacing each inventoried item with a new equivalent article.
 - 2. Of the probable remaining term of usefulness of the item.
- 3. Of the probable term of usefulness of the same or an equivalent new article.
- 4. Of the cost of replacing the article with an equivalent article at the time when the article will probably be discarded.
- 5. In the case of valuation for purchase or sale account should also be taken of the accrued depreciation, which will aid in determining the present value.

The Determination of Cost. — The appraiser will therefore begin with an inventory of the property which should be in sufficient detail to facilitate a cost estimate such as an engineer would make who is called upon to construct a duplicate of the property to be valued. He will call to aid in making this inventory the records of the owner and he may find that he need only verify or amplify a list of items already enumerated. He will then estimate the cost of reproduction and here again he should call to aid, if it is available, the record of the cost of con-

struction. The actual cost, intelligent direction and supervision being assumed, is generally a better guide to the cost of accomplishing a result, than the estimate of the expert who, no matter how much he knows about present cost, must make certain assumptions relating to the past or the future which will always brand his conclusions as approximations, even though the approximations are in some cases to be accepted as being quite as dependable and as acceptable as though they were of absolute accuracy.

It is not proposed to introduce into this volume sample sheets enumerating all the various items that may be encountered in valuing the various classes of public utilities and industrial establishments. The blanks in use for these purposes are obtainable, at least for the utilities, from any of the public service commissions and will be found an excellent guide in listing and appraising the physical elements of value.

The Relation of Present Worth to the Cost of Replacement. — After the cost of reproduction has been determined the time must be estimated during which — everything considered — the article to be valued may reasonably be assumed to continue in service. With this time and the probable life new of articles of the same class it will, as elsewhere explained, be possible to estimate the relation which the present worth of the article bears to the cost of replacing it at the time it will probably be discarded. If now the cost of reproduction be so modified that it will represent the cost of replacing the article at the end of its period of usefulness and the present worth or condition factor is applied to this cost of replacement the result will be the present worth.

When the property to be valued includes land, rights-of-way or water-rights, regard should be had to the principles discussed in the chapters on the "Valuation of Land in Eminent Domain Proceedings" and the "Value of a Water-Right and of Reservoir Lands."

Extent of Detail Required. — It has become a common practice in valuing property for rate-fixing purposes to approximate

the legitimate investment or the present value of physical properties by means of a close estimate of the cost of reproduction. Such estimates are a check upon the records of actual cost and are valuable even when used for no other purpose than as a check.

The fashion is growing of making the estimates of the cost of reproduction with attention to the minutest detail. These estimates are frequently made with more care and with more careful enumeration of items, of specials, fixtures and incidentals and of labor and overhead, than would satisfy a contractor bidding on the work. And when this is done, when the sleeves and elbows of the pipe lines have all been counted and weighed and fence posts have been pulled to determine their length, unit prices are applied — sometimes as they prevail at the date of the valuation but more frequently as they have prevailed throughout a series of years 2, 3, 5 or 10, in order to give weight in a measure to the conditions under which the plant came into being. An approximation results. Then, perhaps, the accrued depreciation and deferred maintenance are estimated by some method of approximation and are deducted and finally appreciation of value may be allowed and an addition is made for the "cost of establishing the business" for "going value" or for other intangible elements such as "franchise" or "water-rights." To the original approximation other approximations are added with a result not always entirely satisfactory.

Where this is the case, one may well pause to consider how far into detail the appraisal of the physical elements should go, and whether there are not many cases where actual cost, with such checks as may be necessary to determine whether the cost is strictly reasonable and legitimate, would serve as the best starting point when a rate-base is to be determined.

The expense of making an appraisal with strict attention to every possible detail is not always justified and it may, moreover, defeat its own purpose, because by the time that a first valuation is completed, as in the case of the railroad systems of the country, a revaluation may be necessary if the same standard of approximation to reconstruction cost is to be continuously maintained. At any rate, the public utility commissions of the country should see to it that useless work on these lines be not demanded. It is their province to fix the standard in this matter.

When rates are to be fixed, they are based on a forecast of what the earnings should be. The exact result of charging for the service rendered at any fixed rates cannot be known until the rates have been in effect for a time. A deficiency in the earnings is discouraging to the owner and makes him prone to give inferior service. He may be forced into the courts and will then incur legal and other expenses which in the end will be paid by the consumer. An accumulating surplus, on the other hand, should insure high-class-service and can be used to safeguard the property against unforeseen accident and to make sure of its proper upkeep without giving to the owner, in the long run, any larger amount in dividends than he is legitimately entitled to receive. If there be error, it should be on the side of the adequate return, provided always that a limit is set to the rate of return on the investment and to the compensation for managing the business, and that the charge for the service rendered be a reasonable charge.

Overhead Expenses

Cost Includes Overhead Expenses. — Overhead expenses have already been defined. They include primarily the cost of management, engineering, interest during construction, taxes and insurance.

In the determination of "cost" whether the actual cost or the estimated cost of reproduction, whether at present day prices or at prices applying to a selected period or to a particular time, it will be essential to have due regard to the overhead expense. It is not enough to be able to state item for item what it would cost to reproduce each in its exact present condition of serviceability, but proper account should be taken of the amount of general expense, of the cost of management, engineering and supervision of the expenditures due to unforeseen conditions, of unprofitable expenditures, and of losses by accident, that would ordinarily be anticipated in the execution of similar work under the same general conditions. The finished plant if constructed under better than average conditions is entitled to a credit due to this favorable circumstance, and if constructed under adverse conditions, may not be entitled to an appraisal at full actual cost.

The determination of the allowance which should be made for overhead expense is therefore a matter of considerable importance as will be seen from the quotations presented here at some length, which will give a fair idea, based upon experience, of the actual overhead expense connected with many public enterprises.

The Special Committee of the Am. Soc. C. E. on the Valuation of Public Utilities is Cited on Overhead. — In reference to engineering as an overhead expense the Special Committee of the American Society of Civil Engineers on the Valuation of Public Utilities in the progress report which it submitted in January, 1914, says:

Am. Soc. C. E. Committee, Engineering.—"Under the head Engineering are usually included not only strictly engineering expenses, but those of other technical employes and of inspectors. It is difficult to draw the line between the preliminary engineering and that during construction, and it is probably best in most cases to include the part of the preliminary engineering expenses connected with the preparation of the final design of the works with those incurred during their construction.

"The percentage of the cost of the work represented by engineering differs with the character of the works and with the amount of care and skill exercised in their design and construction. On railroads it is commonly estimated that the engineering cost will amount to 5 per cent of the physical valuation of the property, exclusive of overhead charges. Statistics of the cost of engineering are available in connection with several municipal and other works as follows:

"Metropolitan Water-Works, Massachusetts. This property, costing to the end of 1912, \$42,036,000, is to the extent of \$15,300,000 made up of old works purchased from the City of Boston and others, on which the engineering charge is not known, leaving \$26,736,000, of which the engineering charge was \$2,077,000, equal to 777 per cent. Based on the total cost, exclusive of engineering, the percentage is 8.42. These amounts include both the preliminary engineering and that during construction.

"New York Water-Works, now in process of construction. The disbursements upon this work to the end of September, 1913, amounted to \$103,178,000. The engineering expense directly attributable to the work under construction amounting to \$10,050,000, equal to 9.78 per cent of the total disbursements for this work. In addition, there were engineering expenses relating to investigations of other drainage areas, which are only indirectly attributable to the work under construction, amounting to \$304,000 or 0 38 per cent of the total disbursements, making the total for engineering 10 16 per cent It is to be noted, however, that there is included in this case under the head "engineering" the cost of unusually extensive borings and investigations which were not included as an engineering charge upon the Metropolitan Water-Works. There is not included, however, the cost of expensive investigations by sinking shafts at the Hudson River, which, although originally charged to the engineering account, has been deducted because the shafts afterward became a part of the final construction.

"Boston Subways. The following table has been compiled

from official reports:

"BOSTON TRANSIT COMMISSION

(1895-1912)

	Total cost	Percentage of total cost	Percentage of con- struction cost	
Subway Engineering General expense (inc commission) Construction Total	\$407,475 48 131,681 87 3,586,002 33 \$4,125,159 68	9 88 3 19 86 93 100.00	11 34 3 68 100 00	
East Boston Tunnel Engineering General expense Construction Total	\$191,466 57 161,134 78 2,894,595 01 \$3,247,196 36	5.90 4 96 89 14	6 62 5 57 100 00	
Boston Tunnel and Subway: Engineering General expense Construction Total	\$417,866 25 226,441 57 7,623,206 56 \$8,267,514 38	5 ° 5 2 73 92 22	5 48 3 58 100 00	
Cambridge Connection. Engineering General expense Construction Total	\$96,575 46 62,355 20 1,199,904 39 \$1,358,835 05	7 10 4 59 88 31 100.00	8 05 5 20 100 00	

"CHARLES RIVER BASIN COMMISSION, BOSTON

Dam, Lock, Embankments, Marginal Conduits, Etc., Completed 1909

	Total cost.	Percentage of total cost.	Percentage of con- struction cost.
Administration	\$94,011 97 446,096 03 	2 6 12 4 80 0 5 0	3.2 15.5 100.0 6.2

"Pennsylvania Railroad Tunnels. — The cost of engineering for the East River Division of the Pennsylvania Railroad tunnels amounted to 5.8 per cent of the total cost of the work, including excavation, retaining walls around the station area in New York, the tunnels eastward under the streets and East River to the surface in Long Island, terminal yard in Long Island, and engineering. This is equivalent to 6.1 per cent of the cost, exclusive of engineering.

"KENNEBEC WATER DISTRICT, MAINE New Gravity Water Supply, with Auxiliary Steam Pumping Plant, 1906

	Total cost.	Percentage of total cost	Percentage of con- struction cost		
Administration Engineering Construction	\$6,161 20 17,872 35	2 30 6.69	² 53 7 35		
Rights of way Pumping station Steam plant China Lake pipe line	926 97 20,833 39 6,275 59 215,302 93	0 36 7.80 2.34 80 51	100 00		
Total	\$267,408 43	100 00			

"LOUISVILLE, KY., SEWERAGE WORKS, 1906 TO 1912
This Work Consisted of Intercepting Sewers, Trunk Sewers, and a Small
Proportion of Lateral Sewers

	Total cost.	Percentage of total ex- penditures.	Percentage of pay- ments to contractors.
Administration Engineering Rights of way Castings and other metal work Damage suits (exc. of rights of way) Amounts of payments to contractors.	\$78,025 03 336,544 87 12,319 36 15,461,29 8,307 52 3,289,330 89 \$3,739,988 96	2 09 9 00 0 33 0 41 0 22 87 95	2 37 10 23 0 37 0 47 0 25 100.00

The above total is exclusive of the cost of preliminary engineering, which involved an expense of approximately \$57,000, equivalent to 1.7 per cent of the construction cost (\$3,289,330.89) shown above.

"Watertown — Mechanical filters built in 1903; cost, \$97,065;

engineering cost, 5 9 per cent.

"Ogdensburg. — Sand filters built in 1910; cost of work \$167,694, engineering charges, 7 per cent, excluding cost of preliminary report, which amounted to \$500.

"Hudson River State Hospital. - Sand filter plant built in

1904; cost \$36,000; total cost of engineering, $10\frac{1}{2}$ per cent.

"Peekskill. — Sand filters built in 1908; total cost, \$63,304;

cost of engineering, 7.1 per cent.

"Yonkers. — Open sand filters, built 1903; total cost, \$50,165; total cost of engineering, 7.3 per cent. Covered sand filters, built in 1907; total cost, \$106,708; cost of engineering, 8.7 per cent.

"Ithaca, N. Y. — Filters built in 1903 on a percentage basis under rush conditions; cost, \$192,114; engineering cost,

7 per cent.

"Springfield, Mass. — Water-works built 1910; construction of additional supply from Little River, including diversion works, reservoir, filters, and pipe lines; cost \$1,465,393; cost of engineering, 10 per cent. The basis of computation does not include the sum of \$268,000 paid for land, legal and other expenses.

"Springfield, Mass. — Ludlow filters; built in 1906 at a cost of \$43,306, to meet an emergency, and requiring very rigid inspection to secure proper grade of sand and proper sanitary conditions during construction; cost of engineering, including board, livery, cots, bedding, and provisions for inspectors on

work, 17 per cent.

"It is obvious that the cost of engineering varies with the character of the work. For instance, the construction of an important dam or aqueduct, built in place and requiring skill in designing and a careful inspection of every part of the work as it is built, requires a larger expenditure for engineering than a large cast-iron pipe line where the cost of laying the pipes in a trench is but a small percentage of the total cost of the line, and the work progresses so rapidly that the inspection cost is small in proportion to the total cost.

"The cost of engineering varies not only with the class of work but with the character of the design and execution. For instance, works may be built with very little inspection, from crude designs prepared by unskilled engineers, with the result that the cost of works may be large although the percentage paid for engineering may be small. Works skilfully designed and efficiently constructed necessarily involve a larger cost for engineering, which should be recognized in any valuation when

the works give evidence of such skill and efficiency.

"The cost of engineering for additions and extensions of a property may be as large, or even a larger percentage of the total cost than for the original plant, but under a continuous system of regulation engineering for additions and extensions, if charged to current expense, should not be included in the valuation

"It is sometimes suggested that no engineering charge should be made in the acquisition of real estate, and there may be some cases where such a view would be substantially correct. It is frequently the case, however, especially where strips of land are to be acquired, that the engineering cost is as great as for

other portions of the work.

"The record of the cost of the Wachusett Reservoir, where the engineers had little to do with the acquisition of land other than to prepare the plans of the lands and to make appraisals of the mill property and water-rights, amounted to about 3 per cent of the total cost of the property acquired. The percentage would have been considerably larger if nearly all of the property had not been acquired by purchase and very little by condemnation proceedings.

"The records of the New York Water-Works show that the engineering connected with the acquisition of land to the end of September, 1913, amounted to 1.87 per cent of the total expendi-

tures on land account."

Am. Soc. C. E. Committee, General Expense. — The same committee in referring to general expenses says that these include:

"All administrative, legal and other general expenses during the period of construction. Such expenses include general office rent, the salaries of the officers of the corporation and of the secretary, treasurer, legal advisers, clerks and others. Statistics showing the amount of such expenses in the case of corporations are not available to the Committee. The statistics of municipal public service properties throw some light upon the subject, although generally defective in that the offices are frequently in public buildings where no rent is charged and the financial and legal duties are performed to a large extent by those who receive salaries paid out of general funds.

"On the Massachusetts Metropolitan Water-Works, the administrative and expert services, with a small portion of the legal services, amounted to 1.43 per cent of the cost of the work, an unusually low figure, as there was very little litigation in connection with the work.

"On the New York Water-Works the strictly administrative expenses to the end of September, 1913, were 1.12 per cent of the total disbursements to that date, but in addition, advertising and the fees of special counsel and commissioners of appraisal to the end of 1912, all in connection with the acquisition of land, amounted to \$2,966,000, equivalent to 3.39 per cent of the total disbursements to that date, making the total for general expenses, exclusive of police services, 451 per cent.

"On the Boston subways the general expenses, as given by accounts, have amounted to 3.42 per cent of the total, but some items usually classed as general expenses were charged

directly to the various sections of the work.

"It should be remembered that in all cases above cited, a part of the general expenses have been paid out of general

funds and are not included in the above percentages.

"In modern works in populated sections of the country there is a strong tendency toward an increase in general expenses, owing to the greater attention paid to policing and sanitation where large bodies of men are employed.

"On the Metropolitan Water-Works the charge for police services amounted to \$211,000 for works costing \$26,737,000,

equal to 0.79 per cent.

"On the New York Water-Works to the end of 1912, the total disbursements amounted to \$87,551,000, of which \$1,369,000

was for police, equal to 1.56 per cent of the total.

"The Committee believes that original conditions should be considered in determining the proper percentage for general expenses and that the allowance above suggested for police and sanitation should be included only when similar expenditures were actually made in the creation of the property under consideration."

Am. Soc. C. E. Committee, Contingencies.—"In making an estimate of the cost of a projected undertaking, the experienced engineer adopts a policy of liberality with the intention of reaching the probable actual cost of the proposed work, and even under such circumstances the actual cost is as likely to exceed as to run below the estimate.

"This policy of liberality includes an addition to the com-

puted theoretical or geometrical quantities in all cases where the actual quantities are likely to be greater, the adoption of liberal rather than minimum prices for the various items of work, and a further allowance for contingencies. If the plans are incomplete, so that many minor features are omitted, he properly adds more for omissions and contingencies than where the plans are in greater detail.

"In the valuation of a public service property, the same ideas should be kept in view and the percentage or sum to be allowed for contingencies should be governed, to a considerable extent. by the completeness of the inventory and the amount already allowed for omissions, by the extent to which additions have been made to the computed theoretical quantities and by the degree of liberality of the prices affixed to the various items of the inventory, but in no case should the contingencies be omitted or reduced to a small figure. Large contingent expenses are necessarily incurred in practically all important public works. They may occur from very many causes, among which may be enumerated the failure of contractors and the cost and legal expenses incident thereto; to the delay of certain parts of the work caused by such failure; to injunctions or to the inability to obtain possession of the land in due season, thereby necessitating the execution of such portions of the work under winter conditions or other adverse circumstances; to stringencies in the money market, causing a temporary shortage of funds, and a consequent disorganization of the forces employed on the work; to protracted strikes; to the necessity of rebuilding parts of the work which have failed because of improper design or unforeseen causes; to making alterations found to be necessary or desirable after the work is built; to the slipping of earth or rock; and to making changes in plans which increase the cost of the work.

"Such contingent expenses as those above enumerated, with the exception of the last two, should be included as contingent expenses in any valuation of an existing property because the existence of the property does not give any clew to the amount of most of the contingent expenses involved in its creation."

Am. Soc. C. E. Committee, Insurance and Risk. — "There is another subject closely related to contingencies which may be classed as insurance or risk. For instance, if an owner of property is constructing a building he runs the risk that it may be burned. If a fire occurs when a building is nearly finished and the owner has to rebuild it, the cost of the completed structure, if he has

no insurance, will be nearly that of two buildings. In estimating the reproduction cost of the building, only one building would appear on the inventory, but in estimating its value, there should be added to the reproduction cost of the single building as otherwise determined, the sum necessary to insure its whole value against fire, and this sum should be added whether the owner actually paid it to an insurance company for carrying the risk or whether he assumed the risk himself. Similarly any other property which involves risk during its construction and testing should have added a contingent sum representing what the cost of insurance would be were there insurance companies to insure against such risks.

"It is seldom that a large public service property is examined that there is not a disclosure of some large expenditure for works which have been destroyed, reconstructed, or not used because of faulty design or construction. In some cases this is distinctly the result of negligence, but in a majority of cases such expenditures have taken place where the owners were not negligent in that they have taken due care in the selection of engineers to design and contractors to construct the work.

They are the result of human fallibility.

"Many examples of failures have been furnished by masonry dams. A considerable portion of the Quebec bridge failed while in process of construction; there was a great slip in the Necaxa Dam, one of the highest earth dams ever built, when it was far advanced toward completion; the Loetschberg tunnel in Switzerland encountered bad ground, which required the abandoning of the tunnel for about a mile and its relocation through another portion of the mountain; the change in plan of the new Croton Dam of the New York Water-Works involved an additional expense of more than \$1,000,000 for construction and interest. These are only a few instances of many which might be cited. Such disastrous occurrences are not contemplated by engineers when they make a provision for contingencies in preliminary estimates of the cost of works, but it seems proper in the valuation of an existing property which has been completed and successfully tested to recognize that the owner has been required to assume the risk of accident and failure and should be compensated therefor by at least the amount which insurance companies would charge for taking such risks were they doing this kind of business. This feature may properly be included in the valuation by increasing the amount allowed for contingencies and risks."

Am. Soc. C. E. Committee, Interest and Taxes. — Referring to interest and taxes the Committee says:

"Interest upon the capital invested in the plant up to the time when it is first operated and begins to have earning capacity

is an unavoidable expense.

"Under the most favorable conditions it is necessary to raise the money required for the construction of the work months in advance of its expenditure and in many cases the whole amount must be raised before beginning the work in order to insure against a suspension of operations with the large loss neces-

sarily incident thereto

"The rate allowed for interest during the construction should be the prevailing rate at the time of the valuation, having regard, however, to the variations in rate in different localities and the character of the property. The rate of interest on money required for the original plant will generally be larger than on that required for extensions and betterments as the corporation naturally has a stronger financial standing after it possesses a successful operating plant.

"The amount to be allowed for taxes during construction must be determined largely on the basis of local tax rates and other local conditions and in this case, as in others relating to overhead charges, the original plant and subsequent additions should

be treated separately."

Overhead in Railroad Construction. — While as in the case of other items of cost the records of expenditures may be the best guide when overhead charges of an operating property are to be determined, they are not always to be accepted without question; the appraiser must know whether they are reasonable or not. It is for this reason that such data as above presented are of interest to the appraiser. Following this matter further it may be noted that in valuing the railroads of the State of Washington Mr. H. P. Gillette ascertained from their records of cost that their overhead costs were as shown in the following table:

TABLE 1. OVERHEAD COST ITEMS
Railroads in the State of Washington By H P. Gillette

	Gt 955 p ce	M, er		M, er	Gt 84 p	and N M., er nt.	32 P	& S y. M, er ent.	132 1	k N. ly. M, er ent.	R 165.	.P R 4M, er ent	& 501	R N M, er
Engineering General expense Legal expense Insurance		50 28	3 0	23 26		40 08	-	56 55 06	3	50 00	5 1	51 22 01	0	83 48 02
Interest on advances Bond interest during con-	0	93	I	23	3	25	0	00						•
struction Bond expense	2	44 10		84 18					5	25	13	60	2	61
Taxes Undistributed accounts	0	02			0	17					0	91		05 64
Totals	6	27	8	74	7	91	7	17	9	75	21	25	-	63

Gt N = Great Northern

In valuing the railroads of Michigan, Prof M. E. Cooley who had charge of this work adopted the following percentages in estimating the reasonable allowance for overhead costs:

					Per cent.
Engineering					4 00
Legal expense.					0 50
Organization expense					. 1 50
Interest during construction					3 00
					-
					9.00

The Wisconsin Railroad Commission on Overhead Allowances

The following information is found in the records of the Wisconsin Railroad Commission:

In Hill et al vs. Antigo Water Co. (decided Aug. 3, 1909). Overhead charges in the case of water companies are usually 10 to 18 per cent. The staff engineers estimated in this case at 10 per cent or the minimum (engineering, 5 per cent; interest during construction, 3 per cent; and contingencies, 2 per cent).

W. & Gt N = Washington & Great Northern

F & S = Fairhaven & Southern

S & N = Spokane Falls & Northern.

N P = Northern Pacific

O R & N = Oregon Railway & Navigation Co.

In City of Ripon vs. Ripon Light and Water Co. (decided March 28, 1910) the staff allowed 12 per cent (5 per cent being for engineering and superintendence, 4 per cent interest during construction and 3 per cent legal expenses, organization expense and casualties). The respondent claimed 6 per cent interest during construction.

In Dick et al vs. Madison Water Commission (decided Nov II, 1910) the staff allowed 10 per cent instead of the usual 12 per cent because the plant is a municipal plant.

In State Journal Printing Co. vs. Madison Gas and Electric Co (decided March 8, 1910), engineering was estimated at 5 per cent on the cost of the original physical plant, but not on additions to the plant. The staff of the Commission assumed that construction extended throughout one year and allowed 3 per cent for interest but subsequently changed to 4 per cent as the construction period may have been underestimated. The respondent contended for 6 per cent interest during construction, claiming a 2-year period of construction.

In the tentative valuation the staff had allowed 2 per cent for various items including contingencies, omissions, casualties, legal and organization expenses but on revision allowed 3 per cent. The respondent claimed through various experts 4 to 7 per cent, the larger amount being based on 2 per cent for omissions and contingencies, 3 per cent for legal and organization expenses and 2 per cent for casualty liability.

The Commission in this case says it is fair "that rates charged for the service rendered be fixed at a level that is high enough to cover all reasonable costs."

In City of Racine vs. Racine Gas Light Co. (decided Jan. 21, 1911) the staff allowed 12 per cent for interest during construction, engineering contingencies, etc.

In the application of the LaCrosse Gas and Electric Co. the staff allowed 12 per cent while the petitioner claimed 15 per cent for interest during construction, engineering and contingencies.

In City of Milwaukee vs. Milwaukee Electric Railway and Lighting Co. (decided Aug. 23, 1912) 3 Cent Fare Case, the

respondent contended that 22 per cent for overhead is not excessive and that there are cases in which such additions have been as high as 51 per cent.

Mr. W. D. Pence of the engineering staff of the Commission testifying to an appraisal of the property made in 1907 says the basis was:

Four per cent for engineering and superintendence.

Two per cent for organization and legal expense.

Three per cent for contingencies.

Three per cent for interest during construction.

Mr. Beggs, the manager of the Company, stated that contractor's profit would be at least 15 per cent of cost of labor and materials. (The Commission included contractor's profit in unit prices.)

Mr. Beggs also claimed that interest during construction could not be less than 5 to $7\frac{1}{2}$ per cent and engineering from 5 to $6\frac{1}{2}$ per cent.

Prof. M. E. Cooley in revising the staff's figures allowed 5 per cent for contingencies during construction on all items except land, furniture, etc, besides 5 per cent for contingent omissions on the investment not including land, cars and car equipment; 4 per cent for engineering, $\frac{1}{2}$ per cent for insurance, $2\frac{1}{2}$ per cent for organization and legal expense, and 6 per cent interest; total 22 per cent.

Mr. M. G. Starrett on behalf of the Company claimed 4 per cent for engineering and superintendence, 2 per cent for organization and legal expense, 3 per cent for interest, 5 per cent for contingencies of construction, 9 per cent discount on bonds and 1 per cent for working capital; total 24 per cent.

The Commission as the result of a study of 30 cases of actual expenditures finds a range of 4.5 to 23.43 per cent and an average of 10.13 per cent for engineering, superintendence, organization and legal expense and interest during construction. This does not include contingencies. Some of the individual percentages were low because a portion of the cost of supervising extensions was charged to construction.

In re Purchase of the Oshkosh Water-Works plant (Sept. 17, 1913) the City estimated 10 per cent for engineering, superintendence, interest during construction, contingencies, etc., this on the physical plant not including general equipment. The Commission allowed 15 per cent and said:

"It will be noted that the valuation recently placed on the Oshkosh Water-Works property contained an item of 15 per cent for engineering, superintendence, interest, organization, etc. It will also be noted that the report of the previous valuation only contained 12 per cent to cover these items. It should be stated that this change from 12 to 15 per cent represents the results of an extended study of the costs of such work and it is my opinion that where the actual costs are not known for such a property as the Oshkosh Water-Works, 15 per cent of the cost of specific construction represents about a fair amount for these overhead charges."

In re Application Manitowoc Gas Co. (Dec. 4, 1913) the Commission allowed 15 per cent exclusive of contractor's profits which should be included in the separate cost items.

In re Investigation Ashland Water Co. (July 10, 1914) the Commission allowed 15 per cent.

Promotion Expenses

Promotion Expenses. — There is an additional class of expenditures incurred in connection with many of the public service and some private enterprises which has a more remote connection with the cost of these enterprises than the overhead or the cost of developing the business. Capital is timid and needs guidance. Preceding nearly every investment of capital in the development of an enterprise of any magnitude the same has been in the hands of the promoter whose investigations and efforts are supposed to have been directed to a unification or combination of properties that will make them available as the basis for the enterprise. The reward to the promoter should appear in the enhanced value of the unified property when compared with the cost of the individual items that go to make it up. But this is not all that there is to promotion expense.

Advertising may be necessary to find a market for the securities whose sale is to furnish the money with which construction will be undertaken A commission may have to be paid to the agent who buys the properties or who markets the securities. The securities may have to be sold at a discount so that the indebtedness on which interest is paid may exceed the sum that was actually required to construct and acquire all the essential property. Promotion expense of this character does not usually appear in any valuation, nor in the estimate of the investment, though it may nevertheless have been an essential element of cost. The necessity which may have compelled the incurrence of a promotion expense may be deplored and yet it must be admitted that, on the whole, society has benefitted by the promoters' activity and by the carrying out of many enterprises which were not at the outset sc attractive that honds based thereon would sell at par or command a premium.

In this connection it may be noted that the Wisconsin R. R. Commission holds that the cost of marketing bonds should be taken into account as a promotion expense but that all discounts do not necessarily become proper additions to physical value.

The appraiser whether of value or of the investment will rarely be required to make a close estimate of the promotion expense, but the fact that under the prevailing conditions there has generally been some legitimate promotion expense which may not appear in the valuation, should not be overlooked when rates are fixed.

Intangible Elements of Value

The Total Intangible Value. — Whenever the net return exceeds a fair interest rate on the capital invested in the physical elements of any enterprise, there will be other elements of value in the same, not represented by the cost of any physical portion thereof. These values being apart from the actual investment in physical properties are usually regarded as intangible values. Ordinarily the sum of all intangible values will be the capitalized net annual earnings, less the investment in the physical proper-

ties (if the accounting system throughout has been proper and the enterprise has met with no untoward experiences) or what may generally be a safer approximation, the capitalized net earnings, less the cost of reproducing the physical properties.

A taxicab concern, for example, has invested in the business \$100,000. The net annual earnings after allowing for depreciation are \$30,000. If it be assumed that 10 per cent per annum is a reasonable return on an investment of this character and permanency of the business may be assumed the capitalization of the concern might be at \$300,000. The good-will of this taxicab business appears at \$200,000. That is to say, if there were no fear of a reduction of income through competition or other causes, a valuation three times as great as the actual investment might be justified. In such a case the sum of all intangible values connected with the business which may include besides what is strictly good-will, advantageous leases on desirable space for the taxi stands and contracts with railroads or other transportation companies, is the capitalization of the net earnings of \$30,000 per year, or \$300,000, less the value of the physical properties assumed as above stated to be \$100,000.

Ordinarily, however, in a business of this character, the earnings may at any time be cut down by competition. Any intending purchaser taking this into account as a hazard of the business will conclude that, while earnings of 10 per cent a year on the value of the tangible property may be adequate, the return on the value of uncertain intangible elements should be very much greater.

He may find circumstances that will justify him in concluding that the return on any allowance for good-will should be 20 per cent and in this event he will find:

Total annual net earnings	
Net annual return on intangible values	

Consequently, the amount which he, as a prudent purchaser, would be willing to pay for the business would be only \$200,000

instead of \$300,000. In other words, \$300,000 would be an over-capitalization in view of the risks of the business, and an adequate annual net return from a business of this character should exceed 10 per cent on the total value thereof.

As another illustration of relatively high intangible value, a newspaper route may be cited. A thousand subscribers, at \$4 each, would make the value of this route \$4000. The investment in physical elements such as a cart and horse may be insignificant. There are, no doubt, many such routes where the bicycles and hand carts in use by the delivery boys are furnished by these boys and not by the owner of the route. Practically the entire value of such a route is represented by good-will.

Intangible Value Under a Restricted Franchise. — When public utilities are under consideration, the problem may become quite complex and each case will have to be considered by itself. A restricted or limited franchise, for example, may make a monopoly of the enterprise, and its terms may be such that charges for service cannot be reduced by any rate-fixing body during the life of the franchise. Under such circumstances, when the volume of business is determinable, it is possible to forecast with some degree of certainty, on the basis of past experience, the annual net earnings and a capitalization of these at a fair rate of interest leads directly to a determination of value. A comparison between value thus ascertained and the cost of reproduction of the physical elements establishes the aggregate value of all intangible elements of whatever nature connected with the utility. Of course, the capitalization of earnings, as here set forth, may not in all cases be a simple matter, because many factors are to be taken into account:

The allowance for replacements must be correctly determined. The hazards of the business must not be overlooked.

The value of the properties remaining on hand subject to sale at the termination of the franchise may be a factor of some moment in determining how much of the investment is to be amortized within the life of the franchise.

Nevertheless, the fact remains that the restricted, exclusive

franchise usually affords a definite basis for estimating its value. When the rates for service rendered or for commodity furnished are subject to regulation by public service commissions or other similar rate-fixing bodies, this is not so, at least not until rates thus regulated become dependable for a number of years and the policy to be pursued by such bodies in the discharge of their duties can be forecast with confidence.

Over-capitalization. — The cases have been so frequent where exorbitant earnings, real or in some measure fictitious, have been made the basis of enormous over-capitalization of enterprises both private and quasi-public in character, that the general public leans strongly toward extreme restriction of earnings whenever the law and circumstances and the power of the ratefixing bodies will permit of such restriction. Why allow any public utility to earn more than is ordinarily earned by a safe investment? Why allow anything for unprofitable investments even though made under good expert advice? Why take into account any lean years of the past? If the utility was not profitable or is not profitable, why should not the loss fall upon those whose poor business judgment led them into a losing venture? Such questions and others of similar character deserve serious consideration when rates are to be established. which are to be fair alike to the owner and to the rate-payer.

Kansas City Water-Works Case and Intangible Value.—Justice Brewer, in stating the conclusions of the Court in the Kansas City Water-Works Case (Circuit Court of Appeals, Eighth Circuit, July 2, 1894, 62 Feb. Rep. 853), says:

"A completed system of waterworks, such as the company has, without a single connection between the pipes in the streets and the buildings of the city, would be a property of much less value than that system connected, as it is, with so many buildings, and earning, in consequence thereof, the money which it does earn. The fact that it is a system in operation, not only with a capacity to supply the city, but actually supplying many buildings in the city—not only with a capacity to earn but actually earning—makes it true that 'the fair and equitable value' is something in excess of the cost of reproduction. The

fact that the company does not own the connections between the pipes in the streets and the buildings—such connections being the property of the individual property owners—does not militate against the proposition last stated, for who would care to buy, or at least give a large price for a waterworks system without a single connection between the pipes in the streets and the buildings adjacent

It (the city) should pay therefore not merely the value of a system which might be made to earn, but that of a system which does earn."

According to this view of the court there is no question about the inclusion of intangible elements among the properties which have value. The Supreme Judicial Court of Maine in its instructions to the appraisers of the properties of the Maine Water Co. (1902) in the Kennebec Water District Case (97 Maine 185; 54 Atlantic 6) said:

"In addition to structure values, the appraisers should allow just compensation for all the franchises, rights, and privileges to be taken."

Here, too, the court recognizes the fact that franchises and privileges may have value.

Intangibles as a Protection to the Owner. — The courts and the public service commissions must protect the investor whose enterprise is developing the latent resources of the country and who is, therefore, to be encouraged, and they must at the same time prevent as far as may be the robbing of the public by overcapitalization and over-bonding with consequent over-charging for the service rendered. But in protecting the public they should not lose sight of the fact that the owner of the utility is entitled to fair compensation for his time and his business ability, and for the risks which he assumed in embarking upon a venture for general benefit.

To do this adequately, in conformity with the rulings of public service commissions and the decisions of the courts, the appraiser has frequently had recourse to intangible elements of value under such names as "unification of properties," "going value," "solidification of road-bed," "appreciation of land"

or of "conduit values due to paving," "cost of establishing business" or under some other designation. Such an addition of intangible value under the name of "going value" based on the losses during the early years of operation has been seriously advocated. If regard should be had in this connection to the exact amount in each case of early operating losses, of unprofitable investments, such as tunnels or wells for water which turned out to be unproductive, destruction by fire or earthquake and the like, then the allowance for unprofitable expenditures might be greatest in the case of the least worthy enterprise. which would be an absurdity. Nevertheless, some allowance for unproductive expenditures and for early losses is frequently justified and these may be taken care of either in the appraisal of the rate-base or they may be taken care of in the rate of return which when fixed somewhat higher than the return on ordinary safe investments will in the course of time amortize a part or all of the expenditures which do not appear in the ratebase. Under such a procedure there is a recognition of the fact that early losses may have been unavoidable, that no blame may attach for having made certain unprofitable expenditures and that the owner who has invested wisely and under more fortunate circumstances is entitled to the reward which will be brought to him either by giving suitable consideration to "going value" or by allowing him to earn more than ordinary interest on his actual investment.

Example of Intangible Value Created by Earnings. — Take as an illustration the case of a property operated at a loss for 5 years and thereafter at a profit. Suppose that the investment in the property was \$1,000,000 before operation commenced; that cost of operation exceeded the earnings in the first five years of operation by \$100,000 and that money for the entire investment had been borrowed at 6 per cent. During the five years the interest payments, compounded at 6 per cent, amounted to \$338,220, which, together with the operating loss, makes \$438,220 as the actual outlay by the owner at the close of the fifth year in addition to the original investment of \$1,000,000.

No compensation for hazard or management is included in the foregoing figures. The property as assumed will yield more than operating expenses after the first five years. Unless the excess over operating expenses is more than 6 per cent on the total outlay of \$1,438,220 the owner will still be conducting business at a loss and unless it is sufficiently in excess of 6 per cent to compensate him adequately for management and risk he will not realize all that he had a right to expect.

By reason of increase of population increased demand for his output, or for the service which is rendered by the utility it may be possible after the first five years to reproduce the utility or to construct a substitutional plant with established business at a less cost than \$1,438,220.

The question is how to determine what will be fair earnings. Two procedures are open:

- a. The actual cost of developing the business may be added to the cost of reproducing the physical plant and the sum approximating \$1,438,220 may be introduced into the calculation as the rate-base.
- b. The cost of reproducing the physical plant together with actual cost of franchises, water-rights or rights-of-way or about \$1,000,000 is made the rate-base, and the cost of developing the business, in this case approximately \$438,220, is estimated and treated as a business loss subject to amortization in a reasonable number of years.

Any allowance less than will result from these procedures would be confiscation of a part of the investment and therefore unfair to the owner, who is in this illustration assumed to have used good judgment in undertaking and developing the enterprise.

Kennebec Water District Case on "Going Concern."—In this connection, too, the instructions issued by the court in the Kennebec Water District Case to the appraisers of the Maine Water Co. (97 Main 185; 54 Altantic 6) may be cited:

"In estimating even the structure value of the plant, allowance should be made for the fact, if proved, that the company's

water system is a going concern, with a profitable business established, and with a present income assured and now being earned."

"Going Value" and "the Going Concern."—A distinction is to be made between the "value of the going concern" and "going value." The value of the going concern is the total value of the entire property, its market value. "Going value" is the increment of value due to the fact that the business is established and in successful operation. It is at least equal to the reasonable cost of establishing the business, including a reasonable allowance for losses in early, lean business years. If assured earnings are large when compared with the operating cost it may greatly exceed early losses just as value may exceed cost. In the case of unprofitable business, as when a company's properties are in a receiver's hand, going value will be only nominal.

It sometimes happens that the public is responsible in some measure for inadequate earnings as in the case of competing utilities. By permitting the competition and a duplication of works, the cost of bringing the business up to a profitable basis is increased and may be the cause of increased losses.

Going Value in the Decisions of the Wisconsin R. R. Commission. — On the subject of "going value" the Wisconsin Railroad Commission in the Cashton Light and Power Co. case (Wis. R C. R., Vol. 3, p. 85) declares this to be an element of value which must be taken into account. The Commission after stating that it is akin to "good-will" refers to Justice Brewer's views in National Waterworks Co. vs. Kansas City (62 Fed. Rep. 853) and then cites the opinion in Cedar Rapids Water Co. vs. Cedar Rapids (118 Iowa, 234) to the effect that "going value" is "that value which arises from having an established going business . . . and attaches to the business rather than the property employed in such business."

The Commission takes a sound view when it says in the Antigo Water Co. case (Wis. R. C. R., Vol. 3, p. 707) that the question as to whether deficits resulting from operation, or due to

other causes, should be regarded as investment, will depend upon the circumstances of each case.

The Commission discusses methods of ascertaining the "cost of establishing the business" and "going value" in Green Bay vs. Green Bay Water Co. (Wis. R. C. R., Vol. 11, p. 243-252), and in this case says:

"The method which is generally followed by the Commission aims to determine, as far as possible, what the actual cost of developing the business in question has been, and to what extent, if at all, such losses have been recovered in later years of operation. There are a number of difficulties in determining, by this method, what the cost of building up a business has been, among which may be mentioned:

"1. Entire or partial lack of records covering the develop-

ment period.

"2. Difficulty of finding original cost of physical plant.

"3. Difficulty of eliminating from reported operating expenses amounts which are the results of extravagance, inefficiency, or other causes which tended to keep the costs above a nominal figure.

"... This method, where it can be applied to its full extent, enables the investigator to determine what it has actually cost the utility in question to build up its business. This sum, added to the actual investment in the physical plant, gives the total amount which the plant and the business have actually cost... Where it is impracticable to determine what the actual cost of the physical property has been the only method of arriving at the value of that property is to ascertain the cost of reproduction.

"The method of determining going value as followed by Mr. Alvord, and which, for the sake of convenience we will refer to as Alvord's method, is an attempt to fix the amount which it would cost to reconstruct the business of the utility, somewhat as a physical valuation reveals the cost of reconstructing the physical plant. There are two assumptions vital to this method:

"I. A city similar in all respects to the one under consideration, except that there is no public water supply system but in which the people are in a general way cognizant of the advan-

tages of such a water supply.

"2. Capital seeking investment which may be either used to construct a plant and business in the city with no water supply or to purchase the existing plant and business.

"In a computation of going value according to Alvord's method the going value is the present worth of the amounts by which the net earnings of the comparative plant are less than the net earnings of the existing plant during the entire period from the date of the first preliminary work until the earnings of both plants are equal. . . . The loss to capital invested in the comparative plant is not to be measured by the extent to which its earnings during the construction period fall short of the net revenues of the existing plant, but rather by the amount by which they are less than the returns which have been foregone in order to enter the new field of investment. . . . The detailed computation of the cost of developing the amount of business necessary to yield a reasonable return upon the property involves a number of further assumptions which make the accuracy of the result very questionable."

Franchises and Related Matters

The Franchise Value. — Any privilege or right granted by legislative authority to engage in a particular business is a franchise.

Whenever the franchise or privilege to do business is an exclusive privilege and grants some latitude in the matter of the charges which may be made for the service rendered thereunder, it is an element that can be valued. It is well stated by the Supreme Judicial Court of Kennebec County, Maine, in instructions to appraisers that its value "depends upon its net earning power present and prospective, developed and capable of development, at reasonable rates, and the value to be assessed is the value to the seller and not to the buyer."

When the value of a franchise is in question, all circumstances that affect the earning powers of a property must be taken into account and by an analysis thereof the excess of the earnings over a fair return on the capital invested in physical properties must be determined. This excess of earnings when capitalized will give some idea of the total value of all intangible elements and may lead to the determination of the franchise value.

Frequently, the only stipulation in the franchise in the matter

of rates is that these shall be fair and subject to regulation. In such cases the franchise value should not be made a part of a rate-base, determined by investment, except only to the extent that its acquisition has actually required legitimate investment, as is the case, for instance, when a city sells to a Street Railway Corporation the right to operate cars on certain streets for a certain period of time. The franchise value together with the value of other intangible elements is independent of the actual cost of these elements; but a franchise value may result from the establishment of rates which produce earnings in excess of reasonable interest return on the capital invested in the physical properties.

The ascertainment of franchise value and value of all other intangible elements connected with public utilities is similar to the ascertainment of the value of the good-will of a private business but with this difference. . . . In the case of the public utility there is usually protection against competition, while in the case of the good-will of a private business, competition or the possibility of competition is to be assumed and such competition may materially affect the value.

A franchise is sought for and operations thereunder are undertaken for profit. Sometimes, of course, the profit to the party who accepts the franchise and operates thereunder is an indirect one, as in the case of the railroads which make large land holdings valuable. While the privilege to do business is granted in order that certain property may be used for the benefit of the public, the person or corporation undertaking the business must be assumed to be doing so in the hope of reaping an adequate reward.

The Franchise Term.—In many of the states perpetual franchises have been granted. Rights to use water-powers, wharf and water-front privileges, the privilege to supply gas or water and the right to occupy streets for various purposes when thus granted without time limit may acquire relatively high value and may prove a serious obstacle to the ultimate rational development of natural resources. The perpetual franchise

has, however, one feature in its favor. The operator thereunder can look with some confidence into the future and if circumstances justify, will find it to his interest and may be expected to make suitable provision for the growth and future demands of the community. Knowing that the prospective earnings justify this course, he can build far ahead of the immediate requirements. But this advantage is not of sufficient importance to justify the grant of perpetual privileges and such privileges are now rarely, if at all, granted by any of the states.

The term of a franchise should not be too short. When the evils of the perpetual franchise began to be fully realized, the next step was to the other extreme and terms were in some of the states restricted to 25 years. Usually several years are consumed in the construction of works, 5 to 10 years or more in the development of the business and thereafter there should be ample time left within which to make some profit. A term of 40 to 50 years would seem to be about right, not alone to encourage reasonable development under a franchise, but also to give the public opportunity to renew, at not too great intervals, the conditions subject to which the privilege is granted.

Amortization During the Life of the Franchise. — In any event suitable provision should always be made to amortize the capital invested either by a purchase on an agreed basis at the end of the franchise term or by the amortization of a part or all of the investment during the life of the franchise with a view to an acquisition of the property by the public. If the latter plan is followed, there may be danger of neglect in the last years of service, as it will be to the advantage of the owner to expend as little as possible in upkeep, provided only that he can continue rendering the service. The provision which must be made in such a case to amortize the investment may cover the entire term or only a portion of the franchise term.

The Indeterminate Franchise. — The indeterminate franchise is fast finding favor. No definite term of life is fixed but provision is made that at any time after a specified number of years, the community may take over the property either at an agreed

price or at an appraised value. Usually the method of making the valuation is agreed upon in advance in order that there may be as few points of disagreement as possible. The indeterminate franchise may be granted subject to various conditions such, for example, as an allowance of 10 per cent or some other amount on the actual investment if the property is taken over within 10 years; or that after a certain number of years a part of the earnings will be turned over to the community; or that certain requirements relating to the character and quality of the service will be complied with.

The indeterminate franchise has not yet been fully tried out, but in those states in which suitable provision has been made for the regulation of rates, there is good reason to believe that it will prove satisfactory.

Capitalization of the Franchise. — The tendency has been to capitalize the value of the franchise, in other words, to use the franchise as a basis for the issuance of securities. Perhaps there is some reason for this in the case of a perpetual privilege, when thereunder the assured earnings exceed the ordinary fair interest return on other similar investments, but the capitalization of the franchise, except the actual cost thereof, is now quite generally prohibited by the laws which provide for the control and regulation of the public service corporations, and the decisions of the courts are adverse to such capitalization.

The Wisconsin Railroad Commission says on this subject in the Antigo Water Case (Aug. 3, 1909), "That if the municipality required the payment of money or its equivalent, or there was necessary legitimate payment made for the franchise, then the sum which may be reasonably said to have been paid for the franchise may be included in the valuation, the same as money necessarily invested in physical property. But the Commission refuses to consider the claim of some experts and corporations that franchises for which no money was paid may have 'intangible' values which should be considered in the making of rates."

The Public Service Commission Law of New York provides: "The Commission shall have no power to authorize the capitalization of any franchise to be a corporation or to authorize the

capitalization of any franchise or the right to own, operate or enjoy any franchise whatsoever in excess of the amount (exclusive of any tax or annual charge) actually paid to the State or to a political subdivision thereof as the consideration for the grant of such franchise."

Appreciation and the Unearned Increment

Earnings Affect Value. — The earning power of a property determines its value. As its net earnings increase, its value increases. The earning power of public utilities, as a general rule, if rates remain undisturbed, increases as population density increases although not in the same ratio. The appreciation or increase of value which results when net earnings, in their relation to the investment, are increasing is the reverse of depreciation, but it follows no definite law and it cannot be forecast with that degree of certainty which can, with some reason, be claimed for depreciation.

The Unearned Increment. — Appreciation or increase of value without increase of investment is the unearned increment which results from the changing conditions of environment. Usefulness in service does not always decrease, but may increase with age. This may be the case with a dam or with a railway embankment, and this increase of usefulness when it can be expressed in terms of money, or value in exchange, represents appreciation.

Increase in the price of labor and materials, or a change in the conditions under which an enterprise was first established, may add to or may take from its value according to whether it would be more or less expensive to construct and establish the same enterprise under the altered conditions.

As a general proposition, it may be stated that values as expressed in terms of money are increasing. It is also true that, as a general rule, public utilities are to be included among the principal factors which are responsible for the growth of the community, and that, when viewed in this light, the owner of the utility is entitled to participate in the unearned increment

just as the owner of land participates. He does so, of course, in a measure as his business increases, but if held down to earnings which will barely yield the ordinary interest rates on safe investments, the extent of doing this may fall far short of the advance in property value shared in by practically all the owners of the realty in the community.

Application to Public Utilities. — Such considerations as this, although not thus expressed, have led the U.S. Supreme Court to hold that the owner of a public utility is entitled, in most cases at any rate, to have the present value of his property made the basis of the computation when rates are to be fixed. In the Consolidated Gas Co. case (Wm. R. Willcox et al. vs. Consolidated Gas Co. of N. Y., 212 U.S. 19, 29 Sup. Ct. Rep. 192) as already quoted the court says:

"And we concur with the court below in holding that the value of the property is to be determined as of the time when the inquiry is made regarding the rates. If the property which legally enters into the consideration of the question of rates has increased in value since it was acquired, the company is entitled to the benefit of such increase. This is at any rate the general rule. We do not say that there may not possibly be an exception to it where the property may have increased so enormously in value as to render a rate permitting a reasonable return upon such value unjust to the public."

The court has, perhaps, overlooked the fact that unearned increments are earnings and can be allowed without adding them to the investment. It may have been perfectly fair to make the valuation allowance to the full extent of the unearned increment in the Consolidated Gas Co. case and this allowance is not here made the subject of criticism. It is only the practical application of the ruling which is brought into question. It is believed that as these matters are better understood there will be a general acceptance of the view that appreciation had better not be included in the rate-base.

In the case of farm lands the situation may obtain of a greater supply thereof than can be made use of by the inhabitants of

the country. The rental value is then low and the value is fixed rather by the supply of the desirable tracts of land than by the revenue which can be produced by cultivation. unimproved farm and the unimproved town lot may be a source of expense instead of a source of income. The owner of such property expects appreciation to bring him a reward for having acquired and for hold ng the property. If he analyses his investment in such property after he has held it for a number of years, he may find that the first cost at compound interest, at savings bank rates, plus annual taxes amounts to more than other equally desirable property can be obtained for. His investment under such circumstances was not a judicious one, provided, of course, that the property while thus held was not income producing. But when real estate is held and is in use by the owner of a public utility, the intent is, whether always realized or not, to allow him to recover in the earnings at least interest on what this real estate has cost him. He is not in the position of the person who owns an unproductive piece of property. Nevertheless, as already stated, in order to share in the unearned increment which he has helped to create for the community he should be allowed, if the prosperity of the community justifies this course, to earn more than ordinary interest on his investment. In the Kennebec Water District Case, already cited. the court instructed the appraisers of the Maine Water Company's properties that "subject to all the foregoing limitations. the owner is entitled to any appreciation due to natural causes."

If, in any measure, appreciation goes to the owner of a public utility which includes among its properties land holdings, then in the case of other utilities which include no appreciating property there should be a like opportunity for profit. In the case of these other utilities the unearned increment cannot be measured by the appreciation of land. Appreciation does not increase the invested capital. It is not essential that appreciation of land or of any other property be added to the rate-base; but the owner of the utility is, nevertheless, entitled to a reasonable share in the general prosperity which he helps to

create. This can best be allowed him, not in estimating actual appreciation, which is more or less uncertain and irregular, and, therefore, not always dependable, but in a suitable interest return on the original investment.

Interstate Commerce Commission Comments on U.S. Supreme Court Decision. — The difficulty of conforming to the decision of the United States Supreme Court in the matter of allowing the appreciation of real estate was felt by the Interstate Commerce Commission of the United States which says in its opinion in the Western Advanced Rate Case (20 I. C. C. Rep. 344, decided Feb. 22, 1911):

"Certainly if the Supreme Court may decline to lay down the absolute rule that 'in every case failure to produce some profit to those who have invested their money in the building of a road is conclusive that the tariff is unjust and unreasonable' (Reagan vs. Farmer Loan and Trust Co. 154 U. S. 412), it is a conservative statement of the law to hold that a railroad may not increase the rates upon a number of commodities solely because its real estate has risen in value."

"While it is evident, therefore, that each case must be decided upon the facts peculiar to it, the Commission believes it proper in this case to follow the general rule, as stated by Judge Hough of the United States Circuit Court (Consolidated Gas Co. vs. City of New York et al., 157 Fed. Rep. 849, 855), 'Upon reason, it seems clear that in solving this equation the plus and minus quantities should be equally considered, and appreciation and depreciation treated alike.'... Thus land has been taken at its fair value and not at its original cost, and the annual appreciation of land has been treated as a profit. By this method all property is treated absolutely alike, as Judge Hough suggests. No difference is made, except that as depreciation represents a decrease in assets, it is placed as a debit against operation, while appreciation is placed as a credit because it is an increase in assets."

Treatment of Appreciation and Depreciation. — The real difference between the way in which depreciation and appreciation should be treated has apparently been overlooked by the courts but recognized by the Interstate Commerce Com-

mission. The lessening of worth is offset, or at any rate is intended to be offset, by an increase of the earnings. Depreciation then appears on both sides of the account. The owner of the property if fairly treated is no better nor worse off in consequence of deteriorating articles because they are made good to him as they deteriorate. But appreciation is usually not estimated from year to year and is not therefore entered with the annual revenue. When the occasional appraisal discloses appreciation. it appears as a profit. The Interstate Commerce Commission regards it as income. The United States Supreme Court holds that the owner of the public utility is entitled to the appreciation unless the same is excessive in amount in addition to a fair return upon what would, without the inclusion of appreciation, be the reasonable amount of capital invested. How much simpler it would be to grant to the utility some reasonable share in the general prosperity not measured solely by the increase in the value of the real estate which it happens to own and use in the public service.

The Secretary of the Treasury of the United States in decisions relating to the income tax says:

"Profits realized on the sale of real estate during the year, also increase of value of unsold property, if taken up on the books of the corporation, (are) to be included in income."

Appreciation and the Rate-Base. — In weighing the question whether or not appreciation is to be added to the rate-base, consideration may be given to the alternative of the rental value of equivalent property. Suppose for example that among the properties owned by a public utility there is a large tract of land located in a region in which real estate values are advancing normally. If instead of acquiring this land the owner of the utility had entered into a lease thereof based upon an agreement that the rent from year to year should be commensurate with a proper valuation of the land, the amount of the rent increasing from year to year would be included in the cost of operation and rates would be fixed as though, in the case of actual ownership, the rate-base had included appreciation.

While this circumstance may justify the inclusion of appreciation in the appraisal of the rate-base in some cases, there will he others where the appreciation is out of all proportion to the original investment. Furthermore, a strict analysis will show that such inclusion would deprive the rate-payer of any share in the unearned increment to which he, too, is a contributor. It is, moreover, as already stated, difficult and oftentimes impossible to ascertain the rate of appreciation which might fairly be taken into account and finally, if appreciation is included in the appraisal of land, it should also be included in the appraisal of all other kinds of property. Such inclusion would greatly and unnecessarily complicate every calculation relating to rates and would introduce an element of much uncertainty. It will be much simpler and yet fair to all concerned to exclude appreciation from the rate-base but, as already suggested, to allow the owner of every utility to participate in the general prosperity by allowing the rate of return on the rate-base to be higher than it would otherwise be estimated.

If this practice coupled with the Unlimited Life Method of procedure could be made general there would be no revision of a rate-base, once established, except as made necessary by additions to the utility properties or changes therein, nor would there be any need of determining present value, and yet the unearned increment would not be ignored. It would appear in the earnings and not in the rate-base, and the equitable apportionment of the unearned increment to the utility owner and to the rate-payer would be facilitated.

If the unearned increment (appreciation) is estimated from time to time and is treated as income and the total net return on the rate-base does not exceed ordinary interest rates on safe investments, the estimated appreciation should be added to the rate-base in fairness to the owner of the utility.

If appreciation is estimated from time to time and added to the rate-base but is not treated as income, the rate-payer will be denied any share thereof.

If appreciation is ignored, and a rate-base, once determined,

is allowed to remain uninfluenced by appreciation, the owner of the public utility should be allowed a higher rate of return than if appreciation were included in the appraisal of the rate-base.

The endeavor should be to secure a general recognition of the latter principle and thereby secure the same liberal treatment for those utilities which do not include among their properties land holdings, as is now accorded with approval of the courts, to those which do include large amounts of real estate.

If treated in this way the owner will never be in a position to capitalize appreciation, unless the use of the property in the public service be abandoned. This is as it would be if the owner were acting as agent, and is equitable.

In the Minnesota Rate Cases, the United States Supreme Court apparently recognizes the broad principle that the public service property should earn a return upon the increasing value of its properties, provided the appreciation be properly ascertained. The Court in these cases says:

"Assuming that the company is entitled to a reasonable share in the general prosperity of the communities which it serves, and thus to attribute to its property an increase in value, still the increase so allowed, apart from any improvements it may make, can not properly extend beyond the fair average of the normal market value of the land in the vicinity having a similar character."

It would be much better to let this principle, recognized generally by the courts when they say that present value is to be the basis of the calculations when rates are to be fixed, be applied, as already suggested, without reference to or without being dependent upon and restricted to the present value of land, so that the same share in the general prosperity would come to the owner of the utility which owns no real estate as to the other utility which owns broad acres.

Cost to Reproduce New Includes Appreciation. — When reproduction cost is used as a means of approximating the investment or the "rate-base," the basis of the calculation may

be current or comparatively recent prices and the general conditions as they exist at the time of the appraisal. Such reproduction cost will include appreciation whether the same be enhanced value of real estate or increased cost of construction due to higher wages and higher prices of materials, or due to other conditions at variance with those which prevailed when the work was first done. Such appreciation comes up for consideration in the case of a pipe line which was laid before the street was paved but which is to be appraised after the street has been paved. The cost of reproduction in this case includes the cost of cutting through and replacing the pavement. This cost of reproduction may be some indication of present value but it is not a good measure of the investment, and at its best will be more or less unsatisfactory as a method of approximating the amount on which interest should be earned Or a pipe which is to be valued may have been laid along an ungraded street. It may have been laid in a temporary trench to be deepened when the street was graded or it may have been placed in a deep trench. The cost of reproduction, if estimated after the street has been graded, will again fail to indicate correctly the amount invested in the pipe and is not entirely satisfactory because the first cost of a deep trench or the lowering of the pipe during the work of grading were legitimate expenditures which the owner of the public utility, if he had been an agent of the rate-payer, would have been justified in incurring. Such considerations as these show how unsatisfactory any close adherence to the cost of reproduction may be and they show, too, that a wide latitude is sometimes allowable when the legitimate investment is approximated by estimating the cost of reproduction. Furthermore, this method can hardly be applied satisfactorily without adopting some definite rule in the matter of valuing real estate. Suppose the rule to be that real estate and all other property be appraised on the assumption of present day cost of acquisition and construction, disregarding low original cost, donations and all other factors and conditions that may have prevailed at the time of construction and that may have kept cost down. Under such a rule all relation between the capital legitimately invested and the cost of reproduction may be lost. In a private enterprise the former and not the latter would be used as the guide in determining what the earnings should be. But the cost of reproduction is generally held to deserve special consideration when the "fair value," required by the courts, is to be appraised and it is frequently accepted as the starting point. When the courts shall have accepted the view that "fair value" is not the proper starting point, then the importance of close estimates of the cost of reproduction, which includes both depreciation and appreciation, will fall away.

CHAPTER V

ELEMENTS WHICH REDUCE VALUE

Deductions from Value

The Lessening of Worth. — The value of a property may increase or decrease with age. Having in the preceding chapter called attention to the essentials of value and to certain elements which add value, there will be considered briefly in this chapter some of the factors which reduce value.

In the case of any operating plant there will be parts which deteriorate or wear out under the effect of continued use. By ordinary care the rate at which the perishable parts of a plant are consumed in rendering useful service is held at or near the average rate which experience teaches should be regarded as inevitable. If ordinary care is not exercised and a plant by reason of neglect gets into a condition which requires special attention to restore it to an average condition, all circumstances of use and age being considered, the prospective outlay to accomplish this rehabilitation is called deferred maintenance. But even when there is no deferred maintenance there may have been a lessening of worth due to the fact that on account of wear and tear, or by reason of obsolescence, or inadequacy, or from any other cause, the time of probable serviceability of any article in question is being continually lessened. Loss of value from such cause is depreciation. Or again, if cost is made a starting point, it may be found that there is included unused property perhaps held to meet a future demand, but which, when valuations as a basis for rates are under consideration, may have to be omitted or deducted from the aggregate of the listed properties. And, finally, the property under consideration may include items designed of a capacity to meet future requirements and so far in excess of a reasonable allowance for growth in the near future that it must be regarded as overbuilt and not, therefore, entitled to be taken into account at its full cost.

Without attempting a full discussion of all of these factors which must be brought under review by an appraiser, particularly when the establishment of a rate-base is involved, attention will be given briefly to property not required for immediate use, and this will be followed by a discussion of depreciation including some matter relating to amortization and the replacement requirement, here conveniently considered though not all strictly pertaining to reduction of value.

The Overbuilt Plant and Property not Required for Immediate Use

Plant Capacity in Relation to Requirement. — Ordinarily, and with good reason, public utilities are constructed of a capacity for service somewhat in excess of immediate requirements. It is customary in anticipation of the increased demand for service which comes with the growth of the community to give the utility ample capacity. A part of the utility plant is built, in other words, to meet the demands of the future rate-payer. The question may well be asked to what extent this is justified and to what extent the rate-payer of today should be made to bear a burden for the benefit of the rate-payer of tomorrow.

Prudent foresight in such matters is always commendable. It should be exercised by the private owner just as it would be exercised by wisely managed municipal public utility departments. Future requirements should be foreseen and provision should be made for expansion of works. This may sometimes require the acquisition of lands, of rights of way, of waterrights, of reservoir sites, and the construction of works beyond immediate requirements. Outlays in this direction, within limits, are generally more economical than deferred acquisition or deferred construction, at the advanced value of land and the higher cost of construction of the future.

While not endeavoring to set up a rule for the determination of the extent to which it may be proper to take future requirements into account in planning any installation, attention is called to the fact that any such provision for anticipated future service adds to the investment of capital and thereby makes higher earnings necessary than would be required, if, at all times, the plant capacity could be kept exactly at the momentary requirement. Here then is an additional call upon the rate-payers, which, naturally, will fall heaviest upon those of the early years, at which time the disparity between actual demand and the provided capacity is generally greatest. There lies herein a further reason why the rate-payer in the early years of the life of any public utility should be relieved, as elsewhere suggested, of the requirement to meet amortization of capital. These rate-payers will ordinarily have done their full share if they contribute operating expenses, interest on the investment, and whatever may be necessary to anticipate and to meet replacements of discarded parts as such replacements become necessary.

Property not in Use. — Guided by the decisions of the courts and by the rulings of public service boards and commissions, it has become customary to discriminate with care between the property of a public utility which is in use and the property which may not strictly be regarded as in use. Property which is in no way related to the service rendered and which is of no prospective necessity will be left out of consideration in this discussion. But there is another type of property owned, but not in use, as for example a reservoir site, acquired by a water works owner at a favorable moment but not likely to be made a part of the system for an indefinite period, or a terminal right of way for a railroad, or an undeveloped water-power which according to sound judgment must be regarded as necessary for future expansion though not in active use as a part of the present system. The wisdom of early acquisition of such properties at opportune moments can hardly be questioned. How to deal with them in making an appraisal for rate-fixing purposes is the question. If such properties are included in the valuation of the rate-base as of date when purchased, the rate-payer is taxed for the benefit of the future. If, on the other hand, they are not included in valuations until they are actually put into use, then they must ultimately be included at original cost plus interest, or as seems to be a more general practice, at their value at the time they go into use. Sometimes by such procedure a large unearned increment represented by increase of value goes to the public utility owner and generally he is entitled thereto as a reward for his foresight.

As a general principle the holding of a reasonable amount of property which will at an early day be in use, should be considered legitimate and in conformity with the practice that would be followed by any prudent management, and it will be proper to place such property in the same category as property in actual use. This, in its essence, is no different from allowing full value of the pumping plant which has twice the capacity immediately required, but which was planned with due regard to future demand upon it.

When any property which is thus held to meet prospective requirements is not included in the appraisal, the burden of holding the same available for future use falls on the owner of the property. When he thus acquires rights, lands or other elements which are of value to his system of works, but are perhaps of a character from which no return can be expected, he puts the community under obligation to suitably recompense him for his foresight and for the material service which he thereby renders. By judicious investment from time to time in property which a prudent agent under like circumstances would acquire, the public utility owner not only provides for the expansion of his system of works, but he hopes to profit by the increased value that comes to all property located in prosperous sections that are well served by public utilities.

The ideal arrangement would be to have all acquisitions of property of whatever nature that appertain to future service made subject to the approval of competent representatives of those who are ultimately to pay the bills, and, in that case, there would be no question as to the propriety of including property held for future use. The only question would be whether the present rates should yield interest on the full investment in such property or whether the owner should be required to carry the same, in whole or in part, for the benefit of the community until such time as the greater demand for the service and consequent greater earnings will justify the inclusion of the property at cost, plus interest, plus such additional amount as may be thought a fair profit allowance to the owner in each particular case. Approval of this character in the past has been out of the question. To a large extent it will be impractical, too, in the future. It frequently happens that the purpose of the acquisition of property must be kept under cover, because, if disclosed, the prices would become prohibitive. For this reason the publicity incidental to a preliminary approval by representatives of the public would often prove embarrassing. It will no doubt be suggested that condemnation proceedings should be resorted to whenever property is to be acquired for the use of a public utility. No one who is familiar with the conduct and results of such proceedings in the courts of this country would be willing to admit the advisability of this procedure in all cases. The jury which is required to base its findings on the evidence submitted in such proceedings frequently reaches conclusions that are not fair. Not until value of the property taken, and the amount of damage to remaining property, is made determinable by impartial experts, not selected by the litigants but appointed by the courts, will there be any hope of securing through court proceedings, at a reasonable cost, the rights and properties that may be required by public utilities.

Discussion of Overbuilt Plants by the Wisconsin R. R. Commission. — In the discussion of the value of the LaCrosse Gas and Electric Co. properties the Wisconsin R. R. Commission says (Wis. R. C. R., Vol. 2, p. 5):

"Duplication of such plants is a waste of capital whenever service can be adequately furnished by one plant. . . . Competition in this service therefore usually means a bitter struggle and low rates, until one of the contestants is forced out of the field, when the rates are raised to the old level, if not above it, or to a combination of some kind between them which also ultimately results in higher rates."

The Commission in the case of the City of Racine vs. Racine Gas Light Co. (Wis. R. C. R., Vol. 6, p. 286) calls attention to the fact that the investment in physical properties is somewhat greater than the amount ordinarily needed. The investment was made to meet anticipated needs. Latitude must be given in such cases to owners but it is questioned whether the return allowed on the investment should be at such high rates as prevail in other cities of the same size.

Referring to non-operating property the Commission in the LaCrosse Gas and Electric Co. case (Wis. R. C. R., Vol. 8, p. 164) expresses the view that the value of such property can be made a part of the value used in fixing rates only when the income therefrom is added to actual income or deducted from operating expenses, even though future use of the same is anticipated.

Discarded Property.—In Wisconsin it is required by law that discarded property must be left out of consideration in making appraisals of value as a basis for fixing rates. However there are nevertheless cases in which equipment, not actively a part of the plant, but held in reserve for emergency use, may properly be included in the base value (Wis. R. C. R., Vol. 5, p. 24).

Depreciation, Amortization and the Replacement Requirement

Depreciation. — Depreciation is a lessening of worth with age which deserves special consideration in any discussion of rate regulation. The matter of amortization and the making of provision for renewals or replacements is so intimately inter-

woven with the depreciation question, as ordinarily handled, that no attempt will be made to restrict the discussion to depreciation. Information bearing upon amortization and the replacement requirement will here be found intermingled with the general discussion of the depreciation question.

Assumptions Made in the Discussion. — The general presentation of the problems involved in determining what should be the fair earnings of a public utility will be simplified by assuming that the actual useful life or term of service of each of its parts will conform with the probable life term predicted for these parts. This is not in reality the case. In the preliminary presentation of the subject, however, this assumption has been strictly adhered to. How the departure of the actual term of usefulness from the probable term will affect the computation of the annual replacement requirement will be considered later.

It has been found convenient to use an interest rate of 6 per cent throughout this volume for purposes of illustration and this rate is to be understood when no other rate is mentioned.

Hypothetical Case; 20 Year Life, 10 Years Old. — Take the case of a plant, all parts of which have a life of 20 years, all constructed at one time and owned by a prudent owner who sets apart at 6 per cent interest, as an amortization fund, each year 0.027185 for every dollar invested therein. If the plant is one which will actually net 6 per cent on the invested capital, then the apparent excess of the annual earnings, over expenses, should be 6 + 2.72 = 8.72 continuously during the life of the plant, and the owner, in estimating the price at which he can sell it without loss at the end of any period, as, for example, at the end of 10 years, would figure as follows (for each \$100 of original investment):

Investment (original)	\$100 00				
In the amortization fund: being the amount of a 10-year an-					
nuity of \$2.7185 at 6 per cent interest					
Remaining value	\$ 64.17				

A prospective purchaser would figure that the plant should be worth at least as much as the present value of \$8.72 per annum treated as an annuity for the remaining 10 years, which, at 6 per cent per annum, is \$64.17.

At the end of 10 years, the original owner, keeping for his own use the money in the replacement fund, will be satisfied to sell at \$64.17. The purchaser, content in this case with the assumed rate of interest of 6 per cent, will be willing to pay \$64.17, because at the end of the plant's useful life, he will have recovered his investment with 6 per cent interest compounded annually. He will then be under the same necessity of replacing the plant, making a new investment of \$100, as the original owner would have been if he had remained in possession.

During the entire 20 years of usefulness the plant has been rendering adequate service. The efficiency of the service is independent of, and bears no relation to, the useful life of the plant, nor to the fact that some or all of its parts were gradually deteriorating.

Interchange of Terms — Depreciation, Amortization and Replacement. — It cannot be known just how, nor at what rate, the actual deterioration of a plant takes place. This may be rapid at some period of its life, and slow at another, but, as the plant is supposed, at all times during its life, to be adequately performing the service expected of it, variations in this rate of deterioration are immaterial. In other words, the amortization of capital is a question which may be considered without regard to the physical condition of a plant at any period of its life. Nevertheless many engineers and economists have found it convenient to consider the actual, or the theoretical accumulation in an amortization fund as the measure of plant depreciation with a consequent interchange of terms. The term "depreciation" is frequently used when the term "amortization" would be more appropriate.

There is a clear distinction between amortization and replacement. The amortization deals with the retirement of the invested capital. This may be in installments in uniform or in unequal annual amounts, or in a lump sum at the end of useful life. The replacement may mean the substitution of a new

identical plant, but at a cost dependent on new conditions, new prices of labor and material, or it may mean the substitution of new devices rendering equivalent service. In either event the replacement may be at a greater or less cost than the original cost, with, therefore, a corresponding increase or decrease of capital invested. Expenditures for new parts of a plant, which take the place of old parts which are retired for any cause, should be charged to replacement only to the extent of capital represented by the part of the plant thus retired. Any excess of the expenditure for replacement over the cost of the discarded part of a plant should be treated as an addition to, and any less cost as a deduction from, the invested capital. The term "replacement" should not be used in the sense of retirement of invested capital, which deals with the cost of the replaced part and not with the cost of the new equivalent installation. Theoretically, the amount which should go into an amortization fund should be estimated on the basis of invested capital, or cost, and not on the cost of replacement.

In the case of the supposed valuation by a seller and by a purchaser of a plant with a 20-year useful life, at the end of a 10-year period, there is no need of assuming that an amortization fund has actually been created. The amortization annuity, instead of actually appearing in a fund, may be otherwise invested.

Example of Insufficient Amortization. — When the owner of a steamboat which has a limited life and which is yielding 6 per cent per annum of its cost and nothing for amortization, sets apart, out of the 6 per cent, an annual amount, also bearing interest at 6 per cent, to meet its replacement at the end of the steamboat's life, he will have invested not only the original cost of the steamboat, no part of which comes back to him in the annual 6 per cent return, but also a gradually increasing sum which in the life of the steamboat will become adequate to replace it. At the end of the steamboat's usefulness, after replacing it with a new one, the total original investment will be doubled without any increase of earning capacity, and the owner will have, in effect, lost his original investment.

It follows from this that a return of 6 per cent per annum, without anything for amortization, or for replacement, on an investment in a perishable article, when money is worth 6 per cent, is inadequate. The excess of earnings over expenditures must be at least equal to the current interest rate on safe money investments plus an increment depending on the useful life of the plant. This increment must be such that, within the life of the plant, it will either return to the owner his original investment or will be adequate to replace the article in service with a new one.

Had the owner borrowed money for the acquisition of the article, and were he paying interest on the borrowed money at 6 per cent, this fact would be self-evident. The 6 per cent earnings would then be required to meet interest payments, and, at the time when the article has reached the end of its life and must be replaced with a new one, he would find himself, not only in debt for the original article but would have to duplicate the indebtedness to make the replacement.

Amortization and the Value of Stock.—The amortization increment is ordinarily expected to appear in the earnings as that sum which, at compound interest during the life of the article, will be adequate to retire the original investment.

To illustrate these points further, let it be supposed that ownership is represented by capital stock of a corporation. If a plant owned by the corporation and built with funds contributed by the stockholders earns just enough to net 6 per cent without any allowance for amortization, the stock which at the outset may have been worth 100 per cent will gradually decrease in value until, at the end of the plant's usefulness, it will be worth nothing.

The situation is quite different when the earnings net 6 per cent plus an annual amortization increment here supposed to be paid into a special fund. In this case, the stockholder receives 6 per cent each year, and the amortization grows while the plant depreciates in value. The stock, if fully paid up, will be at par from the beginning to the end of the plant's usefulness,

and the money in the fund at the end of the period is available either for distribution to the stockholders, being a return of the money advanced by them or it is available for reinvestment in a new plant to replace the original one. Should a sale be made at any time while the plant is in service, with due allowance for its depreciation (offset by the amortization fund) and, this value being recognized by a purchaser and the price paid, there would again be 100 per cent available for distribution to the stockholders, the deficiency of the selling price being made up by the accumulation in the amortization fund.

In the case of net earnings amounting to less than interest on the invested capital plus current depreciation, the valuation of the plant by a purchaser would be at all times less than the value determined by deducting accrued depreciation from cost. In the case of earnings amounting to a proper interest return on the investment plus an adequate allowance for amortization or for replacement the valuation would be, as already explained, capital invested (or the replacement cost) less depreciation.

The Use of an Amortization Fund. — Theoretically, then, a part of the earnings each year may be placed in an amortization fund as a repayment of capital invested, and this fund may be used for the replacement of the parts of the plant as they go out of service or of the entire plant when it has reached the end of its life.

The accumulation of an amortization fund for such use, however, while theoretically sound policy, is a measure not always adopted in actual practice, particularly when the properties owned are of a complex character — when they are made up of numerous parts of various periods of probable usefulness. Municipalities, State and National Governments, do not set apart funds for the replacement of worn-out or antiquated buildings, parts of water-works, street pavements, sewers, and the like, until the replacement is necessary. They do not maintain funds at interest representing accrued depreciation out of which to reconstruct their public works. The sinking fund required to retire bonds which may have been issued to construct these

works originally must not be confounded with a replacement fund. The one may be necessary to pay for the works in the first instance, the other to maintain them for all time. The annual contribution to the sinking fund is a partial payment for the original work. The contribution to a replacement fund, in the case of a plant which is to serve without time limit, is for the purpose of perpetuating the work, because in that case the replacement fund, as far as it will go or as far as it is required, will be used for making replacements.

The Wisconsin R. R. Commission in the case of the Superior Commercial Club vs. Duluth Street Railway Co. (Wis. R. C. R., Vol. 11, pp 1 to 21) elsewhere quoted makes clear the distinction between depreciation and amortization. Referring to depreciation the Commission in the matter of the Fennimore Mutual Water & Light Plant, in 1913 (Wis. R. C. R., Vol. 12, p. 209), warns against the confusion of the depreciation fund with the depreciation reserve. The "fund" is actually created by setting a part of the income aside. The "reserve" is merely a book account which designates the amount and character of various transactions bearing upon depreciation and replacement expenditures.

Though it may be difficult to make satisfactory forecasts with reference to necessary reinvestments to replace discarded parts of a plant, the requirements for amortization, being based on cost, are usually readily determinable with some degree of precision.

Application to Complex Plants. — Thus far, the plant is assumed to have been constructed and put into use all at once, and is of such a character that all its parts have the same life. The same principles will apply when a plant is made up of many elements or parts having various periods of usefulness. The amortization or the replacement annuity is, in such case, determined for each part and from the sum of the annuities thus ascertained the minimum earnings which will prevent loss are determined.

Mathematical Determination of the Replacement Fund.— The following problem presents itself: In the case of a plant of gradual development but of full growth and mature age, composed of numerous units, the useful life of all the units or parts of which is n years, it is desired to know what amount is in the replacement fund at any time, that fund being assumed to receive such an increment at the end of each year that, during the life of each unit, this annuity, with interest, will amount to the original cost of this unit.

Being composed of a large number of elements — each year having added new ones — the addition to it per year will be taken for the purpose of this illustration at one-nth of the total plant as it stands at the end of the nth year.

For each dollar invested on this assumption in the first year, there will be \$r\$ invested in each succeeding year, and for each dollar thus invested there will be n dollars of total investment.

Let a represent the annual contribution to the replacement fund for each dollar invested.

Assume this contribution to be available at the end of each year.

Then after n years, na will be the annual contribution to the amortization fund for each dollar of the annual investment.

Let m represent any number of years greater than n.

Let i represent the interest rate expressed in hundredths, *i.e.*, for 6 per cent, i = 0.06.

During the first n years, after beginning the construction of the plant, there will be no replacements, and the replacement fund continues to grow. At the end of the nth year the replacement requirement, assuming permanency in character and cost, will be \$i\$ for each dollar of annual investment, and this replacement requirement will continue at this rate thereafter.

At the end of the *n*th year the replacement fund will contain: For each dollar invested the first year:

$$a (\mathbf{i} + i)^{n-1} + a (\mathbf{i} + i)^{n-2} + \cdots + a \text{ dollars}$$
or
$$\frac{a}{i} [(\mathbf{i} + i)^n - \mathbf{i}] \text{ dollars}.$$

or

For each dollar invested the second year:

$$a (\mathbf{1} + i)^{n-2} + a (\mathbf{1} + i)^{n-3} + \cdots + a$$
 dollars
$$\frac{a}{i} [(\mathbf{1} + i)^{n-1} - \mathbf{1}]$$
 dollars.

For each dollar invested the nth year: a dollars.

Therefore the total amount S_n in the replacement fund at the end of the *n*th year, after deducting the \$1 replacement requirement of that year:

$$S_n = \frac{a}{i}[(1+i)^n - 1 + (1+i)^{n-1} - 1 + \cdots + (1+i) - 1] - 1. (1)$$

$$S_n = \frac{a}{i} \left\{ \frac{\mathbf{I}}{i} [(\mathbf{I} + i)^{n+1} - (\mathbf{I} + i)] - n \right\} - \mathbf{I}.$$
 (2)

$$S_n = \frac{a}{i^2} [(\mathbf{1} + i)^{n+1} - (\mathbf{1} + i) - ni] - \mathbf{1}.$$
 (3)

There will be in the replacement fund for each dollar annually invested:

At the end of the (n + 1)st year:

$$S_{n+1} = S_n (r + i) + na - r.$$
 (4)

At the end of the (n + 2)d year:

$$S_{n+2} = S_n (\mathbf{1} + i)^2 + (na - \mathbf{1}) (\mathbf{1} + i) + na - \mathbf{1}.$$
 (5)

At the end of the (n + 3)d year:

$$S_{n+3} = S_n (\mathbf{I} + i)^3 + (na - \mathbf{I}) (\mathbf{I} + i)^2 + (na - \mathbf{I}) (\mathbf{I} + i) + na - \mathbf{I},$$
 (6)

and so on; and at the end of the mth year:

$$S_m = S_n (\mathbf{I} + i)^{m-n} + (na - \mathbf{I}) (\mathbf{I} + i)^{m-n-1} + (na - \mathbf{I}) (\mathbf{I} + i)^{m-n-2} + \cdots + na - \mathbf{I}.$$
 (7)

Substituting the value of S_n and summarizing the series:

$$S_{m} = \frac{d}{i^{2}} [(\mathbf{I} + i)^{m+1} - (\mathbf{I} + i)^{m-n+1} - ni (\mathbf{I} + i)^{m-n}] - (\mathbf{I} + i)^{m-n} + \frac{\mathbf{I}}{i} [(\mathbf{I} + i)^{m-n} - \mathbf{I}] (na - \mathbf{I}), \quad (8)$$

which may be reduced to

$$S_{m} = \frac{a}{i^{2}} [(1+i)^{m+1} - (1+i)^{m-n+1}] - \frac{1}{i} [(1+i)^{m-n+1} + na - 1]. \quad (9)$$

For the interest rate of 6 per cent: i = 0.06 and

$$S_m = \frac{a}{0.0036} \left[1.06^{m+1} - 1.06^{m-n+1} \right] - \frac{1}{0.06} \left(1.06^{m-n+1} + na - 1 \right).$$
 (10)

For m = n and i = 0.06

$$S_n = \frac{a}{0.0036} (1.06^{n+1} - 1.06) - \frac{1}{0.06} (0.06 + na)$$
 (11)

or
$$S_n = \frac{a}{\cos 36} (1.66^{n+1} - 1.66) - \frac{na}{\cos 6} - 1.$$
 (12)

If now the total amount in the replacement fund be compared with the total investment which, on the assumption made, will be n dollars for each dollar of annual investment, the relation between the replacement and the investment expressed in percentage and called R will be found to be:

$$R_{m} = \frac{100 a}{ni^{2}} [(1+i)^{m+1} - (1+i)^{m-n+1}] - \frac{100}{ni} [(1+i)^{m-n+1} + na - 1]$$
(13)

and for the interest rate of 6 per cent or i = 0.06,

$$R_{m} = \frac{100 a}{0.0036 n} (1.06^{m+1} - 1.06^{m-n+1}) - \frac{100}{0.06 n} (1.06^{m-n+1} + na - 1)$$
(14)

and for m = n and i = 0.06,

$$R_n = \frac{100 a}{0.0036 n} (1.06^{n+1} - 1.06) - \frac{100 a}{0.06} - \frac{100}{n}$$
 (15)

or
$$R_n = 27,778 \frac{a}{n} (1.06^{n+1} - 1.06) - 1667 a - \frac{100}{n}$$
 (16)

Table 2 is based on the foregoing formulæ. For comparison two rates of interest 6 per cent and 4 per cent per annum have been used in estimating the amount which should be in the replacement fund.

TABLE 2. THE REPLACEMENT FUND

For plants made up of numerous parts, all having the same probable life new, all serving their full probable term of usefulness and no longer. The plants are assumed to have an age equal to or greater than the number of years in the probable life term of their parts. The total invested capital in each case will be n dollars for each dollar of annual investment, when n represents the number of years in the probable life term

	Amount in the replacement fund expressed in percentage of the total investment								
At the end of the year	Prob life 5 years		Prob life 10 years		Prob life 20 years		Prob life 40 years		
	4 per cent	6 per cent.	4 per cent	6 per cent	4 per cent	6 per cent.	4 per cent.	6 per cent	
5 10 15 20 30 40 60 80	38 4 38 6 38 4 38 4	37.6 37.7 37.6 37.8	41 7 41 8 41 8 41 8	40 2 40 2 40 0 40 3	41 1 41 0 41 0 41 1 41 1	38 0 37 9 37 9 38 0	36 I 36 O 36 I	 31 3 30 9 30 9	

In the foregoing mathematical analysis, a plant has been assumed which has reached its full growth and which has an age equal to or greater than the number of years in the useful life term of its parts Moreover it has been assumed that all of its parts have the same probable life when new.

The same formulæ will apply to any number of articles of the same probable life installed at a uniform rate per year, even when the plant of which they form a part is still being extended, because in this case the articles may be separated into two groups, one being composed of all articles n years of age and less, which have not yet been replaced and the other group of those articles which have replaced discarded articles. To each of these groups taken separately the above formulæ apply.

It is noteworthy in the assumed case of a plant which has attained full growth and is made up of numerous parts that, when the replacement requirement is computed from the beginning by the compound interest method, the amount in the replacement fund should theoretically vary between comparatively narrow limits; at 6 per cent interest from 31 to 40 per cent for life terms ranging from 5 to 40 years. But in reality there can never be absolute agreement between the actual useful life and the probable life of all parts of the plant. The formulæ noted in this chapter are not therefore strictly applicable. They are nevertheless valuable in illustrating a principle.

Application of Earnings to Replacement and Amortization.— The demands upon the replacement fund usually begin long before the end of the probable life term is reached and may be quite irregular in amount. The non-existence of a replacement fund in the full amount indicated by mathematical and theoretical consideration does not, therefore, prove that the deficiency has been distributed as profit, nor yet that there has been any waiver of the right to have the earnings cover a fair replacement increment.

Furthermore, if the earned annual replacement increment be treated as amortization of capital and be immediately applied for this purpose, it will thereby be removed from all further consideration. The interest on any increment thus applied is not available to retire more capital. Treated as an annuity and remaining in the business, interest may be compounded so long as the fund is held for its intended purpose, that is, for retirement of capital at the end of the useful life of the item which is being retired. Interest ceases to accumulate the moment the fund is applied to retire the investment in whole or in part. Consequently, if a uniform annual amortization increment bearing interest compounded annually be determined from amortization tables based on the probable life of a new article and if it be covered by the earnings from year to year, even though the amortization increment as earned be reinvested in the property, it cannot rightfully be classed as a repayment of

invested capital until the end of the probable life term. If the fund resulting from the accumulation of such increments be applied at any earlier date, a new amortization annuity, based on the remaining value and the remaining life, must be computed.

Incomplete Amortization. — It will be seen that if an amortization annuity thus determined from probable life when an article goes into service be deducted from the investment from year to year, the result will be incomplete amortization. In the case of an article with a probable life of forty years, the amortization rate thus computed at 6 per cent interest would be 0.6462 per cent per annum. The amount of capital returned in forty years would be \$25.85 on each \$100 of capital invested and there would remain \$74.15 still to be made good at that time.

These facts make clear the point that, whenever amortization in lieu of replacement is accomplished by annuities bearing compound interest, the appraisal for rate-fixing purposes must be of the entire investment without deduction of accrued depreciation.

Second Mathematical Determination of the Replacement Fund. — The foregoing mathematical demonstration that the accumulation in a replacement fund for a plant of mature age, when computed by the compound interest sinking fund method, and actually earned should amount to a considerable sum, confirms a conclusion which can be reached in a more direct way.

In the assumed case of a plant which has a life of n years, and of which one-nth has been constructed each year, after n years there will have to be replaced one-nth thereof each year. Because the annual investment in the installation has been uniform there will be, for each dollar invested per year, a total investment of n dollars.

The annual replacement after n years, for each dollar annually invested, will be \$1. If now the annuity to replace the several parts of the plant in n years is a dollars for each dollar of the annual investment, then after n years the annual amount re-

ceived as annuity will be na, and this will fall short of meeting the actual expenditures by an amount expressed by $(\mathbf{r} - na)$ which, at 6 per cent per annum, is the interest on $\frac{100}{0.06}$

dollars; or, expressed in percentage of the cost, is $\frac{100^2 (1 - na)}{6 n}$ per cent of the total investment in the plant.

For a plant not subject to further growth, with a uniform useful life of all its parts, and constructed progressively, there should be, at 6 per cent interest, an unexpended interest-bearing balance in the replacement fund as follows:

When the useful life is five years:

$$\frac{100^2 (1 - 0.8870)}{30} = 37.7 \text{ per cent of the total investment.}$$

When the useful life is ten years:

$$\frac{100^{2} (1 - 0.7587)}{60} = 40.2 \text{ per cent of the total investment.}$$

When the useful life is twenty years:

$$\frac{100^2 (1 - 0.5437)}{120}$$
 = 38.0 per cent of the total investment.

When the useful life is forty years:

$$\frac{100^2 (1 - 0.2585)}{240} = 30.9 \text{ per cent of the total investment.}$$

If earnings have been adequate to provide an interest-bearing replacement fund, then these percentages represent the probable accumulation in such a fund.

Some amount such as shown by these figures, depending on the expectancy, represents the accumulation of replacement annuities during that period of the plant's life during which the actual replacement expenditures were less than the annuity. If the annual allowance for maintenance in the past has been based on the requirements of operation and repair without surplus to meet future replacements and if there has been no special allowance for amortization, the current allowance for amortization and replacements should not be determined by the interest bearing sinking fund method based on original probable life but should be otherwise determined, as hereafter shown.

When, in other words, opportunity is not given to accumulate the 40 per cent of the invested capital (approximately), which, for ordinary periods of useful life of perishable properties, should in the course of time be in a replacement, depreciation or amortization fund, any amount estimated from amortization tables on the original full period of useful life will fall short of the real replacement requirement.

Illustration of the Replacement Requirements. - Let it be assumed that a conduit, such as a cast-iron pipe, used for any purpose, has a length of 40 miles. Let it be also assumed that the pipe is not being further extended, that the life of this pipe is 40 years, no more and no less, and that it was constructed progressively, one mile each year. It took 40 years to install the pipe, and at the end of this time the first mile of pipe laid was ready for replacement - it had served its time. Each year thereafter, one mile of pipe has to be replaced, and the replacement at this rate will continue indefinitely. The annual replacement expenditure during the first 40 years is nothing. but, thereafter, it is the cost of installing one mile of pipe. prices of labor and material have remained constant, and if conditions have otherwise remained as they were when the first mile of pipe was laid, then the annual replacement expenditure will be one-fortieth of the total amount invested in the pipe line.

Provision for this replacement must be made if the pipe is to continue in service. If, now, the extension of the pipe progresses beyond the 40-year period at the same rate, before assumed, of one mile per year, there will be no changes in the annual replacement requirement during a second period of 40 years, but at the end of this second period — at the end of 80 years — there will be 80 miles of pipe in service, and thereafter during the third 40-year period there will have to be replaced annually 2 miles of pipe, or one-fortieth of 80 miles, or twice the amount of pipe extension per annum.

Determination of the Replacement Requirement. — It is possible, by such analysis, when a plant is growing at a fixed rate and has attained an age exceeding the life of its perishable parts, to prescribe a rule for determining the replacement requirement; but it must be remembered that a rule thus determined can be strictly correct only for the impossible hypothetical case of service in exact conformity with the assumed probable life, and that for practical application a rule thus determined may require some modification as explained in Chapter VI.

For each group of parts having the same length of life, there is to be determined: first, the average annual capital invested, using, however, replacement cost instead of the actual investment; and second, the full number of times that the age of the plant is greater than the useful life of the particular group of parts under consideration. The replacement requirement (for the hypothetical case, in which actual service conforms throughout with the assumed probable life) is then ascertained by multiplication.

A pipe line may again serve as an illustration: Suppose it is desired to know the replacement requirement for a pipe line 300 miles long, which has been extended 2 miles each year, the age of the oldest portion of which, therefore, is 150 years.

The life of the pipe being taken at 40 years, the full number of times this is contained in 150 years is three. The annual replacement requirement will be three times two, or 6 miles of pipe.

The 6 miles of pipe requiring replacement were constructed 40 years ago, and the conditions under which this was done may have been materially at variance with those prevailing at the time of their replacement. Consequently, in the determination of the replacement requirement, expressed in dollars instead of in miles of pipe, the replacement cost of the system and not the original cost of capital invested should be taken into account. Expressed as a percentage of the total length of pipe in service, or of the total cost of replacing the entire pipe line, this would be 2 per cent.

By the compound interest annuity method of computation, in the selected illustration at 6 per cent interest, the allowance for replacement would be 0.646 per cent of the cost of the system, which is only one-third of the actual requirement, and this allowance, as already explained, would only then be justified if amortization had covered the entire period in the life of each part of the pipe during which there was no expenditure for replacements, so that the inadequate annual allowance could be supplemented by the earnings of an accumulated replacement fund.

In a plant which is made up of a multiplicity of parts of various periods of usefulness, those which have the same expectancy should, as before stated, be grouped together. For each group, the replacement requirement can then be estimated separately, and from the several amounts thus ascertained the total requirement is determined.

The rule previously laid down for a hypothetical case is not strictly applicable under the conditions as they actually present themselves. There can be no absolute conformity between the assumed period of usefulness of the various parts of a plant and the time during which they actually prove useful.

The probable useful life or expectancy is merely the average life, which is often not reached and is just as often exceeded. Thus, again referring to the pipe line, it is to be assumed that while some of it may serve beyond the average period of usefulness of such pipe, other parts thereof, from one cause or another, will require replacement early in its life. Consequently, any rule such as that previously laid down, which indicates a uniform replacement requirement in successive periods, with a sudden rise in the requirement at the beginning of each new period, if the plant be one that is steadily growing, will require some modification.

The simplest modification of the foregoing rule is to assume gradual changes in the annual replacement requirement as the age of the plant increases, instead of the sudden changes, and then to call this requirement at all times inversely proportional

to the useful life of any group of parts. This is sometimes referred to as the "Straight Line Method." It might with equal propriety be called a direct percentage method, as the inverse ratio is usually expressed in percentage.

Under this direct percentage method, there would be allowed 2.5 per cent per annum of the replacement cost of all parts of a plant having a 40-year life; 3.33 per cent per annum of the replacement cost of all parts having a 30-year life; 5 per cent per annum of the replacement cost of all parts having a 20-year life, and so on.

This method, applied to the hypothetical case of a pipe line, constructed and extended one mile per year, and each mile thereof having a useful life of exactly forty years, would, at the end of the fortieth year, make the replacement requirement 25 per cent per annum, or one mile of pipe. At the end of the sixtieth year, the requirement thus determined would be 2.5 per cent of the 60 miles of pipe then in service, or 1.5 miles of pipe. This would be 50 per cent in excess of the amount actually replaced, which at that time would be only one mile. This would also apply for any time before the pipe first laid has reached the limit of its usefulness, as at 20 years. In the assumed case there is no replacement requirement at 20 years; yet the straight percentage method indicates 2.5 per cent of 20 miles of pipe, or 0.5 mile of pipe. It follows from this illustration that the Straight Line Method would give results somewhat too high.

By further analysis of this problem, the following formulæ have resulted, which are free from this objection and fulfill every ordinary requirement. In devising these formulæ, the fact was taken into account that there may be some replacement requirement in the early years of a plant's life, and that this requirement gradually increases. These formulæ apply only to plants which have been developed gradually and are being extended at a uniform annual rate.

Using the notation already introduced, and designating with C the total cost of replacing the group of items, the probable

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useful life of which, when new, was n years, with c the annual renewal requirement, and with g the average annual investment in extensions, the formulæ are:

For m less than n,

$$c = \frac{mg}{2n} = \frac{C}{2n}. (17)$$

For m greater than n,

$$c = \frac{C}{n} - \frac{g}{2}.$$
 (18)

For very large values of m in relation to n (n being the years of probable usefulness), the value of this expression approaches $\frac{C}{n}$ which is the mathematical equivalent of the Straight Line Method.

However desirable it might otherwise appear to introduce a method of computing the replacement requirement by recourse to amortization tables, to do this satisfactorily, in the case of a complex plant, is usually out of the question, when past earnings have been inadequate to accomplish the desired amortization. In such cases the use of some formula, as above noted, for estimating the probable replacement requirement is to be recommended and its application would be equitable from the standpoints of both the owner and the rate-payer.

The Interest Bearing Annuity and the Replacement Requirement. — When an annuity, bearing interest compounded annually, is allowed to accumulate in a fund to retire invested capital, the demand upon the rate-payer is in annual installments. It would be equally proper to make the demand upon the rate-payer for the replacement of each individualized article, at the full cost of replacement, at the time when the article is discarded. In this case the lump sum cost of replacement, equivalent to the amount of the annuity, will take the place of the installments. If the annuity installments are forthcoming as they are due, then the annuity method is adequate. If the owner of the property does not get them, recourse should be had to the lump

sum allowance due at the time of failure if capital is to be kept unimpaired.

It is perfectly reasonable, moreover, to assume, unless there is evidence to the contrary, that the method of estimating and providing for replacement requirements, which prevails in any case, has been introduced deliberately. The owner of the public service property may be perfectly willing to waive collection of the annuity installments if he knows that what they will amount to, that is, the actual annual replacement, will be covered by the gross earnings when the time comes for discarding parts of his plant. In other words, he may be willing to accept the amount of an annuity in lieu of the annuity itself; and the rate-payer may desire such an arrangement, because, in the early days of the plant's life, he may be unable to pay a sufficient amount for the service to cover the replacement annuity. It must be remembered, however, that such an arrangement burdens the future rate-payer to some extent for the benefit of the rate-payer in the early days of a plant's life. Not more so, however, than when, as is done by some appraisers, early losses are used as a measure of "going value."

It follows directly from the foregoing that even when earnings cover current average annual replacement requirements, the appraisal for rate-fixing purposes may still be the entire investment without deduction for depreciation. This will be the case whenever it can be shown that past earnings were inadequate to permit the accumulation of a fund, out of earnings in excess of reasonable interest on the investment, which, if it existed, would offset in whole or in part the so-called accrued depreciation.

CHAPTER VI

THE EFFECT OF NON-AGREEMENT OF ACTUAL WITH PROBABLE LIFE UPON THE DETERMINATION OF THE DEPRECIATION OR REPLACEMENT REQUIREMENT

Depreciation Estimates are Approximations. — Consideration is now to be given to depreciation as a factor affecting the required earnings. It is not enough to know what the theoretical depreciation will be if estimated from the probable life of any article. Any article in use may be considered as being gradually consumed in the service. When no longer useful, it must be replaced. The replacement requirement therefore must be estimated. To do this properly something more must be known besides the cost, probable life, and age. It will be necessary to take the condition of the article into account — to give consideration in other words to the question of whether it will outlast its probable term of usefulness or not. To disregard this fact results in crude approximation and loose methods of accounting which are undesirable.

Probable Life is Based on Experience. — The sum of all experience, so far as the same has been made a matter of record, fixes the probable life of various classes of articles. Some articles in every class will fail early, others will survive their probable life term. When any system of accounting is adopted under which the capital invested in individualized articles is to be retired during their probable life, some articles will fail before their cost is completely amortized and there will be others continuing to render efficient service after their cost has been completely amortized. The anomaly results of having to carry in the accounts a part of the cost of articles no longer in use but still in the process of amortization and also of having

wiped out the entire cost of others still in service, which judged by the accounts should have no value.

Effect of Expectancy upon Present Value. — In order to weigh the advantages and disadvantages of these methods in their practical application some attention must now be given to the probable life and to the expectancy of the various elements that go to make up a public service property

When it is desired to know the present value of any article, the question is not "how old is it and what did it cost?" but "how much longer will it serve and what will it cost to replace it?". The first question might be asked when the accrued amortization of capital is to be estimated. The second question is to be answered when the current depreciation or the current replacement requirement are to be estimated.

How then shall the expectancy be determined?

All estimates of annual depreciation and of accrued depreciation are based on premises which cannot be determined with accuracy. The probable life of any article when new and the life expectancy of any article which has been in use for some time cannot be determined with any great degree of precision. Consequently estimates of depreciation are only approximations.

Academic Discussion Justified. — There is much uncertainty in such estimates under the ordinary conditions under which public utilities are operated. A wide range in the method of making the estimates has been the result. When, therefore, the correctness of methods is under discussion, this difficulty of making close estimates should not be lost sight of. Nevertheless, the academic discussion which is being indulged in by the engineer and the economist relating to best and most convenient methods of procedure is justified, because the same will lead to an ultimate standardization of methods and finally to the general adoption of the most convenient and generally best method, fair to both the rate-payer and the owner.

Although this limitation be recognized as well as the uncertainties that result from imperfect knowledge relating to the actual and to the probable life of the elements of any public

service plant, and to the difficulty of determining the expectancy of those articles which have already been in use for some time, it is nevertheless important that the whole question be fully considered in order that a framework may be constructed into which the best data furnished by experience can be fitted.

The non-agreement of actual life of individual items with their probable life and the extent to which this lack of agreement should be taken into account in estimating present worth and in estimating replacement requirements, in line with this thought, has been studied on various assumptions with interesting results. These will be briefly referred to, and the resulting tables are presented for use until, in the light of larger experience, they can be replaced with better ones.

Assumptions Relating to Departure of Actual from Probable Life. — Unfortunately there are no records available from which absolutely dependable tables of expectancy could be prepared for each class of perishable articles in use in connection with public service properties, such as have been prepared by actuaries for human beings. Any assumption in this regard is more or less conjecture. Nevertheless, it is interesting to note what the expectancy would be at various ages, if certain definite reasonable assumptions are made.

When any large number of articles which have the same probable life, as, for example, ten years, is under consideration, there will be as many service years in the aggregate, represented by the failures to reach the probable life term of ten years, as there will be service years represented by those articles which outlast the ten-year term. It may also be accepted as a certainty that there will be a greater number of articles per year to go out of use in the years just preceding and just following the term limit than at any other time. This suggests conformity with the law of probabilities.

All articles in a group having a ten-year probable life might fail (a) exactly at the end of ten years, but this is highly improbable. The individual articles might fail (b) at a uniform rate

per year, one-twentieth each year, the last going out of service in the twentieth year. Or (c) there might be no failures at all for a number of years, as, for example, during the first half of the probable life term, and thereafter a uniform or an irregular rate of failures until some time after the end of the probable life term. Or (d) there might be a gradually increasing number of failures per year from the beginning to the end of the term of the probable life, and thereafter a gradually decreasing number of failures.

Of these various possible distributions of failures to a series of years the most probable one is unquestionably (d). For the sake of a definite basis for calculation it has been assumed that the increase in the number of annual failures up to the maximum and thereafter the decrease in the number of annual failures is uniform, that annual increase before the maximum and annual decrease thereafter are the same and that the maximum rate is one-tenth of the whole number and occurs in the tenth year. On these assumptions studies have been made to determine the expectancy of any article which has reached any age with rather interesting results. These are presented not only as an improvement in the method of estimating probable remaining life or expectancy of any article which is no longer new, though still in good condition, but also to encourage further study along these lines, in order that, where necessary, closer approximation of the actual accrued and annual depreciation can be made than has heretofore been attempted.

It should be stated that other hypotheses in the matter of the rate of failures were tried, but that no other gave results that appeared as reasonable as those based on the hypothesis as just stated. Thus, for example, the assumption was tried that $\frac{1}{2}$ per cent of all articles of ten-year life fail in the first year, $1\frac{1}{2}$ per cent in the second year, and so on, $9\frac{1}{2}$ per cent in the tenth, $9\frac{1}{2}$ per cent in the eleventh, $8\frac{1}{2}$ per cent in the twelfth, etc., and $\frac{1}{2}$ per cent in the twentieth, but with less satisfactory results for application in estimating remaining value and replacement requirements.

And, again, the failures were estimated for each year on the assumption that they will occur according to the law of probabilities, coupled with the assumption that all failures will occur within a period twice as long as the probable life term. The results on this assumption will be again referred to. In so far as the results relating to replacement requirements obtained by the law of probabilities is concerned, it may be stated that these, while not at great variance with the adopted hypothesis, were yet too irregular in amount to warrant the use of the law of probabilities in place of the other more readily applied hypothesis. Either hypothesis of rates of failure may be used as a fair basis for approximation to actual conditions. There has appeared no good reason, therefore, for confining the study to the law of probabilities which after all would have to be applied with some such assumptions as made, of practically no survival beyond twice the probable life term and 50 per cent of all failures within a period equal to four-tenths of the probable life term, or with other assumptions which would also be more or less of a conjectural nature.

Tabular Illustration of Expectancy for 10,000 Articles. — In order to further consider the question of the expectancy of any article which is no longer new, let it be assumed that 10,000 articles are installed at the same time and that all of these have the same probable life. Let the probable life term be divided into ten periods. Then according to the hypothesis relating to the annual failures, as already stated, there will be 100 of these articles going out of use during the first period; 200 in the second period; and so on to 1000 in the tenth period; thereafter 900 in the eleventh period; 800 in the twelfth period, etc., and 100 in the nineteenth period.

On this hypothesis results are readily found as shown in Table 3, in which years may be regarded as representing periods.

TABLE 3 EXPECTANCY

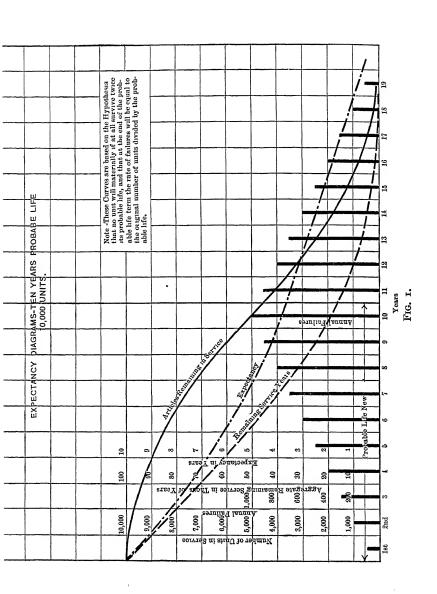
The probable life of each article is 10 years or periods. For terms other than 10 years, each year in the table may be regarded as a period equal to one-tenth of the probable life term

(Based on the special hypothesis of failures as e 'plained in the text)

		For 10,000 articles		Single article
Year or period	Number of failures	Remaining num- ber of articles at beginning of year	Remaining service years at begin- ning of year	Expectancy at beginning of year or period
I	100	10,000	100,000	10 0
2	200	9,900	90,000	9.09
3	300	9,700	80,100	8.27
3 4 5 6	400	9,400	70,400	7.46
5	500	9,000	61,000	6 77
	600	8,500	52,000	6 12
7 8	700	7,900	43,500	5.5I
	800	7,200	35,600	4 95
9	900	6,400	28,400	4 44
10	1000	5,500	22,000	4 00
II	900	4,500	16,500	3 67
12	800	3,600	12,000	3 33
13	700	2,800	8,400	3 00
14	600	2,100	5,600	2 67
15 16 •	500	1,500	3,500	2 33
	400	1,000	2,000	2 00
17	300	600	1,000	1 67
l l	200	300	400	I 33
19 20	100	100	100	1 00
20	0	0	0	0

Merit of the Assumed Law of Failures. — Although, under the hypothesis of failures on which Table 3 is based, there may still be considerable departure from the actual number of failures in any year, there can be no question that this hypothesis is, as already stated, a much nearer approach to the truth than the other hypothesis heretofore generally accepted as a basis for calculation, that each article will fail theoretically at exactly the end of its probable life term.

The last column of Table 3 shows that on the hypothesis as explained, an article which has a probable life when new of ten years will, if it is still in service and in good condition at the beginning of the tenth year, have an expectancy of four years and at the beginning of the fifteenth year, its expectancy will still be 2.33 years.



If the reasonableness of the assumption on which Table 3 is based be admitted, or if it should be possible to prove by actual records of failures that these assumptions are near enough to the truth to be accepted as giving results substantially correct, then a further analysis will show that the actual replacement requirements under various conditions of investment will be as shown in Tables 4 to 6. In the preparation of these tables account has been taken of the failures that will occur among the replacements as well as among the units of the original installations.

Diagrammatic Illustration of the Assumed Rate of Failures.— The basis for the results in Table 3 for articles with a probable life of ten years is shown diagrammatically in Fig. 1. The expectancy is found by dividing the remaining service years at any time by the corresponding number of surviving units. The reversed curve marked "Articles remaining in service" clearly indicates the hypothesis of failures on which the table is based. It is to be noted that under this hypothesis there is no serious departure from the results that were obtained by assuming that the law of probabilities would apply.

Tabular Presentation of Replacement Requirements for Groups of Articles. — The replacement requirements, as shown in Table 5, for numerous articles which when new have a probable life of ten years, if failures occur substantially as assumed, and if each failing article be at once replaced, would increase from \$1 in the first year to about \$10 in the ninth year for \$100 of original investment, fluctuating thereafter between \$9 and nearly \$12 per year and gradually settling down to \$10 per year. For an annual investment of \$100 per year (i.e., for a growing plant), the replacement requirements would gradually increase from \$1 per year in the first year to \$463 in the fiftieth year, or from \$1 per \$100 of investment in the first year to \$6.01 in the tenth year to \$8.16 in the twentieth and to \$9.27 in the fiftieth year.

In practical application, in other words, the annual replacement requirement in the case of a plant of full growth all parts

of which have a probable life of n years, after the plant is n years older than any of these parts, will be about one nth of their replacement cost.

In a plant which continues to grow, the theoretical annual replacement requirement will gradually approach but can never quite reach one *n*th of the total replacement cost. (See Tables 4 to 6.)

TABLE 4 REPLACEMENT REQUIREMENTS NUMEROUS ARTICLES. PROBABLE LIFE 5 YEARS

Each article is replaced as it goes out of use For an original investment of \$100 with no betterments or additions. Also for an investment growing at the uniform rate of \$100 per year.

(For the special hypothesis as stated in the text)

\$ 4 00 8 16 12 65 17 64	Replacements per year \$ 4.00 12.16	Replacements per \$100 of investment
8 16 12 65	12 16	
23 34 21 97 21 16 20 54 19 73 18 31 19 73 20 37 20 45 20 21 19 90 19 87 19 91 20 03 20 06 20 02 20 00 20 00 20 00	24 81 42 45 65 78 87 75 108 91 129 45 149 18 167 49 187 22 207 60 228 05 248 25 268 16 288 03 307 94 327 92 347 95 368 00 388 02 408 00 448 00	6 08 8 27 10 13 16 14 62 15 56 16 18 16 55 17 30 17 54 17 788 18 00 18 11 18 22 18 30 18 18 18 48 18 55 18 61 18 .72
	19 73 20 37 20 45 20 21 19 90 19 87 19 91 20 03 20 06 20 .02 20 00 20 00	19 73 20 37 20 76 20 45 20 27 60 228 05 20 21 248 25 19 90 268 16 19 87 288 03 19 91 307 94 20 03 327 92 20 06 347 95 20 02 368 00 20 00 388 02 20 00 408 00 20 00 428 00 20 00 448 00

TABLE 5. REPLACEMENT REQUIREMENTS

NUMEROUS ARTICLES. PROBABLE LIFE 10 YEARS

Each article is replaced as it goes out of use. For an original investment of \$100 with no betterments or additions. Also for an investment growing at the uniform rate of \$100 per year.

(For the special hypothesis as stated in the text)

Year	Plant of full growth, original investment \$100	Growing plant, annu	ual investment \$100.
	Replacements per year	Replacements per year	Replacements per \$100 of investment.
I	\$ 1 00	\$ 100	\$1 00
2	2 01	3 OI	1 51
3 4 5 6	3 04	6 05	2 02
4	4 10	10 15	2 54
ž	5 20 6 36	15 35	3 07 3 62
-		21 71 29 28	3 62 4 18
7 8	7 57 8 8 ₇	38 15	4 77
9	10 25	48 39	
10	11 73	60 12	5 38 6 oi
11	11 33	71 46	6 50
12	11 03	82 48	6 87
13	10 79	93 27	7.18
, 14	10 60	103 88	7 42
15	10 46	114 34	7 63
16	10 28	124 61	7 80
17	10 09	134.70	7 93
18	9 85	144.55	8 08
19	9.53	154 08	8.12
20	9 10	163.18	8.16
21	9 53	172.72	8.23
22	9 84	182 56	8.30
23	10 05	192 60	8.37
24	10 17	202.77	8 45
25	10 22	212 99	8 52
26	10 22	223.21	8.59
27	10 17	233.38	8 64
28	10 11	243 49	8 69
29	10 03	253 52	8.74 8.78
30	9 97	263 49	
35	9 99	313 31	8 95
40	10 01	363 40 413 40	9 09 9.19
45	10.00	463.40	9.19
50	10 00	403.40	7 -1

TABLE 6. REPLACEMENT REQUIREMENTS

NUMEROUS ARTICLES. PROBABLE LIFE 20 YEARS

Each article is replaced as it goes out of use. For an original investment of \$100 with no betterments or additions. Also for an investment growing at the uniform rate of \$100 per year.

(For the special hypothesis as stated in the text)

	(For the special hypoth	esis as stated in the text)
Year.	Plant of full growth, original investment \$100	Growing plant, ann	ual investment \$100.
	Replacements per year.	Replacements per year	Replacements per \$100 of investment
1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 4 5 6 7 8 9 0 1 1 2 3 4 4 5 6 7 8 9 0 1 1 2 3 4 4 5 6 7 8 2 2 2 2 2 2 2 2 2 2 2 2 3 3 5 0 5 5 6 6 5 7 0	\$\\ 0 \cdot 0 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 3 \cdot 6 \cdot 9 \cdot 9 \cdot 9 \cdot 5	\$ 0 25 0 75 1 50 2 51 3 77 5 29 7 08 9 14 11 40 11 40 23 66 20 17 23 66 31 58 36 07 24 46 31 58 36 07 27 46 31 58 32 07 45 87 51 38 57 9 51 17 96 03 101 34 106 18 107 22 210 54 226 00 261 20 228 00 228 10	\$0 38 0 50 38 0 50 0 63 89 1 1 1 55 1 1 2 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 1 2
	5.00	311.04	4 45

TABLE 7 THE REPLACEMENT REQUIREMENT IN THE CASE OF NUMEROUS ARTICLES AND A COMPARISON OF METHODS OF PROCEDURE

PROBABLE LIFE 10 YEARS

With due regard to expectancy at various ages. Numerous articles. All installed at same time No additions to plant. Articles replaced as worn out. Original investment \$100. Interest 6 per cent. Net earnings 6 per cent.

(Based on the hypothesis of failures as explained in the text.)

	Prob-	Straigh	nt Line N	lethod		Equal	Ann Me	ual F thod	aym	ent	Unlımı	ted Life l	Method.
Year	able repl't requ't	Rem'g value	Amorti- zation allow- ance	Req'earn-		Rem'g value.	za ali	orti- tion ow- ice.	Rec ear ing	n-	Rem'g invest- ment.	Repl't allow- ance.	Requ'd earn- ings.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 6 17 18 19 20 1 22 23 24 25 6	\$1 00 2 01 3 04 4 10 5 20 6 36 7 57 8 87 10 25 11 73 11 33 11 03 10 46 10 28 10 05 9 85 9 53 9 85 10 17 10 22	6 55566666665554333333666666666666666666	\$10 10 10 10 10 10 10 10 10 10 10 10 10 1	15 4 4 5 14 5 14 5 14 5 14 5 14 5 14 5	16 18 18 18 18 18 18 18 18 18 18 18 18 18	\$100 93 87 81 76 71 68 66 64 65 66 67 68 69 	\$78889999999999999999999999999999999999	59 01 37 73 03 33 51 63 75 69 63 57 51 45	\$13 13 13 13 13 13 13 13 13 13 13 13 13 1	999999999999999999999999999999999999999	\$100 100 100 100 100 100 100 100 100 100	\$1 00 2 01 3 04 4 10 5 20 6 36 7 57 8 87 10 25 10 26 10 28 10 9 53 10 17 10 22 10 22	\$7 00 8 01 9 04 10 10 11 20 12 36 13 57 14 87 17 73 17 33 17 03 16 79 16 46 16 28 15 85 15 53 15 10 15 58 16 17 16 16 22 16 22 16 22
27	10 17	63	10	13	78		.		13	- 59	100	10 17	16 17
28 29	10 11	1 2	10		78 34			• •	13	59 59		10 11	16 11
30	9 97		10		34		.			.59		9 97	15 97

Departure of Actual Failures from Assumed Laws.—While it may be granted that in the long run the failures of individual articles in any class will follow some definite law (perhaps a law

similar to that which has above been cited as more probable than failure always at the end of the probable life term), the fact remains that in no particular case, no matter how large a plant may be, will there be absolute conformity with any assumed law. Consequently figures determined on the basis of any reasonable hypothesis of failures can be used to prepare smoothed-out curves and from such curves, tables for general use can be prepared. It is enough to know for the present that ordinarily it may be assumed that the replacement requirement of a large number of articles with a probable life of n years should increase progressively year by year to about one nth of the cost of effecting complete replacement of all articles and that this rate of one nth would be reached at about the nth year.

Table 7 has been prepared to show for numerous articles, all of which when new have a probable life of 10 years, the replacement requirements on the assumption that failures actually occur according to the hypothesis already explained, and to show their relation to the remaining value, the current amortization, and the required annual earnings estimated by the Straight Line and by the Equal Annual Payment methods of procedure, also the remaining investment, the replacement allowance, and the required earnings if estimated by the Unlimited Life Method. These methods of procedure, when rates for public utilities are to be fixed, are explained in Chapter IX.

Expectancy of an Equivalent Single Article.—When numerous articles are under consideration, it may be desirable to know the expectancy of an equivalent single article, representing the aggregate of all the separate articles, which may be used to simplify the estimates of accrued and current depreciation. If the Straight Line Method of procedure is adopted and if numerous articles of the same kind and same individual cost are involved, and if it be assumed that actual life in each case will agree with the estimated probable life, then the expectancy of the equivalent single article will be the average of the remaining ages of the individual articles. But under all other methods of

procedure, even with the assumption of agreement between actual and probable life, this will not be the case. A full discussion of this fact would be superfluous, but it is here stated as a caution against a possible erroneous assumption.*

However, in view of the fact that the termination of service or failure of any article does not occur at the exact end of its probable life term, there would be no sense in attempting to establish the remaining probable life for group values unless hard and fast appraisal rules are laid down by those charged with the regulation of public utility rates coupled with adequate assurance that such rules will be adhered to.

The Assumed Hypothesis of Failures.—It may be repeated that the foregoing Tables 4 to 7 are based on the hypothesis that failures of any group of articles of the same probable life will be most numerous at the end of the probable life term; that there will be a gradual uniform increase in the number of annual failures from the beginning to the end of this term; and that but few, if any, of the articles will have a life in excess of double that of the probable life of the article new.

It may be repeated, too, that this hypothesis is not based on adequate experience, that it probably departs further from average results than would be found under strict adherence to the law of probabilities, which is the basis of Table 13, and that it remains subject to modification as experience may determine. Until suitably modified, it offers, however, a better means of approximating remaining value of any article than is afforded when calculations are made from probable life tables without regard to the condition of the article at the time of the valuation.

Failures and Expectancy according to the Law of Probability.— A study has been made, as already stated, to see where the law of probability would lead, and the result of the comparison will be of interest.

^{*} The remaining life of an equivalent single article, when computations are made by the Sinking Fund Compound Interest Methods, is noted in a paper in Transactions American Society of Civil Engineers, Vol. XXV, p. 836, with examples on a 4 per cent interest basis.

Let it again be assumed that practically no article of a large group, all of which have a 10-year probable life, will survive 20 years or twice the probable life term, and that one-half or very nearly one-half of the failures occur within the two years just preceding and the two years just following the end of the probable life term. Then, according to the law of probability, and on the assumption that the failures may be bunched at the end of the successive years, there will be failures in each successive year as shown in Table 8. These are noted only to the nearest 5 in 10,000, and in other respects are offered only as approximations to demonstrate a law rather than the result of accurate computation.

TABLE 8 FAILURES AND EXPECTANCY ACCORDING TO THE LAW OF PROBABILITY

On the Assumption that no Article Survives Twice the Probable Life Term. 10,000 Articles Probable Life = 10 Years

		For 10,000 articles					
Year	Number of failures	Remaining number of articles, beginning of year	Remaining service years, beginning of year	Expectancy at be ginning of year.			
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	15 35 85 180 330 550 805 1065 1265 1340 1265 1065 805 550 330	10,000 9,985 9,950 9,865 9,685 9,355 8,805 8,000 6,935 5,670 4,330 3,065 2,000 1,195 645 315	100,000 90,000 80,015 70,065 60,200 50,515 41,160 32,355 24,355 17,420 11,750 7,420 4,355 2,355 1,160 515	\$10 00 9 00 8 05 7 11 6 22 5 40 4 78 4 04 3 52 3 08 2 71 2 42 2 18 1 97 1 .80 1 64			
17 18 19	85 35 15	135 50	200 65	1 50 1 25 1.00			
20	15	15	15	0			

In addition to the probable failures from year to year, Table 8 also shows the number of original articles in a group of 10,000 that will be still in service in any year; the probable remaining service years in the surviving articles of the original group; and the expectancy of any single article at any age.

TABLE 9 REMAINING VALUE AND REPLACEMENT REQUIREMENT

REMAINING VALUE OF NUMEROUS ARTICLES WHICH HAVE ALL COME INTO USE AT THE SAME TIME AND ALSO THE REPLACEMENT REQUIREMENT REMAINING VALUE OF A SINGLE ARTICLE ORIGINAL INVESTMENT \$100. PROBABLE LIFE TO YEARS

Failures assumed to occur according to the law of probability. 6 per cent interest

		Single article		Numerous articles			
Year.	Expectancy	Remainii	Remaining Riplacem				
	years Begin- ning of year	Straight Line Method Be- ginning of year	Equal Annual Payment Method Be- ginning of year	value Equal Annual Pry- ment Method	requirement Unlimited Life Method		
I 2	\$10 00 9 00	\$100 00	\$100 00 92 41	\$100 00 92 55	\$ 0.15		
3	8 05	80 50	84 57	84 94	0 85		
4	7 11	71 10	76 79	78 00	1 80		
4 5 6	6 22	62 20	68 80	69 61	3 31		
6	5 40 4 78	54 00 47 80	61 06	63 34 59 63	5 53 8 12		
7 8	4 04	40 40	55 00 47 49	56 23	10 80		
9	3 52	35 20	41 92	56 56	12 95		
10	3 08	30 80	37 18	58 9ģ	13 96		
II	2 71	27 10	33.01		13 62		
12	2 42	24 20	29 70		12 23		
13 14	2.18	19 70	26 96 24 53		9 01		
15	1 97 1 80	18.00	24 53 22 50	8r 67	9 01		
16	I 64	16 40	20 55		8 04		
17	I 50	15.00	18 87		8.53		
18	I 25	12 50	15 84		9 33		
19	I 00	10 00	12.82	62 86	10 16		
20 21				02 80	10 07		
22					10 87		
23	1				10.56		
24					10 18		
25				65 85	9.82		
26					9 58		
27 28					9 56		
29		1			9.82		
30	1	1	1	64 47	9 92		

Remaining Value and the Replacement Requirement according to the Law of Probability. — Table 9 has been prepared to show for numerous articles, all of which have a probable life term of 10 years, the probable annual replacement requirement on the assumption that failures occur according to the law of probabilities, all articles going out of use within 20 years. There is also shown in the table the remaining value of these articles if estimated by the Equal Annual Payment Method, and also the expectancy of a single article with a 10-year probable life and its remaining value if computed by the Straight Line Method and by the Equal Annual Payment Method.

Computation of Annuities to Meet Actual Replacement Requirement. — The computation of the annuities which would replace each lot of annually failing articles, if the same be assumed to fail on any hypothesis similar to those already suggested, can readily be made and will prove instructive. It will be found that in every case the sum of all such annuities will exceed the annuity computed in the ordinary way from the average or probable life. If the computation be then extended to cover all articles remaining in service from year to year and to include also the new articles which have been added to replace the failures, it will be found that in the early years the sum of the annuities is larger than the annuity computed by the use of probable life, in the ordinary way, that after a period in excess of the probable life term, the sum of the annuities will be a minimum and somewhat less than that computed in the ordinary way, and that thereafter it will increase again to about the amount computed by the Straight Line Method.

It will be unnecessary to introduce a complete calculation to illustrate this point and only the results for articles with a probable life of 10 years will be briefly referred to.

Let it be supposed that of 10,000 articles, which all have a probable life of 10 years, 100 fail at the end of the first year; 200 at the end of the second, 300 at the end of the third and so on to 1000 at the end of the tenth, 900 at the end of the elev-

enth, 800 at the end of the twelfth, and so on to the last 100 at the end of the nineteenth.

By substituting dollars for articles and computing the annuity at 6 per cent interest required for the replacements of each year it will be found that an annuity of \$94 will be required to replace the failures of the first year, an annuity of \$97 to replace those of the second year, an annuity of \$94 to replace those of the third year, an annuity of \$91 to replace those of the fourth year, and so on to the nineteenth year. The sum of these annuities is \$1101 or 11.01 per cent.

If the articles which fail each year are replaced and the required annuities for these replacements are also taken into account then the sum of the annuities at 6 per cent interest will be: 11.01 per cent the first year, 10.17 per cent the second, 9.41 per cent the third, 8.78 per cent the fourth, 8 26 per cent the fifth, 7.86 per cent the sixth and so on to a minimum in about the fourteenth year of a little over 7 per cent, and thereafter gradually increasing to about 11 per cent in the thirtieth year.

Had the determination been made in the usual way, based on the assumption that no distinction need be made between the actual and the probable life, the allowance for replacements would have been 7.59 per year, continuously from the beginning. Conceding a moderate or even a wide range of error due to an almost arbitrary though reasonable assumption of the probable annual failures when average life is known, the result of the computation of what should be set aside annually on the annuity basis, to meet the replacements, when compared with the annuity determined from probable life new, shows that the latter is insufficient.

The computation also shows that the required earnings based on this determination would be undesirably high at the beginning, in the early years, when they should be low.

No further demonstration than the above will be needed to show the futility of depending on the Sinking Fund Method and the Equal Annual Payment Method as usually applied, when estimating the replacement requirements.

122 VALUATION, DEPRECIATION AND THE RATE-BASE

The legitimate and always applicable method of procedure when public utility rates are to be fixed, is that which the author calls the Unlimited Life Method. This does not involve any estimate of accrued depreciation, and only reasonable care in determining the annual replacement requirement. The replacement fund is kept apart and if error has been made in the assumption of the life of the items and the fund becomes either too large, or inadequate, the necessary correction can be applied in subsequent years.

CHAPTER VII

THE PURPOSE OF THE APPRAISAL

General Statement

Various Purposes of Appraisal. — The owner of an operating property is interested more in knowing what it has cost him than he is in the value of the property, until the time comes when a transfer of ownership is to be made or the business is to be capitalized. Generally, however, when any enterprise of considerable magnitude is involved the owner should, at all times, have a clear conception of its value. This can only be obtained by making the analysis of the investment and of operating expenses in relation to revenue, and involves much more than a study of financial records. The condition and adaptability of a plant to the uses to which it is being put are involved and must be considered by the appraiser. Valuation, therefore, may be required:

- a. As a basis for a purchase and sale transaction.
- b. When the property is to be pledged as a security for a loan or as the basis for a bond issue.
- c. When the rates of a public utility are to be fixed or regulated, because the earnings resulting from the rates should be adequate to bring a suitable return on the investment and because the charge for the service should be reasonable.
 - d. As a basis for taxation.

The taxation value bears or is intended to bear some definite relation to market value. This is not only true in the case of ordinary taxation, for the purpose of carrying on government, but also in the innumerable cases in which it becomes necessary to apportion to the property which is benefitted, the cost of some improvement in proportion to the benefits which it confers.

Elements of Value. — No definite and final rules can be laid down as a guide for the appraiser in reaching his conclusions relating to value. He is directly concerned with the ascertainment, as definitely as circumstances will warrant, of the net revenue both present and prospective. He must give consideration to:

The cost of construction.

The cost to reproduce the property new.

The relation of the property to actual or possible competing properties.

The condition of the property, including its adaptability to the intended purpose

The weight to be given to these items and the method of applying them in making valuations will be discussed in the proper chapters of this publication.

When property with a salable output is valued for purchase or sale, or for capitalization, or bonding, the investigation must be extended to cost of operation and the market value, present and prospective of the service rendered or of the commodity furnished.

In the regulation of rates, also, consideration should be given to the value that will result from the earnings and this likewise involves a comparison of operating costs with earnings, present and prospective, from whatever source.

Valuation for Purchase or Sale

When property is to be valued for purchase or sale, both the seller and the purchaser desire to know the value as determined from the excess of the earnings over the cost of operation.

First Step — Determination of the Cost. — The first question to be answered in determining the amount that may reasonably be assumed to be invested in the property will be what has the property cost or what may it reasonably be assumed to have cost. Perhaps this can be ascertained from the cost records with due consideration of losses from unprofitable operation and with proper allowance for excessive promotion costs, for excessive salary payments, for unprofitable or useless expenditures and losses by accident. When cost cannot be thus ascertained or whenever there is a doubt and the importance of a close approximation warrants such procedure, the cost of reproducing the property is to be estimated. Proper allowance must be made, too, for all expenses of whatever nature connected with construction and with the establishment of the business.

Second Step — Deferred Maintenance and Depreciation. — The second step will be to ascertain the deferred maintenance, if any, and the accrued depreciation. The accrued depreciation will be the difference between the cost of reproduction new and the present or remaining value of the items which make up the property and which are subject to depreciation.

Third Step — Cost of Operation. — The third step will be the determination of the cost of operation. According to the nature of the business this may be directed to the aggregate output of commodities or service, or it may cover a segregated analysis of cost of operation for a variety of services or commodities. But the essential fact to be ascertained is the total outlay including interest on the investment, salaries and wages, supplies, maintenance and repairs, current depreciation or replacement requirements, and in the case of a franchise, or patent right with a limited life term, or in the case of an oil well or a mine with a limited oil or ore body, including also an amortization increment.

Fourth Step — Earnings, Present and Prospective. — The fourth step will be the determination of the earnings, current and prospective, that will result from such charges for service or for the commodity as may reasonably be assumed to be proper and dependable. It is here, in the case of the public utility, that there may be much uncertainty due to the difficulty of forecasting the attitude of the rate-regulating authorities toward the public service corporation. For while it is true that private property may not be taken for public use without due process of law and that there must be no confiscation through inadequate earnings, there is yet some uncertainty relating to the

compensation which the owner may expect for establishing and managing the utility and also as to whether, in every case, the value of the service rendered has really justified the enterprise. There is much uncertainty, in other words, in relation to the earnings in excess of interest on the investment that will be permitted by the public service commissions, and, when value depends largely upon prospective business, there may be uncertainty, too, relating to the forecast of earnings on which the estimates of profits are based.

Fifth Step — Determination of Profits. — The fifth step involves a comparison of the cost of operation with the anticipated gross income. The excess of the income over cost of operation represents the profits of the business. The profits may be actual or they may be prospective. The profits both current and prospective in excess of interest on the investment, when capitalized, create an increment of value in excess of the investment. This increment of value may, according to circumstances, be apportioned to "franchise," "going value," "good-will," "patent right," or any other classification of value that may seem appropriate.

Real Estate Value. — The valuation of real estate may acquire special consideration. But here, as in the case of other revenue-producing property, the real test of value is the amount of the revenue which the property will produce. What is the rental value of the property? This is the question to be answered. The appraiser will direct his inquiry to the gross income, immediate or prospective, which is dependable. He will ascertain the taxes and probable assessments for improvements to which the property is liable, and he will determine from these various items the net revenue which the property will produce. This net revenue, immediate and prospective including appreciation, will then determine the value.

When the right of eminent domain is exercised to acquire real estate for some special purpose due consideration must be given to its serviceability for the intended use. This subject is further considered in Chapter X.

Appraisals as a Basis for Fixing Rates

Protection of the Investment. — In determining the investment on which the investor in public service properties should be allowed a reasonable return, all attendant circumstances must be duly considered. It may be stated, however, that, apart from the determination of the rate of interest which should result from the investment, it will be equitable and fair to consider the public service corporation as the agent of the State or municipality, as the case may be, and to determine in what situation the State or municipality would have found itself had there been no intermediate owner or public service corporation.

Let it be assumed that the owner of a public service plant has made his investment under good expert advice, and that the plant is in every respect the same as, or equal to, what the people would have constructed for themselves. Let it be further assumed that the plant is free from debt, that it will have no residual value, and that it and all its parts will have an actual useful life of n years. The owner will then be entitled:

First. — To a reasonable interest on his investment;

Second. — To operating expenses; including maintenance, repair and replacement expenditures;

Third. — To an annuity which, in n years, at compound interest, will amount to his investment;

Fourth. — To reasonable compensation for managing the business which may be based on the volume of business transacted, and on the general prosperity of the community.

Purpose of the Accumulating Replacement Fund. — If it be now supposed that the owner actually received these amounts, estimated on a proper basis, and that he allows the annuity at compound interest to accumulate so that amortization will be an accomplished fact at the end of n years, then, as he has command of the amortization fund, he will have a decreasing amount of capital actually tied up in the plant. This decreasing capital or remaining value of the plant is the complement of the growing amortization fund. This fund, in the case of a

plant which is to continue in service beyond the term of n years, is supposed to be held inviolable for the replacement of the plant at the end of its life. The owner reaps no benefit from it whatever, beyond holding it as the means for replacing a worn-out plant.

The value of the plant in its varied stages of depreciation, plus such amortization fund, should at all times be at least equal to the original investment. The owner, even if he gets an annuity, as here assumed, is entitled at all times to the interest, not on a plant valued at first cost or investment less depreciation, but on the entire first cost. Had he determined, instead of building the plant, to keep his funds invested in safe securities at ordinary interest rates, he would, at the end of n years, have been in possession of his entire capital plus interest on the full amount thereof for the entire time. If, under the assumed facts, he were not allowed interest on the full amount invested in the public service plant, an injustice would be done.

This is true even when replacement takes the place of amortization. The owner in this case is entitled to interest on the entire capital invested in the plant, and, at the end of the plant's usefulness, he is also entitled to a return of the capital itself. Suppose that a city constructs a plant, paying cash for it, and collects rates which will just yield a fair rate of interest on the investment. At the end of n years the plant is replaced with a new one of the same capacity. As the city has not included in its rates, theretofore charged, an increment for amortization, it now finds itself in possession of a new plant and a total investment twice as great as the cost of the first plant. Applying the same principles to the second plant, as to the first, rates should be doubled. This, of course, would be an absurdity.

A charge by a city for service rendered which is less than sufficient to amortize the cost of the plant within its life may yet be equitable and proper, when the cost of the plant is intentionally put upon the whole community and not upon the rate-payer.

Rate-Base with and without Deduction of Depreciation.— The application of the fundamental principles, elsewhere noted, to any public utility plant will show that the determination of a rate-base without deduction from the capital actually but reasonably invested is a proper proceeding, provided, of course, that the replacement requirements are not overlooked and are computed by some proper method.

It may be repeated that, when depreciated value or investment less accrued depreciation is taken into account as a basis for computing necessary earnings, the current depreciation or amortization must be computed on the basis of depreciated value or investment and the *remaining* life of the plant or of its parts.

This can best be made clear by an illustration: Let it be supposed that the passenger rates and the freight tariff on a steamboat line are subject to regulation, and that some one going into the steamboat business builds a steamer for the service. Let it be assumed, too, that in connection with this business he requires no capital investment other than the cost of the steamer; that terminal facilities, office space, and whatever else he needs, are obtainable by rental. For the purpose of this illustration, let it be further assumed that the volume of business is such that there is no doubt about the income, so that the element of hazard is eliminated.

If the steamboat has a life of 20 years, it will gradually depreciate in value and will go out of service at the end of a 20-year period. Ignoring its possible scrap value, which is immaterial for the purpose of this illustration, the following questions are to be considered.

At the end of 10 years, with interest at 6 per cent per annum, and earnings just sufficient to yield interest plus an amortization, figured for a 20-year life at \$0.027185 on each dollar of the investment:

- I. What will be the value of the steamboat to the owner at the end of 10 years?
- 2. What will be the amount that a purchaser can afford to pay for the steamboat at the end of 10 years?

- 3. What should be the earnings during the time the steamboat is in possession of the original owner?
- 4. What should be the earnings during the time the steamboat is in the possession of a purchaser after 10 years of service?

The first and second questions are answered elsewhere. The owner, by one line of reasoning, finds the remaining value in the steamboat to be 64.17 per cent; the purchaser, by a different line of reasoning, finds the same value.

The third question, too, is answered elsewhere. The original owner is entitled to a net return during the entire period of his ownership of 6 per cent on his investment, which is at all times 100 per cent. No deduction is to be made for depreciation because the fund which results from the accumulation of the amortization annuity, together with its interest, is available for no other purpose than the replacement of the steamboat at the end of its period of usefulness. It is dead capital, and remains dead until the property is disposed of or until required to replace the worn-out steamboat. The original owner, therefore, is entitled to a return of 6 + 2.72 = 8.72 per cent per annum on his investment.

In considering the fourth question, it may at first appear as though the purchaser, having invested only \$64.17 on each \$100 of original cost, could claim a return on this investment alone — that he should be allowed, in addition to the amortization as above determined, net earnings of \$3.85 (6 per cent on \$64.17) per annum on what he paid for each \$100 of the original cost of the steamboat; that the valuation for rate-fixing purposes, in other words, should be the original investment less depreciation. Under the adoption of this view, it will be seen that, if the steamboat were sold repeatedly, there would be a constantly decreasing appraisal for rate-fixing purposes.

In the last year of its service the valuation entitled to consideration in fixing earnings would be only 8.23 per cent. This view is unfair to the owner of the property, who should be assumed to be planning a continuation of the steamboat business. When he takes possession of the steamer, its value to

him, as already set forth, is \$64.17 on each \$100 of original cost, but, as owner, he at once finds that, of his capital ordinarily available for other purposes, an amount equal to 35.83 per cent of the cost of a new steamboat is, to all intents and purposes, tied up in his steamboat business. It has become dead capital, for all purposes except replacement, so long as he remains in the steamboat business. This 35.83 per cent at interest at 6 per cent is necessary to supplement the annuity regularly going into the amortization fund, together with which at the end of the 20-year period it will just replace the steamer. Whether or not the 35.83 per cent is actually set apart is immaterial; the fact remains that ownership of the depreciating steamer renders this amount of capital as already stated unavailable or dead for any purpose other than replacement, and the new owner is entitled to interest on this 35.83 per cent just as well as on the 64.17 per cent which he paid for the steamer.

The demonstration of this fact may be made as follows: The purchaser of the steamboat, who buys the boat when it has a remaining period of usefulness of 10 years, invests, as has been explained, \$64.17 for each \$100.00 of the original cost of the steamboat. He is unquestionably entitled to interest on this sum, together with amortization, which at the assumed interest rate of 6 per cent will be:

Interest at 6 per cent on \$64.17 per annum	\$3.85
Amortization annuity for the remaining 10 years, during which	
the investment of \$64.17 is paid back to the purchaser (\$7.59	
on each \$100)	4 87
Total	\$8 72

This is exactly the same as though, instead of the value of the steamboat, the capital originally invested had been taken into account, in which case the original owner or purchaser would be allowed:

Interest at 6 per cent on the investment of \$100 per annum	\$6.00
Amortization annuity to retire \$100 of the investment within the	
life of the steamboat, that is, within 20 years	2.72
Total	\$8.72

Although it may be superfluous, one more illustration of this principle will be given. Let it be supposed that the owner borrows money from a bank at 6 per cent per annum to build a steamboat, and that he earns 6 per cent plus the amortization increment of 2.72 per cent.

Of the \$8.72 to his credit at the end of each year's business for every \$100 of capital invested, he pays the bank \$2.72 on account of principal and so much of the remaining \$6 as may be necessary to meet the interest then due. This will be all of the \$6 the first year, and a decreasing amount thereafter until the end of the 20-year period, when his steamboat is retired. He then finds that he has paid back to the bank on account of the borrowed capital twenty annuity increments of \$2 72, amounting to \$54.40, and that there is still due to the bank \$45.50. He also finds that the various amounts remaining in his hands from year to year, \$0.16 at the end of the second year, \$0.34 at the end of the third year, \$0.52 at the end of the fourth year, and so on, together with interest thereon at 6 per cent, when computed for the 20-year period, will amount to the \$45.50, the balance due at the bank. The owner finds he has earned nothing. He has invested no money of his own and has received no return, which is as it should be in this hypothetical case. The rates, however, throughout the entire 20 years were fixed on the principle that 6 per cent per annum should always be allowed on 100 per cent of the capital invested, together with the amortization annuity, but without any deduction for depreciation. They could not have been fixed lower without entailing loss to the owner.

Unlimited Life. — The value of a revenue-producing property when the earnings thereof include an amortization annuity has already been discussed. It remains to consider the case of a property which, in addition to the accepted reasonable rate of interest (net), is without limit as to its time of serviceability, earning the estimated annual replacement requirement determined by some formula, as above explained, instead of the annuity computed from amortization tables.

In this event the plant may be regarded as having unlimited life. Each part thereof as it wears out is replaced out of current earnings. The owner does not maintain an amortization fund, neither is any of his capital rendered dead or unavailable. To him the value of the property is at all times 100 per cent, so, too, in the case of a purchaser. Knowing that all replacement requirements are fully covered by the earnings, the purchaser is willing to pay 100 per cent for the plant, regardless of the amount of accrued depreciation.

The case may be considered of a public service property whose earnings have been inadequate to supply any amortization increment, but which will in the future be able to earn the actual annual replacement requirement. The original investment in this case having been 100 per cent and there having been no amortization annuity in the past, there can be no transfer of the property at less than 100 per cent without loss, but if, by reason of inadequate returns, the market value could not be maintained at 100 per cent, and a sale has been made at less than this sum, the new owner will be compensated and protected if, on his investment which is not original cost, he earns reasonable interest and an adequate amount for replacements. This must be so, because, in the future, actual replacement requirements being covered by the earnings, the worn-out parts will be replaced without cost to the owner. This replacement neither increases nor decreases his investment; but, if the property is extended and new parts are added, such additions represent newly invested capital to the full amount of their cost, and in such a case his investment, expressed as a percentage of the total cost, will gradually increase.

At all times, however, without causing loss to the new owner, that part of the plant which he bought at a depreciated value could be valued at his purchase price, while all extensions subsequent to the purchase, on the assumption that replacements are met out of earnings and that there is no amortization of capital, should, for rate-fixing purposes, be appraised at 100 per cent. Such a course, however, would deprive the new owner of

the opportunity for profit, of which he probably thought to avail himself when he bought a plant of depreciated value, and would place the rate-payer in the position of having made a profit at the expense of the original owner. This fact explains why the market value of stocks and bonds is cited so frequently as an indication of value.

Improper Use of the Term "Value."—It may be held that a determination of value for rate-fixing purposes, on the principles herein set forth, is not a determination of value at all. This may be true, but it then becomes a matter of defining "value," as used by the courts in order that a distinction may be made between value and the appraisal of the investment on which rates may be properly based.

The term "value" has been very generally used in matters involving the fixing of rates in the past. When fundamental principles are better understood, more attention will be paid to the capital reasonably and properly invested.

Under a system of permitting the owner of public service properties to earn from year to year the actual average replacement requirements, the necessity for a close distinction between repair and replacement disappears. This is of some advantage, as it is at best difficult to discriminate between small items of replacement and large repair items.

Basis of Rates in the Knoxville Case. — The United States Supreme Court in the Knoxville case Knoxville vs. Knoxville Water Co. (212 U. S. 1; 29 Sup. Ct. Rep. 148) says:

"A water plant, with all its additions, begins to depreciate in value from the moment of its use. Before coming to the question of profit at all the company is entitled to earn a sufficient sum annually to provide not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. The company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. It is entitled to see that from earnings the value of the property invested is kept unimpaired, so that, at the end of any given term of years the original investment remains as it was at the beginning. It

is not only the right of the company to make such a provision, but it is its duty to its bond and stock holders, and, in the case of a public service corporation, at least, its plain duty to the public. If a different course were pursued, the only method of providing for replacement of property which has ceased to be useful would be the investment of new capital and the issue of new bonds or stocks. This course would lead to a constantly increasing variance between present value and bond and stock capitalization — a tendency which would inevitably lead to disaster either to the stock-holders or to the public, or both. If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon over issues of securities, or of omission to exact proper prices for the output, the fault is its own. When, therefore, a public regulation of its prices comes under question, the true value of the property then employed for the purpose of earning a return cannot be enhanced by a consideration of the errors of management which have been committed in the past."

According to this statement by the Court, the owner of a public utility should be required to exact prices for his output, which, at the beginning of operation and at all times thereafter, will cover the current depreciation of the physical elements of his plant. Yet, as elsewhere explained, in the early years, this is frequently impossible, and in nearly every case is inadvisable and would work unnecessary hardship upon the consumer. Losses in the early years when rate-payers are few are oftentimes unavoidable and these losses deserve consideration. history cannot be ignored if rates are to be so fixed as to be fair alike to the owner and to the rate-payer. In other words, not all shortage of earnings in the past is to be ascribed to errors of management. It is difficult to reconcile the language of the Court with this principle even as it is difficult to understand why so many of the Courts have held that value which results from earnings must be made the starting point when rates are to be fixed.

"Value" Defined in the Minnesota Rate Cases. — Justice Hughes in delivering the opinion of the U. S. Supreme Court in the Minnesota Rate Cases (June 9, 1913) (230 U. S. 352) said:

"The depreciation in question is not that which has been overcome by repairs and replacements, but is the actual existing depreciation in the plant as compared with a new one. It would seem to be inevitable that in many parts of the plant there should be such depreciation, as, for example, in old structures and equivalent remaining on hand, and when an estimate of value is made on the basis of reproduction new the extent of existing depreciation should be shown and deducted. . . . And when particular physical items are estimated as worth so much new, if in fact they be depreciated, this amount should be found and allowed for. If this is not done, the physical valuation is manifestly incomplete. And it must be regarded incomplete in this case."

In the Minnesota Rate Cases the Master, in ascertaining a basis for rate fixing, had allowed the cost of reproduction new without making any deduction for accrued depreciation. The Master did not deny that there was depreciation but found that the same was more than offset by the appreciation in value of certain items. It was this finding which the Supreme Court refused to approve.

The Supreme Court in its decision as usual makes value the starting point. Consequently the court says that depreciation must be deducted from the cost of reproduction new. But cost of reproduction is not "value." Not even the value of the property new is exactly determined by its original cost. The opinion of the court becomes illogical if it be conceded to be unnecessary to make value the starting point when rates are to be fixed. But the Master, too, seems to have overlooked this point and sought to find something, appreciation in this case, to offset depreciation. He was quite as much at fault as the Supreme Court. And yet the allowance of appreciation in determining present value as a basis for fixing rates or for the issue of bonds is not uncommon. Railroads claim appreciation of their road-beds. Though it may be granted that the compacted road-bed, long in service, is worth more than a recently constructed bed, this should have nothing to do with the fixing of rates, no more than accrued depreciation should have. The new road-bed requires more careful watching and greater expense for upkeep. Consequently the net earnings are lower than they would be with the same volume of business, the same rates and a well-compacted old road-bed. If the resulting earnings are inadequate in the early years of business, the deficiency may be treated as a temporary investment to be amortized out of the larger net earnings of later years.

Depreciation in the Pocatello Water Company Case. — The Supreme Court of Idaho in a recent decision (1915) in the Pocatello Water Company case, Murray vs. Public Utilities Commission (150 Poc. Rep. 47, p. 50), reversing the Public Service Commission of that State says in reference to depreciation:

"So far as the question of depreciation is concerned, we think deduction should be made only for actual tangible depreciation and not for theoretical depreciation, sometimes called 'accrued depreciation' In other words, if it be demonstrated that the plant is in good operating condition and giving as good service as a new plant, then the question of depreciation may be entirely disregarded."

This decision is in substantial accord with the minority report of the Commission which had been made by A. P. Ramstedt and in which he said:

"A person having invested his money in a continuous business enterprise for the benefit of others must always be ready to replace the constructive portions of his plant as they wear out. A person having embarked on such an enterprise is justly entitled to compensation to cover this depreciation in addition to a fair return, over and above expenses, upon the reasonable value of the property. Allowance for depreciation cannot, in my judgment, be considered as profit or an earning factor in the business . . . The fact that in an investigation of the petitioner's property . . . it is found that the market value of his physical property employed for public use has depreciated \$77,188.39 does not, in my judgment, justify the commission, in its determination of a fair value for rate purposes, to deduct the amount of depreciation from the present estimated cost of reproducing the property new, and thereby reduce the earning power of his property."

The fact is evidently being recognized in Idaho as elsewhere, that the service rendered should always be 100 per cent good even though rendered by a plant whose physical parts are perishable and that if a 100 per cent valuation for rate purposes is fair when the plant is new it is equally fair when the plant is old.

Wisconsin Railroad Commission on Investment as the Ratebase. — The view which the author takes relating to making the necessary original cost and not "value" the starting point when rates are to be fixed finds some support in the following statement by the Wisconsin R. R. Commission in the City of Appleton Case (Wis. R. C. R., Vol. 5, p. 220).

"For rate-making purposes the actual total investment, subject to certain qualifications, seems to be the basis for determining the reasonableness of the charges that may be exacted of the public for the services rendered or product furnished in certain jurisdictions. Of course where such information is not available, the reasonable value of the investment would have to be ascertained by some method of appraisement, and in such event the 'actual total investment' doctrine would be inapplicable."

Appraisals for Taxation Purposes

The Taxation Value. — The tax value is usually intended to bear some definite relation to the market value. In some of the older Eastern cities of this country the tax value is intended to represent the full market value. Generally, however, this is not the case. Real estate will usually be found to be valued for taxation purposes at from 50 to 75 per cent of its market value and personal property sometimes still lower.

The appraiser of tax values is therefore interested in market value and must give consideration to all the elements that go to establish market value, such as supply and demand, earning capacity present and prospective, rate of appreciation and the like. But this is a subject which is foreign to the purposes of this volume and may be dismissed with the above broad statement.

Appraisals for the Purpose of Capitalization

Capitalization. — The primary matter to receive consideration when a property is to be valued for the purpose of capitalization is its earning capacity. What are the net earnings, present and prospective? When this question can be satisfactorily answered the problem has been solved. Inquiry must therefore be made relating to the total annual outlay involved in conducting the business enterprise. Operating expense of every kind including an adequate annual replacement requirement and an amortization increment if the life of the enterprise is limited, must be weighed against the assured present and prospective income. The resulting net earnings capitalized at the interest rate, which an enterprise of the nature under consideration should yield, will indicate the upper limit of value for capitalization.

When a public utility is under consideration the problem may be complicated by the fact that rates and therefore the earnings are subject to regulation and in consequence of the control and regulation exercised by the public through its public service commissions, any great precision in estimating the gross present and prospective earnings may be out of the question. This may prove embarrassing particularly in those cases where the actual investment of capital is small but the volume of current business is large. In all cases the actual properly invested capital will be a first approximation of value for capitalization purposes and usually the lower limit of such value if the property is one in successful operation.

CHAPTER VIII

THE FIXING OF RATES

Public Utilities and the Regulation of Public Service

Ownership of Public Utilities. — In the older countries the public utilities are generally owned and managed by the State or municipality. In the countries, on the other hand, which are but sparsely populated and in which the potentiality of natural resources is large, private enterprise is usually depended upon to make the development.

Certain utilities are almost universally publicly owned. This is true of city streets, very largely of country roads and bridges and, in most countries, of the sewers. There is probably not a city in the United States in which a charge is made for the service rendered by the sewer system. The benefit, in this case, to the community as a whole, of properly disposing of human excrement and such domestic waste as can be floated away with water, is generally recognized. There is no apportionment of cost to individuals, in other words, according to the value of the service. The cost of establishing and maintaining the system is raised by taxation.

The streets and public roads, too, are generally built at public expense. But there are other utilities such as water-works, electric light and power works, gas-works, telephone systems, railroads and other transportation systems which may be either publicly or privately owned. The private ownership is usually exercised through a corporation. That this should be so is natural for the reason that stability of management is thereby secured, the element of uncertainty in the matter of the life of the owner being eliminated.

Quasi-public Character of Public Utilities. — While the public service corporation is subject to the same general laws which

control other corporations operating for profit, they are accorded certain rights and privileges which give them a quasi-public character. They have the right of eminent domain, *i.e.*, the privilege of taking, under due process of law, private property for their uses (subject, of course, to the making of adequate compensation therefor) and they may be accorded the privilege of occupying streets, roads and public places subject to reasonable regulations.

The tendency of the day is to give the public service corporations more and more monopolistic character and to trust to regulation and control for the protection of the public interest. This seems wise and should make for good service at reasonable rates, always provided that regulation be not too stringent and onerous and that the public service corporation be allowed to make a fair profit.

The Disadvantage of Frequent Modification of Rates.—In some of the States, as until within recent years in California, the right to regulate has found its main expression in laws requiring the rates of certain utilities to be fixed annually. The water and gas rates have thus been under annual investigation in San Francisco, and the occasional establishment of rates which were thought inadequate has led to much long drawnout litigation which has rarely resulted satisfactorily to either of the litigants.

As the result of such litigation relating to water rates established by San Francisco for 1902-03 and for the years subsequent thereto, the court has allowed, temporarily, the collection of rates by the Spring Valley Water Company in excess of the limit fixed by the city authorities and the excess collected has been impounded and now (1915) amounts to over \$1,500,000. It is only fair to say, however, that by recent amendment of the charter of the city, the rates will hereafter be subject to regulation by the California State Railroad Commission (Public Service Commission).

When the established rate is made subject to frequent change and the owner of the utility is not allowed to earn more than some predetermined interest return on his investment, the incentive to improve the service falls away, because, if the owner succeeds in economizing, in producing his output at a lower cost, the immediate result will be lower rates and less earnings. The gross income may be reduced though, perhaps, the value of the service to the rate-payer may remain unaffected or may even be increased. Rates should not therefore be subject to change too frequently. If there must be a term during which the established rates will apply, let it be at least 5 years.

Control and Regulation. — The acceptance of the privilege to operate a public utility carries with it a submission to such regulation as may be demanded by the public which grants the privilege. This regulation must be reasonable. The public utility owner is in business for profit. He invests his capital and applies his ability and experience to the development of enterprises that would otherwise remain dormant for a longer or shorter period. He brings within reach of the community, means of transportation, an adequate water supply, light, power and heat, telegraph and telephone facilities which all contribute to the growth and prosperity of the community. He does this because he considers his enterprise a good business venture.

In the case of the commercial and industrial enterprise, too, a tendency may be noted toward monopoly which is likely to lead in time to regulation and control similar to that of the public utility. Whenever the operation of any concern acquires such magnitude that it can and does destroy the business of its competitors, it becomes monopolistic in character and the state is bound, sooner or later, to exercise its right to regulate and control not only the method of conducting business, but in the end, perhaps, the prices to be charged for its output.

Excessive and Fictitious Profits. — Where, in the rapid transition from a frontier region to a state with stable industrial and commercial conditions, the advance in the value of real estate has been rapid and the opportunity for profit in many lines of business has been large, the owner of the public utility has frequently taken advantage of the opportunity and has capitalized

inordinate profits, sometimes real and sometimes fictitious. Where there were no public service commissions — and these are a relatively new institution, — the accounts were kept in a fashion to suit the manipulator. Apparent profits were used as the basis for a distribution of dividends and for the issuance of stocks and bonds, and the rate-payer was charged to the limit.

The Right to Regulate Rates. — The right to regulate rates is now quite generally recognized. Without such right every corporation which has a monopoly would have the public at its mercy and could charge for its service all that the traffic will bear.

The right to regulate, if properly exercised, should in the long run prove of advantage to the owner of the utility. Fair regulation will insure to the owner what he is entitled to and will protect him against unfair competition such as has frequently been the result of opportunity to capture with a relatively small outlay the cream of the business.

Under proper regulation the owner of every public utility whose existence is justified by all attendant circumstances, should get the protection of his legitimate reasonable investment, also a fair interest return on this investment and should recover all the necessary operating expenses including replacement requirements. He should receive such treatment that at some time, by amortization of the capital which he has invested, or by a sale to some purchaser, he can withdraw from the business without loss, provided of course, that he has exercised due foresight and has not met with unavoidable losses such as may be caused by floods or earthquakes.

Regulation Essential. — The necessity for regulation has now become apparent. The practice of granting perpetual franchises with unwise privileges has fallen into disfavor. This first became manifest in laws permitting the periodical regulation of public service rates by local, municipal and county authorities. Thus, for example, in California the Constitution of the State (in effect Jan. 1, 1880 but now modified) provided:

"The use of all water now appropriated, or that may hereafter be appropriated, for sale, rental, or distribution, is hereby declared to be a public use and subject to the regulation and control of the State, in the manner to be prescribed by law; provided, that the rates or compensation to be collected by any person, company or corporation in this State for the use of water supplied to any city and county, or city or town, or the inhabitants thereof, shall be fixed annually by the Board of Supervisors, or City and County, or City or Town Council, or other governing body of such city and county, or city or town, by ordinance or otherwise, in the manner that other ordinances or legislative acts or resolutions are passed by such body, and shall continue in force for one year and no longer. Such ordinances or resolutions shall be passed in the month of February of each year, and take effect on the first day of July thereafter. (Art. XIV, Sec. 1) (1880)

"The right to collect rates or compensation for the use of water supplied to any county, city and county, or town, or the inhabitants thereof, is a franchise, and cannot be exercised except by authority of and in the manner prescribed by law." (Art. XIV. Sec. 2) (1880)

"In any city where there are no public works owned and controlled by the municipality, for supplying the same with water or artificial light, any individual, or any company duly incorporated for such purpose, under and by authority of the laws of this State, shall under the direction of the Superintendent of Streets, or other officer in control thereof, and under such general regulations as the municipality may prescribe for damages and indemnity for damages, have the privilege of using the public streets and thoroughfares thereof, and of laying down pipes and conduits therein, and connection therewith as far as may be necessary for introducing into and supplying such city and its inhabitants either with gas light or other illuminating light, or with fresh water for domestic and all other purposes, upon the condition that the municipal government shall have the right to regulate the charge thereof." (Art. XI, Sec. 10) (1884)

San Francisco Charter Provisions relating to Rates. — The Charter of the City and County of San Francisco (in effect January 8, 1900) mentions, among the powers of the Supervisors:

"To fix and determine by ordinance in the month of February of each year, to take effect on the first day of July thereafter, the rates of compensation to be collected by any persons, company or corporation in the City and County, for the use of water, heat, light or power, supplied to the City and County or to the inhabitants thereof, and to prescribe the quality of the service."

The enforcement of such laws as these has, at times, proven embarrassing to the public service corporations, and has given rise to charges of corruption, to wrangling and to litigation, unfortunate alike for the corporations and for the public. The remedy has been sought in national and state control, this control being exercised through National and State Commissions.

The Interstate Commerce Act. — The Interstate Commerce Act in line with this idea was approved on February 4, 1887. The purpose of this Act may be stated as follows:

"The principal objects of the Interstate Commerce Act were to secure just and reasonable charges for transportation, to prohibit unjust discrimination in the rendition of like services under similar circumstances and conditions; to prevent undue or unreasonable preferences to persons, corporations or localities; to inhibit greater compensation for a shorter than for a longer distance over the same line; and to abolish combinations for the pooling of freight." (Interstate Commerce Commission vs. Baltimore & Ohio Railroad Co., etc., 145 U. S. 263.)

Public Service Control in Massachusetts. — The creation of a Board of Railway Commissioners in Massachusetts in the early sixties may be noted. A Gas and Electric Light Commission followed in 1885. The Board of Highway Commissioners in that State controls the Telegraph and Telephone Companies.

Public Service Control in California. — The Constitution of California, which went into effect on Jan. 1, 1880, contained a provision for a Railroad Commission. But its powers were limited and its work was not effective. By an amendment to the Constitution in 1911 the powers of the Commission were more clearly defined and its work speedily met with deserved approval. By further amendment the powers of the Railroad Commission were extended (Mar. 28, 1911) to all other public utilities so

that the Railroad Commission of California is now in effect a public service commission.

Sec. 23 of Art XII of the California Constitution now provides:

" Every private corporation, and every individual or association of individuals, owning, operating, managing or controlling any commercial railroad, inter-urban railroad, street railroad, canal, pipe line, plant or equipment, or any part of such railroad, canal, pipe line, plant or equipment, within this State, for the transportation or conveyance of passengers or express matter, or freight of any kind, including crude oil, or for the transmission of telephone or telegraph messages, or for the production, generation, transmission, delivery or furnishing of heat, light, water or power or for the furnishing of storage or wharfage facilities, either directly or indirectly, to or for the public, and every common carrier is hereby declared to be a public utility subject to such control and regulation by the Railroad Commission as may be provided by the Legislature, and every class of private corporations, individuals or association of individuals hereafter declared by the Legislature to be public utilities shall likewise be subject to such control and regulation. The Railroad Commission shall have and exercise such power and jurisdiction to supervise and regulate public utilities, in the State of California, and to fix the rates to be charged for commodities furnished, or services rendered by public utilities as shall be conferred upon it by the Legislature, and the right of the Legislature to confer powers upon the Railroad Commission respecting public utilities is hereby declared to be plenary and to be unlimited by any provision of this Constitution."

The California Public Utilities Act by an amendment in effect August 10, 1913, provides:

"The Term 'public utility,' when used in this act, includes every common carrier, pipe line corporation, gas corporation, electrical corporation, telephone corporation, telegraph corporation, water corporation, wharfinger and warehouseman, where the service is performed for, or the commodity delivered to, the public or any portion thereof,' as herein used, means the public generally, or any limited portion of the public including a person, private corporation, municipality or other political subdivision of the state, for

which the service is performed or to which the commodity is delivered, and whenever any common carrier, pipe line corporation, gas corporation, electrical corporation, telephone corporation, telegraph corporation, water corporation, wharfinger or warehouseman performs a service or delivers a commodity to the public or any portion thereof for which any compensation or payment whatsoever is received, such common carrier, pipe-line corporation, gas corporation, electrical corporation, telephone corporation, telegraph corporation, water corporation, wharfinger or warehouseman is hereby declared to be a public utility subject to the iurisdiction, control and regulation of the commission and the provisions of this act. Furthermore, when any person or corporation performs any service or delivers any commodity to any person or persons, private corporation or corporations, municipality or other political subdivision of the state, which in turn either directly or indirectly, mediately or immediately, perform such service or deliver such commodity to or for the public or some portion thereof, such person or persons, private corporation or corporations and each thereof is hereby declared to be a public utility and to be subject to the jurisdiction, control and regulation of the commission and to the provisions of this act."

Public Service Control in New York. — In New York there has been for many years a Railroad Commission. In 1905 a Commission was established for the control of gas and electric corporations. In 1907 these commissions were superseded by two public service commissions, which were not however given control over water companies.

Public Service Control in Wisconsin. — In Wisconsin, too, the need for regulation of transportation rates first found expression in the creation of a railroad commission and, as in the case of California, the powers of this Commission were subsequently extended so that it is now a public service commission. This Commission exercises the same control over municipally owned utilities as over those which are privately owned. It has accomplished much of the pioneer work in standardizing methods of procedure and its work is of great value in establishing the fundamental principles of valuation as a basis for the fixing of rates.

Public Service Control in Other States. — Many other states have public service commissions or railroad commissions with

duties and powers of a public service commission. Georgia, Oregon, Nebraska, Oklahoma, Maryland, Colorado, Florida, Illinois, Indiana, Iowa, Maine, Missouri, Pennsylvania, Rhode Island, South Carolina, Vermont and other states started off with a railroad commission. Arizona, Kansas, Nevada, New Jersey, New Hampshire and Oklahoma have their public service commissions more or less liberally endowed with powers of regulation and control.

Supreme Court of Wisconsin on the Powers of the Railroad Commission. — All the powers which a public service commission has are conferred upon it directly by the people or by legislative enactment. Its function is to make administrative regulations. Its powers are not legislative. On this point the Supreme Court of Wisconsin says (Minn. St. Paul and Sault Ste Marie Ry. Co. vs. Railroad Com. of Wis., 136 Wis. 146):

"The division of the governmental powers into executive, legislative and judicial, while of great importance in the creation or organization of a State, and form the viewpoint of institutional law and otherwise, is not an exact classification. No such exact delimitation of governmental powers is possible. In the process of enacting a law there is frequently necessary the preliminary determination of a fact or a group of facts by the Legislature, and it is well settled that the Legislature may declare the general rule of law to be in force and take effect upon the subsequent establishment of the facts necessary to make it operative or to call for its application. . . . The Legislature may delegate any power, not legislative, which it may itself rightfully exercise. This power to ascertain facts is such a power as may be delegated. . . . This law established, and thenceforth assumes the existence of rates, charges, classifications and services, discoverable by investigation but undisclosed, which are exactly reasonable and just. It commits to the Railroad Commission the duty to ascertain and disclose that particular rate, charge, classification or service. The law intends that there is only one rate charge or service that is reasonable and just. When the order of the Commission is set aside by the Court, it is because this reasonable and just rate, charge, classification or service has not yet been correctly ascertained. When the order of the Commission has been rescinded or changed by the Commission because of the changed conditions it is because there is a new reasonable rate to be ascertained and disclosed, applicable to such new conditions and fixed by force of law immediately when the new conditions came into existence. But the theory and the mandate of the law is that this point is always discoverable although not always discovered. Until it is discovered and made known the former rates and service prevail. The order of the Commission is prima facie evidence that the rate, charge or service found and fixed by it is the particular rate, charge or service declared by the Legislature in general terms to be lawful and to be in force. If it were conceded that the Commission had power or discretion to fix one of several rates, either of which would be just and reasonable, it would be hard to say that this was not a delegation of pure legislative power to the Commission. But the theory of this law is to delegate to the Commission the power to ascertain facts and to make mere administrative regulations.

"The notion that commissions of this kind should be closely restricted by the courts, and that justice in our day can be had only in courts, is not conducive to the best results. Justice dwells with us as with the fathers; it is not exclusively the attribute of any office or class, it responds more rapidly to confidence than to criticism, and there is no reason why the members of the great Railroad Commission of this State should not develop and establish a system of rules and precedents as wise and beneficent within their sphere of action as those established by the early common-law judges. We find the statute well framed to bring this about."

Competition in the Public Service. — Before the advantage of cooperation and combination leading to monopolistic control was fully realized by the owners of business ventures, it was common belief that free and unrestricted competition should be encouraged and would result in permanent advantage to the public. The fallacy of this view when large public service concerns are involved is now generally recognized. The competition in the case of such concerns may result in a momentary lessening of rates but in the long run the rate-payer or the security holder will be the sufferer. The temporary advantages may become a permanent burden.

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Coöperation Between he Public and the Owner. - The treatment of the public service corporation should be such as to encourage the undertaking of other public service enterprises. It should not be a hard and fast rule that a fixed rate of return will be allowed in all cases. The owner should feel that good management will be rewarded. There is perhaps no better way of securing the desired hearty and cordial cooperation between the owner of the utility and the public than by the introduction of a system of profit sharing. This has been done with apparently great success in Chicago and other cities where revenue in excess of an agreed interest return on an appraisal of street car lines is divided on a reasonable basis between the city and the owner. So, too, in Boston, for every decrease of 5 cents per 1000 cubic feet in the charge for gas the Gas Company is permitted to increase its dividend rate one per cent. The consumer benefits by the lower rate, the Gas Company by the larger dividend.

Attitude of the Public Service Commissions. — Public service commissions were not created to assume an attitude hostile to the public service corporations. While they must protect the public against excessive charges for the service rendered and must see to it that there will be no discrimination against individuals, they are likewise charged with the duty of allowing the corporations to do business on a fair basis. It must not be expected, however, that mistakes will not be made. As the records to date are read, such mistakes will be found in both directions. On the whole the attempt to be fair is generally recognized and the confidence of the people in their public service commissions is growing from day to day.

The Rate-Base

Net Earnings as a Basis of Value. — When there is no competition and no restriction by law or otherwise upon the charges which may be made by public utilities for commodities furnished or for service rendered, it is safe to assume that these charges will be fixed on the broad principle of "all the traffic will bear." This does not mean all that the rate-payer would be willing to

pay rather than to be deprived of the commodity or of the service, but that amount which the owner finds to be best adapted to maintain and increase his aggregate profits. He will in determining this amount give consideration to all elements of cost, to his investment, to the hazards of the business, to the possible profitable extension of his business, to the value of the service and to profit. When rates are thus established by an owner, the net earnings bear no definite or fixed relation to the investment and the value of the business is to be determined from a consideration of the present and prospective net earnings and of those elements which determine the permanency of the business. In such a case, in other words, it is proper to capitalize assured net earnings to determine value. When value thus ascertained is in excess of cost, the excess represents profit and such excess has been all too frequently used as a basis for "watering" stock and for over-bonding. Not only has this been the case, but, by ignoring depreciation, or by making inadequate provision for the replacement of worn-out parts, it has sometimes been possible to make the net earnings appear greater than they actually were, and thus to further swell the apparent though, in part, fictitious value.

It is fundamentally sound policy to encourage the development of the natural resources of any country, state or region by permitting those who establish public utilities to earn a reasonable profit on the investment made for the benefit of the public, and to earn a fair interest rate, provided that the investment has been wisely made.

The Public Utility Affects the Value of Other Property. — No necessary public utility can be established without at once adding to the value of all real property in the district which it serves. The profit resulting from the establishment of the utility is general. No water-supply system can be put in to serve a municipality, no work for the improvement of a water-supply, such as high pressure fire protection can be added to the system without adding value to vacant lots as well as to those already built upon. Value is added when gas and electricity are brought

within reach, when streets are improved, when sewers are built, when tunnels or bridges for better intercommunication are constructed. The addition of this value cannot always be immediately capitalized but it is nevertheless an asset and should not be entirely lost sight of when rates are to be fixed.

This point is here brought out in order that the reasonableness of allowing the owner of the public utility a fair profit in addition to ordinary interest on his investment may be realized.

Public Contribution to the Revenue of Public Utilities. — The owner of real estate or other property even when not served by the utility, may reasonably be required to make a direct contribution toward its earnings. Not all of the revenue should, in other words, come from the rate-payers. It is frequently practicable to secure a contribution from the property owner who is not a rate-payer by establishing a high rate for the service rendered to the public and thereby reducing the rate for the service to the rate-payer, or by a remission of local taxes in whole or in part, or by granting a bonus when a new installation is involved.

When an effort was made some years ago by a rival telephone company to secure permission to operate in San Francisco, the inducement of free service for all city departments was held out. The advisability of accepting the offer of the outside company was passed upon by the author, who was at that time City Engineer of San Francisco, and the disadvantage of competition in the telephone service was pointed out. The new company did not acquire the requested privilege at that time, though it did so under a later administration. The acceptance of the offer to render service free to the City, while attractive, was unfair to the rate-payer who is thereby made to carry, not only the full burden entailed by the telephone service of which a part should be carried by the vacant lot, but also the additional amount which is saved to the City by free service.

While it would be correct in principle and economically sound to let a suitable part of the revenue of every public utility be provided by a general tax, the application of this principle is not as simple nor as easy as appears at first view. The main difficulty lies in the fact that tax rates are already high and that the layman sees the force of the argument that the payment for the individual service rendered should be measured by the cost of the service. It is convenient to ignore the unearned increment that goes to the parties who are not served by the utility and it has become customary to ignore the same, consequently, the practice of letting the rate-payer carry the full burden may be expected to continue with only rare exceptions.

Cost of Rendering the Service as a Modifier of Rates. - In the case of the service rendered by certain utilities special effort is made to determine the cost of serving various classes of customers in order that the charge for the service may bear some direct relation to the cost of rendering the service in each class. This may be unwise. The prime consideration is what should the rate-payer in each class be asked to pay when his ability to pay is weighed against that of the rate-payers in other classes. Perhaps the establishment of class service is not always the best procedure, but it is legitimate and at present apparently the only practicable way of conducting certain business enterprises. There is no intent to find fault with the system except only to say that, if cost to serve the individual rate-payer were made the criterion of the charge for the service, there would be practically as many rates as there are rate-payers. In the application of the principle, therefore, whether always sound or not, it has become customary to classify the service and within the established classes to make the charge uniform.

Consideration is to be given, then, when rates are to be fixed, not alone to the total amount of the earnings, but also to their distribution to various classes of consumers. This is a subject which will not be taken up in this volume, in which the problem under discussion is concerned with aggregate earnings and not with the distribution thereof to the individual rate-payer.

Deficient Earnings in Early Years. — Nearly every public utility begins operating at a loss. There may be a number of years during which the earnings will not be sufficient to cover

ordinary operating expenses (including depreciation) and these years are generally followed by others in which the return is not commensurate with what the investment should earn. The deficiency in the earnings of the early years in the case of any legitimate public utility should be made good to its owner in some way at some time. This may be done by treating any ascertainable deficiency in the earnings as a permanent investment representing cost of establishing the business or by making suitable provision to amortize it in a reasonable time. When, therefore, the appraisal of a rate-base or the fixing of rates for a long-established public utility is in question, it should be assumed to be quite as likely that past history will show a deficiency as that it will show an excess of earnings over what should be considered a reasonable return. It follows that past history relating to insufficient earnings should be brought under review and that there may be cases in which the contrary opinion of the U.S. Supreme Court, as expressed in the case of Knoxville vs. Knoxville Water Company (212 U.S., p. 14) (20 Sup. Ct. Rep., p. 148), does not strictly apply. In this case, quoted at greater length on p 134, the Court says:

"If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon overissues of securities, or of omission to exact proper prices for the output, the fault is its own."

The omission to exact prices for the output that would keep the investment unimpaired may be due to the fact that the service rates necessary to accomplish this would have been greater than the service would bear. The failure to secure adequate earnings is not always the fault of the owner of the property. It may even be the deliberate program to let the ample returns of future years make good the early deficiency as, for example, when the Unlimited Life Method of procedure is adopted.

In the Kennebec Water District Case (97 Maine 185; 54 Atlantic 6) the Court in its instructions to the appraisers of the properties of the Maine Water Co. said:

"The actual rates which may have been charged heretofore, and the actual earnings, are both admissible and material in determining the value of the plant. The value of the evidence, however, will depend upon whether the appraisers shall find that the rates charged have been reasonable."

The Basis of the Calculation when Rates are to be Fixed.— Enough has been said to show that, when an appraisal is to be made as a basis for establishing the rates to be charged for the commodity or the service output of a public utility, something else is needed besides value. The starting point in the last analysis is not the value of the property, but the amount of the legitimate investment necessary to establish the business supplemented by the volume of the business transacted. The legitimate investment is not necessarily the actual cost of constructing and developing the property and bringing it to a paying basis, but it is the amount which may reasonably be assumed to have been properly and legitimately invested in the public service. This amount may for convenience be called the "ratebase," and as here defined is to be distinguished from "value."

It is to be understood that this definition of the rate-base is more or less academic and has not yet obtained general recognition. The ordinary practice has been to attempt to make "present value" the basis of the calculation when fixing rates. The endeavor to do this, with inclusion in the amount on which an interest return is to be computed, of certain elements of value which can have no existence unless created by adequate earnings, has met with only indifferent success and has led the rate-fixing bodies into much confusion from which there now appears to be a strong desire to escape.

The "rate-base," as thus defined, may include some or all of the cost of establishing and developing the business. It would be unwise to lay down an inflexible rule that the entire cost of establishing and developing the business should be included in the rate-base because any such rule would result in placing a premium upon inefficient management. The aim should be to include a reasonable allowance for this item which in one case may be determined from cost records while in other cases it may have to be estimated from less dependable data and assumptions.

The rate-base should be made the basis of the calculation. The earnings present and prospective should be adequate to yield a proper return on the rate-base and they should moreover also be adequate to create some value in addition thereto which will be compensation to the owner for having established and for managing the utility and may be in lieu of appreciation which under this method of procedure would not always appear in the rate-base.

The Effect of a Bonus upon the Rate-base. — Attention has already been called to the fact that when the owner has received a bonus, in order to encourage construction, the original cost to him is reduced by the amount of the bonus. This circumstance should not be overlooked in determining the rate-base.

How the bonus paid to the owner may affect the rate-base can be made clear by an illustration. The case can readily be conceived of an irrigation system constructed at an average cost of \$30 per acre. The right to take water from this canal system is made the matter of contract, each land owner who takes water for a given tract paying to the canal owner a bonus of \$20 per acre. When all the land within the area to be served by the canal has acquired the right to take water from the canal, the owner will be out of pocket only \$10 per acre and not \$30. Full justice will be done to both the owner and to the irrigator if the rate-base in this case be estimated at \$10 and not at \$30 per acre, due consideration being given to all other elements that should be included in the rate-base.

Two Procedures may be Followed in Determining a Ratebase. — There are two standpoints from which the matter of determining the rate-base may be approached and the procedure in the matter of making an appraisal which is to serve as a basis for fixing rates will vary according to the standpoint taken.

In the one case, the investment is regarded as unimpaired, and no deduction is made for amortization or depreciation, and the amount of the investment properly determined is made the rate-base.

In the other case the amounts earned as depreciation and remaining unexpended are considered as being applied to the retirement of capital, in which event the rate-base is the properly invested capital less the accrued depreciation (it being assumed that current depreciation has actually at all times been earned and collected).

These two views have led to various procedures when the fair earnings of a public utility are to be determined — these will be given further attention in following pages.

The late Commissioner J. W. Eshleman, in writing the decision of the Railroad Commission of California in the matter of the application of James A. Murray and Edward Fletcher for an increase of water rates in the county of San Diego, Cal., says: "My own view is that the nearest and fairest approximation which may be made to a correct 'value' upon which a public utility shall be allowed to earn is the amount of the investment wisely made, and this view is not at all in conflict with the position of the courts in this regard." The Commissioner as appears from this statement recognizes the desirability of making "investment" and not "value" the basis of the calculation but apparently feels the necessity of reconciling the use of the amount invested with the decisions of the courts, which, for the most part, still hold "value" to be the starting point.

Factors to be Considered in Establishing the Rate-base. — In determining what should be regarded as the legitimate reasonable investment in any public utility to serve as a basis for computing rates, some consideration may be given to certain factors which aid in determining what the value of the property resulting from net earnings should be. Such factors are the capitalization of the business enterprise, the original cost, the cost of reproduction new, the depreciated or present value, the taxation value and the reasonable charge for the service rendered. There should be taken into account, too, in the broad problem of determining net earnings, the past history,

sacrifices made by the owners to establish the business, financial aid by the public, the quality of the service, the cost of like service rendered elsewhere and other similar matters. The aim will be to establish, first, a rate-base on which the owner is to get a fair interest return and, second, to ascertain what the net earnings should be in addition to this return to create such a value as will be fair to both the owner and to the public.

Comments on Base Physical Cost by the Wisconsin Railroad Commission. — Referring to the use of the original cost of the properties of the LaCrosse Gas and Electric Co. as the basis of the calculation the Wisconsin Railroad Commission states (Wis. R. C. R, Vol. 2, p. 16) in substance that it is quite likely that, because of local conditions or the manner in which the plants have been acquired, the cost of these plants to the present owner is considerably higher than the figures given (cost of reproduction). Owing to progress in the electrical field most plants have had to be rebuilt or re-equipped. The cost to present owners of such plants is likely therefore to be much greater than either the cost of the original plant or the cost of reproduction new.

The Commission in the Antigo Water Case (Wis. R. C. R., Vol. 3, p. 631) says:

"The original cost, the cost of reproduction new and the present value bear a very close relation to the physical property of the plants and are therefore of the greatest importance in determining the value of the same. As to which one of these three elements is of the greatest importance in fixing this value, is a matter that largely depends upon the circumstances in each case and may also be more or less affected by the purposes for which the valuation is intended. . . . No matter what the differences between these three cost values may amount to, each one of these items constitutes evidence of the true value and should therefore be carefully considered."

The Commission also says that the burden of unreasonably high construction cost should not be charged to the ratepayers. Comments on Value and Depreciation by the Wisconsin Railroad Commission. — Substantial recognition of the principle that present value is not a proper rate-base is found in the findings of the Wisconsin Railroad Commission in the case of the Superior Commercial Club vs. Duluth Street Railway Co. (Wis. R. C. R., 1912, Vol. 11, p. 1 to 21). The Commission makes the following statement:

"A valuation of the physical property of the Superior division of the company as of June 30, 1911, showed a cost new of \$717,538 and a present value of \$487,236. When the present value of the physical property of 1911 is increased by the present value of that part of the property located in Duluth, but chargeable to Superior and which cannot greatly exceed \$70,000. when additions of about \$10,000 are made for working capital and when proper allowances for depreciation and going value are added, it will be found that the total amount does not quite reach the cost value new. In fact, it does not greatly exceed \$700,000 This sum finds support in the cost of reproduction of the plant and the business as well as in their original cost. . . . As under normal conditions investors are entitled to have their property or investment kept intact, it follows that the amounts, which have been properly set aside for such purposes or for depreciation, in accordance with the provisions of the law and the rules of the Commission, should in the instant case be included in the amount on which returns are allowed. On the other hand amounts earned for depreciation but withdrawn or used for other purposes than provided by law should not be so included."

Again in the case of the Stevens Point Lighting Co. relating to service and rates the Commission says (Wis. R. C. R., Vol. 14, p. 364):

"The failure of a utility to make allowance for depreciation if the earnings have been sufficient is tantamount to a withdrawal of capital from the business and the cost of reproduction new must be diminished in determining the fair value upon which the reasonable return allowed is to be based when an adequate reserve for depreciation has not been provided. The utility is, however, entitled to earn an amount sufficient to offset future depreciation."

Other Factors Affecting Rates. — While it is important to establish a rate-base whenever rates are to be fixed, there may be cases in which other circumstances are of equal moment with the rate-base as a guide to the allowable earnings. It may happen that the public service requires only a small investment of capital compared with the volume of the business that is transacted, and it may then be more desirable and equitable to bring the compensation of the owner into some relation to the volume of business transacted rather than to the capital which is invested in the business.

The case may readily be conceived of a concern such as an express company which rents its office facilities and operates under contract with railroad and steamship companies and which, outside of its trucks and other vehicles for the local distribution of the parcels entrusted to its care, has made no investment of any moment. It would be vain in such a case to attempt a regulation of rates based solely upon a fair return upon the invested capital. The whole field must be brought into view. The volume of business transacted, and the value that would be created if earnings are allowed which exceed, in some definite fashion, the cost of conducting the business, should receive due consideration. If earnings are thus allowed which exceed the cost of operation by 10 to 15 per cent, this would not seem unreasonable unless the resulting rates are, in fact, more than the traffic can or should bear.

Compensation for Hazard. — The risk of loss assumed by the owner when he undertakes the development of a public service enterprise is an element for consideration when a limit is to be set upon the allowable earnings. This fact is generally recognized. The compensation for this risk should be determined on the basis of the risk that may be justly contemplated by those who enter upon similar ventures under like conditions. This principle is clearly set forth by the Supreme Judicial Court of Maine in its instructions to the appraisers of the Maine Water Company's properties in the Kennebec Water District Case (97 Maine 185; 54 Atlantic 6):

"The reasonableness of the rate may also be affected, for a time, by the degree of hazard to which the original enterprise was naturally subjected; that is, such hazard only as may have been justly contemplated by those who made the original investment, but not unforeseen or emergent risks. And such allowance may be made as is demanded by an ample and fair public policy. If allowance be sought on account of this element, it would be permissible at the same time to inquire to what extent the company has already received income at rates in excess of what would otherwise be reasonable, and thus has already received compensation for this hazard."

Earnings in Relation to the Rate-base and to the Volume of Business. — If it can be accomplished without making rates unreasonable, there should be not only an interest allowance upon the rate-base equal to the current rate on money invested in industrial and related enterprises, preferably about 6 per cent per annum, but also some additional allowance, perhaps generally less than 5 per cent, unless conditions are unusual, as a reasonable participation in the general prosperity of the community (this in lieu of appreciation), plus some percentage allowance on the reasonable cost of operation (not including in this cost the interest on the invested capital). This latter allowance should, perhaps, be so graded that it will be small when the investment is large in relation to cost of operation and that it will approach 10 per cent or, in some cases, be even more, if the investment in relation to the cost of operation is small. The objection which may be urged to such an arrangement, that it will make it to the interest of the owner to inflate the cost of operation, is to be weighed against the danger of discouraging investment in public utilities if any less liberal policy is pursued. Moreover, under the control which is now exercised over the public utility by the public service commissions, this objection will lose much of the force which it might otherwise have.

The capitalization of the percentage allowance on the cost of operation, plus any other allowance (except amortization) in excess of a fair interest on the investment, will represent the sum of intangible values, in excess, of course, of such portions thereof as have found a place in the rate-base.

The Franchise Value and the Rate-base. — While recognizing that earnings may be so high that a franchise has value the Wisconsin Railroad Commission places itself squarely on record against the inclusion of such value in the value which is made the basis for fixing rates. The Commission in the Antigo Water Co. case (Wis. R. C R., Vol. 3, p. 727) says in substance that though earnings may be high enough to yield a surplus that may be made the basis for determining a franchise value, properly subject to taxation, this fact by no means implies that these earnings or any value based thereon should also be the basis for rate-fixing. Taxation, according to the Commission, is based on the ability to pay. Rates should be based on the cost of service (measured by expenses) including a fair return on the investment. The Commission apparently recognizes, what the author endeavors to make clear, that "taxation value" and therefore "market value" really depends on the earnings and therefore, too, upon the rates.

In the case of the City of Appleton vs. Appleton Waterworks Co. the Commission again says (Wis. R. C. R., Vol. 5, p. 282) that in estimating capital upon which return is to be made the franchise has no value, because franchise value is produced by the rates charged. It is proper, however, to tax the company on its franchise value which would be paid for if the business were sold.

The Commission in the case of the State Journal Printing Co. vs. Madison Gas and Electric Co. (Wis. R. C. R., Vol. 4, p. 586) declares franchises to be monopolistic in their nature. They belong to the community. Their value has been created by the growth of population and by economic and social developments rather than by individual effort. Belonging to the public, the right of control and of the disposal thereof also rests in the community. Under these conditions according to the Commission, there appear to be no good grounds upon which the value of exclusive privileges of this kind should become private property.

CHAPTER IX

POSSIBLE PROCEDURES WHEN THE RATES FOR A PUBLIC SERVICE ARE TO BE FIXED

The Prime Consideration. — When a method of procedure is to be selected under which the necessary earnings of a public utility are to be determined, the first consideration will be the fairness of this procedure both to the rate-payer and to the owner of the utility. Although there are various methods of procedure which have been adopted throughout the country, and which may be shown to be correct under various restrictions as to application and under the assumption that their application is continuous from the beginning, it will not do to accept any particular one as equitable or advisable in every case.

It is elsewhere shown that when capital is assumed to be retired at a rate which keeps pace with theoretical depreciation, serious consideration must be given to the departure of actual life from the probable life of the various parts of the utility which have a limited period of serviceability. It is made clear that a proper accounting system must be adhered to and that a change of procedure may result to the disadvantage either of the owner or of the rate-payer.

The Preferable Method of Procedure. — Apart from the fairness which is essential, the adopted method of procedure should not impose unnecessary burdens upon either party. That method which requires the least earnings in the early years of their life will be preferable in the case of newly established enterprises. In the case of properties that have been long in use, past history must be taken into account, as it will not do to arbitrarily assume that the business has been a profitable one from the beginning. The presumption should rather be the

other way, the probability that there were losses in the early years should not be overlooked.

The various methods of procedure have had their origin in the various plans that have been adopted for making provision for amortization of capital and the replacement or renewal of discarded articles. When the distinction between amortization and the replacement requirements is disregarded, such methods of procedure as the Straight Line Method and Equal Annual Payment Method are the result. When amortization is treated as a matter apart, such methods as the Sinking Fund Method and the Unlimited Life Method are the natural outcome.

The Sinking Fund Method.—The "Sinking Fund Method" of making appraisals and of determining necessary earnings is a method under which the annual allowance for replacements is uniform in amount. This replacement allowance (frequently referred to as depreciation) is the annuity which, together with compound interest at the rate of the net earnings of the property, will during the probable life of an article amount to its replacement cost. The replacement, it is assumed, is to be accomplished at the end of the probable life of that article. There is no repayment of capital. The investment remains undiminished. This is, therefore, a 100 per cent valuation method.*

The amount which goes into the replacement fund does not retire capital. It generally remains in the business and may be assumed to earn the same amount as any other capital used in the business. Nevertheless, it is held by some authorities that this fund should be treated as though invested in absolutely safe securities and that, therefore, the interest rate introduced into the calculation should be about 4 per cent. But whether the fund be invested in outside securities or in the business, it is a fund which should be separately accounted for. Its earnings are earnings of the business and make the amount which must be collected from the rate-payer correspondingly less. When the fund remains in the business, it should be charged with the regular interest rate of return and this amount of the earnings

^{*} Trans. Am. Soc. C. E., Vol. LXXV, p. 828.

should be added to the fund and will not be available for distribution as profit.

To illustrate — if the investment is \$1,000,000 and out of surplus earnings \$100,000 are invested in betterments, the total investment becomes \$1,100,000 of which \$100,000 is due to the replacement fund. If the net earnings are 6 per cent or in the aggregate \$66,000 per annum, only \$60,000 of this will be available for distribution to the owners and the other \$6000 must be added to the replacement fund.

Equal Annual Payment Method. — The " Equal Annual Payment Method," as described in the report of the American Society of Civil Engineers' Special Committee on Valuation, presented to the Annual Meeting of January 21, 1914, refers to a method which makes the annual depreciation, or amortization increment an amount increasing, from year to year according to a definite law. The annual depreciation is estimated by sinking fund methods. It is equal to the annuity which will retire the remaining value in the remnant of the original probable life term and, when added to the interest on the remaining value (uniformity of interest rates being assumed), the sum will be uniform from year to year throughout the probable life term. When depreciation allowances computed by this method are actually earned, they are considered as refunds of invested capital. The remaining investment in that case decreases as the earned depreciation decreased by expenditures for replacements accrues. Depreciation earnings may be regarded as being thus applied to retire capital, but this method of estimating what should be earned from year to year is undesirable, because it involves frequent, elaborate and cumbersome re-estimates of remaining value, requiring the service of experts, while as usually applied, it is identical in results with the Sinking Fund Method, which has the advantage of simplicity.

The Straight Line Method. — The "Straight Line Method" of estimating the annual depreciation or the annual amortization installment is that method which makes the annual amortization of capital uniform throughout the probable life of an

article. The annual amortization installment is estimated from the original cost of an article less residual value and its probable life, by dividing the cost less residual value by the number of years of probable life. This method, like any other amortization method, is justified on the theory of the immediate application of the annual depreciation installment to the retirement of capital. In this respect both this method and the Equal Annual Payment Method differ essentially from the Sinking Fund Method and from the Unlimited Life Method, under neither of which any retirement of invested capital is taken into consideration.

Unlimited Life Method. — The "Unlimited Life Method" of procedure, when rates are to be fixed, is justified by the fact that public utilities may generally be regarded as having perpetual life and that, ordinarily, capital need not be retired unless the property is to be purchased. A public service property taken in its entirety may be treated, therefore, as though it had unlimited life. The property is kept in good condition by the repair of its parts and by replacements, as these parts become useless and have to be discarded. No part of the investment, if there be unlimited life, need be returned to the owner, but as the plant grows old and one part after another has to be replaced. he must be allowed to recover in the earnings the cost of each article as replaced It is on this principle that many complex and, particularly, publicly owned properties are operated. In the early years of the life of a property made up of many parts, such as rails and ties, the replacement requirements will be small. As the property acquires age, the replacements - provided that extensions are relatively unimportant and negligible in comparison with the extent of the property under consideration - will gradually increase to nearly the amount indicated by the Straight Line Method. The departure from this amount will be dependent upon the annual extensions of the system in relation to the entire investment. There will be ultimate agreement between the replacement requirement and the amount estimated by the Straight Line Method if the plant is one that has ceased to grow. The valuation for rate-fixing purposes under the Unlimited Life Method of procedure will be the amount of the investment without deduction of depreciation, and the replacement requirements, until definitely ascertained by experience, will be approximated from the estimated cost of effecting the replacements, with due consideration of the age and expectancy and the probable life new of the individual parts of which the property is made up.

Sinking Fund Method Illustrated. — Some of the principles that govern the establishment of rates may be made clear by the use of an illustration. An electric generator with a 20-year life will serve the purpose. Assume, in the absence of any accepted method of procedure, that remaining value is determined by deducting from cost the accrued depreciation estimated by compound interest sinking fund methods. Suppose the generator to be a part of a light and power system and suppose. further, that it has reached the last year of its useful life and will have no scrap value. A purchaser will value the generator, if he estimates interest on a 6 per cent per annum basis, at about 8 23 per cent of its original value and this is all that he will pay for the same. He takes upon himself, when he buys the light and power plant, an obligation to replace the generator with a new one at the end of another year. He must then renew the investment represented by this article. The obligation which he voluntarily takes upon himself at the time of his purchase to replace the generator in a year is 91.77 per cent of the cost of a new generator. At the end of the life of the generator, he will have received in his earnings the last increment of the original investment in the generator, or 8.23 per cent of its cost, and he will meet his obligation to continue in business by acquiring a new generator and thereby renewing the full original investment in this particular appliance.

It makes no difference whether there is only one generator, or whether there are 20 in use. The principle is always the same. Moreover in such a simple case as that of 20 generators of all possible ages, each with a useful life of exactly 20 years, the accrued obligation to replace these generators when worn

out (6 per cent per annum interest being again used for purposes of illustration) will be 38 per cent as shown by the formula on page 93.

In other words, the remaining value of the 20 generators (cost less depreciation, as ordinarily noted) will be about 62 per cent. No purchaser would include the 20 generators in his valuation of the property at more than 62 per cent of their aggregate cost; but he would, nevertheless, and with reason, expect to be allowed to earn interest on 100 per cent of their cost new, claiming rightfully that he is entitled to the same rate of income return on unexpended annual replacement increments as he is entitled to earn on the rest of his invested capital. He will justify this claim by pointing out that the earnings on any accumulated replacement fund are not available for any other use than the replacement of worn-out property; that such earnings do not, therefore, represent income; and that it is for this reason he is entitled to have the aggregate annual replacement allowance, together with accumulated interest, treated as interest-bearing capital.

California Law Restrictions upon the Depreciation Fund. — In this connection attention may be called to the laws of California (Statutes of 1911, First Extra Session, Ch. 14, Sec. 49), which prescribes that the Railroad Commission (having the duties of a public service commission):

"may from time to time ascertain and determine and by order fix the proper and adequate rates of depreciation of the several classes of property of each public service utility. Each public service utility shall conform its depreciation accounts to the rates, so ascertained, determined and fixed, and shall set aside the moneys so provided for out of earnings, and carry the same in a depreciation fund, and expend such fund only for such purposes and under such rules and regulations, both as to the original expenditures and subsequent replacement as the Commission may prescribe. The income upon investments of moneys in such fund shall likewise be carried in such fund."

It is here apparently recognized that the depreciation fund should be used for the sole purpose of replacing worn-out parts and that in regulating rates, consideration need be given only to the amount of capital reasonably and properly invested without any deduction for depreciation.

The earnings, in other words, to be adequate, must include at least operating expenses of every character, a proper allowance for present and prospective replacements (so-called current depreciation) and a reasonable income computed at the proper interest rate on the invested capital.

Comparison of Sinking Fund and Equal Annual Payment Methods. — Enough has already been said to show that the so-called Sinking Fund Method of procedure, when no distinction is made between actual and probable life, is exactly equivalent in its results to the Equal Annual Payment Method. The former, in which the rate-base is 100 per cent of the investment, involves a single computation of the annuity which at compound interest will amount in the probable life term to the capital invested in any article. The latter, under which accrued depreciation is deducted from the investment, requires a re-estimate of the current amortization (depreciation) increment from year to year and a new annual estimate of accrued depreciation.

Amount of Amortization or Amount in the Replacement Fund.

— If under any method of procedure the owner of a plant of numerous parts and all possible ages is required to place all sums earned as "depreciation" or earned to meet "replacement requirements" into a special fund, and the depreciation or replacement requirement has been actually earned from the beginning, the amount of amortization accomplished out of such a fund or the theoretical balance on hand, after meeting all current replacement requirements, will be as follows:

Under the Straight Line Method of estimating amortization for old plants of full growth, the theoretical accumulation should be 50 per cent of the investment in perishable property—this accumulation is independent of the interest rate and this fund's earnings are of no concern to the rate-payer.

Under the method of estimating the "accrued obligation to

replace" by the Sinking Fund Method, for old plants, with interest at 6 per cent and the hypothesis that actual life will conform to probable life, the theoretical accrued obligation to replace or what is equivalent, the theoretical accumulation in the fund as shown in Table 2, page 94, will be:

						F	er cent
For	articles	having	5	year	life,	about	38
	"	"	10	"		"	40
	"	"	20	"		"	38
	"	46	30	"		"	34
	"	"	40	"		46	31
	"	"	50	"		"	28
	"	"	60	"		"	25

These percentages apply strictly, of course, only to cases in which the age of the plant is greater than the life of its parts. They indicate the extent, however, to which an accumulating replacement or depreciation fund may exceed the demands upon it. In some measure, approximating this unexpended surplus, such a fund is available for outside investments. Interest on the same, when it represents the obligation to replace, must be earned by the owner for the benefit of the plant, consequently, it makes no difference to what purpose he applies this surplus, provided only that ample assurance remains that replacements will actually be made when necessary. Interest is to be added to this fund even when the same is invested in the plant and by the amount of such interest, the earnings will not be available for distribution as profit.

When the capital is being amortized, the accumulations in the amortization fund are the absolute property of the owner. They are not subject to control by the rate-payer, yet, here, too, proper assurance may be expected that replacements will be duly made and it would not be unreasonable to expect an adequate special fund to be maintained to meet emergencies.

Soundness of 100 Per Cent Appraisal as Rate-base. — In order to show beyond all question that the method of valuing at the investment without deduction of the depreciation in fix-

ing rates, under continuous application from the beginning, is correct and proper, an electric generator with a life of 20 years, which has served 15 years, may again be taken as a basis for an illustration.

The usual assumption is made, for the purpose of this illustration, that there is no change in the cost of this article during its life, that it has no scrap value and that it will go out of service when exactly 20 years old. Interest in this illustration is taken at 6 per cent per annum.

By the "Equal Annual Payment Method" (correct if applied from the day the article went into use and if there were, in fact, agreement between actual and probable life):

Original investment		\$100.00	
Life (new)	20 years		
Time in service .	15 years		
Remaining life	5 years		
Accrued depreciation (amortization to date)		63.27	
Remaining value		36 73	
Interest on remaining value			\$2 20
Annual depreciation or annual amortization in	acrement for s	ixteenth	
year			6 52
Required net earnings .			\$8 72

In this case the depreciation in the 16th year is that amount which, invested annually at 6 per cent, will retire the remaining value \$36.73 in the remaining 5 years of life.

By the "Sinking Fund Method" (correct under continuous application from the beginning and agreement between actual and probable life) the computation is as follows:

Permanent investment			\$100	
Life (new)		20 years		
Time in service		15 years		
Remaining life		5 years		
Interest on the investment				\$6.00
Annual depreciation or annual replace	cement i	increment	for any	
year				2 72
Required net earnings				\$8.72

While it can be shown that for each year the earnings should be \$8.72 on \$100 of original capital investment, a new cal-

culation is necessary, as already stated, for each perishable article and for each year under the Equal Annual Payment Method, while for the more rational Sinking Fund Method, a single simple calculation suffices for each group of articles of the same probable life term.

For comparison the application of the other methods of procedure to the same generator may be of interest.

By the "Straight Line Method" (correct under continuous application from the beginning and agreement between actual and probable life) there will be:

Original investment	. \$100
Life (new).	20 years
Time in service	15 years
Remaining life.	5 years
Accrued depreciation (amortization to date)	75
Remaining value	75 \$25
Interest on remaining value	\$I 50
Annual depreciation or amortization incre	ment for sixteenth
year	. \$5 ∞
Required net earnings	\$6 50

By the "Unlimited Life Method" on the assumption that there is only a single generator and that the annual replacement increment is estimated by the compound interest annuity method:

Original investment	 		 \$100
Life (new)	 	20 years	
Time in service	 	15 years	
Remaining life	 	5 years	
Interest on the investment.			 \$6.00
Annual replacement increment			 2 72
Required net earnings			 \$8.72

The foregoing comparisons, as above stated, are based on the assumption, which can never be fully realized, that there is absolute agreement between the actual and the probable life of each article.

Effect of Departure of Actual Life from Probable Life. — To illustrate the application of the various methods of procedure with some regard to the fact that of many articles having the

same probable life when new, some will actually serve a shorter and others a longer time, let it be assumed that a group of articles is under consideration whose average life, and therefore whose probable life, is 5 years. Let it be further assumed in order to give definiteness to the problem that there will go out of use and be replaced 4 per cent of these articles at the end of the first year; 8 per cent at the end of the second year; 12 per cent at the end of the third year; 16 per cent at the end of the fourth year, 20 per cent at the end of the fifth year; 16 per cent at the end of the sixth year; 12 per cent at the end of the seventh year; 8 per cent at the end of the eighth year; and 4 per cent at the end of the ninth year.

This hypothesis of failures has already been referred to and its basis explained in Chapter VI. Two courses are open. Either the allowance for amortization or so-called depreciation is extended for each article throughout the period of its probable life, regardless of whether the article fails early or survives, or, as an alternative, the annual allowance of amortization or depreciation is continued throughout actual life and stops with the failure of each article. Each article which takes the place of another which is discarded is supposed to receive the same consideration and the same treatment in the accounts as an original article.

The result is shown in Table 10.

Explanation of Table 10. — In Table 10 there is noted for the Equal Annual Payment Method in column A the amortization requirement computed from amortization tables, entering the tables not with age, but with probable life new less the expectancy; and in column B, the amortization requirement as determined from Table 20 by taking the difference between the remaining value at the beginning of successive years, giving, as explained, consideration to the new articles introduced each year to replace discarded articles. When amortization of capital continues during the actual life of each article, terminating when the article goes out of use, the Equal Annual Payment Method is no longer, strictly speaking, an equal annual payment method

because the interest on the remaining value plus the annual depreciation is no longer constant in amount.

TABLE 10. COMPARISON OF RESULTS

THE SINKING FUND, EQUAL ANNUAL PAYMENT AND UNLIMITED LIFE METHODS

When consideration is given to the requirements for periods determined either by probable life or by the actual life of articles

FIVE-YEAR PROBABLE LIFE NUMEROUS ARTICLES Failures as noted in the text — 6 per cent interest

	New invest	Sinking Fu	nd Method	Equal Annual Payment Method		Unlimited Life Method	
Year.	for each \$100 of orig invest beginning	Repl.	Repl allowance	Amort during	Amortizat actua	ion during il life	Replace- ment re-
	of year	during prob life	during actual life	prob life	A	В	quirement
1 2 3 4 5 6 7 8	\$100 00 4 00 8 16 12 65 17 64 23 34 21 97 21 16 20 54 19 73	\$17 74 18 45 19 89 22 13 25 26 11 66 14 85 17 17 18 58 19 95	\$17 74 17 74 17 74 17 74 17 74 17 74 17 74 17 74 17 74 17 74	\$17 74 19 51 22 13 25 72 30 40 12 63 16 32 19 11 20 98 21 46	\$17 74 18 66 19 31 20 09 19 89 19 64 19 69 19 34 20 41 20 44	\$15 10 17 46 19 65 17 30 19 40 20 30 20 60 21 10 19 70 18 30	\$ 4 00 8 16 12 65 17 64 23 34 21 97 21 16 20 54 19 73 18 31

For comparison with the Sinking Fund Method and the Equal Annual Payment Method, there are noted in the last column of Table 10 the annual replacement requirements, which are the amounts to be provided if the Unlimited Life Method of procedure is adopted. By the plan of amortizing the cost of each article or for providing a fund for its replacement in the exact term of its probable life, there will be a rapid accumulation in the replacement or depreciation fund, or a rapid amortization of capital in the early years, which is an undesirable feature of operation. To find the necessary earnings by each of the several methods, interest on the capital remaining as an investment is to be added to the amounts noted in the table.

Amortization During the Probable Life Term. - When only a single article is involved, as in the case of a steamboat, the disadvantage of adopting the plan of amortizing its cost or of providing a replacement fund in the exact term of its probable life is apparent The steamboat may meet with an accident in the early years of its life. If it does and is replaced with a new one, amortization of the remaining value of the first steamboat plus the amortization of the cost of the new steamboat will be necessary The burden will fall upon the rate-payer. Under the alternative plan of continuing a uniform annual amortization allowance during the actual term of the steamboat's service there will be no effect apparent upon the required earnings by an early failure of the steamboat. The loss of the first steamboat will fall upon the owner, but, if rates are equitably established, the loss will be made good to him in the course of time by reason of the survival of other steamboats and the continuance of the depreciation or amortization allowance after original cost has been amortized.

The wise plan is the one in which there is the least disturbance of the rates and in which, so far as may be, the required earnings will be least in the early years.

The Book Accounts Under Various Methods of Procedure.— The book accounts relating to the foregoing tabular illustration when numerous articles all with a probable life new of 5 years are involved would show the following for each \$100 of original investment:

a. Sinking Fund Method with a replacement allowance for the term of the probable life of each article, regardless of the time of its actual failure. (Each article which replaces a failing article is here treated as a new article.)

	Dr. side of Ledger		Cr. side of L	edger
ıst yr.	To depr. or amort. allowance	\$17 74	By renewals	\$4 00
2nd yr.	To int. on balance \$13 74	0 82		
	To depr or amort. allowance	18 45	By renewals	8.16
3rd yr.	To int. on balance 24 85	I 49		
	To depr. or amort. allowance	19.89	By renewals	12.65
4th yr.	To int. on balance 33 58	2 OI		
	To depr. or amort. allowance	22.13	By renewals	17.64

5th yr.	To int. on balance 40 08	2 40		
•	To depr or amort allowance	25 26	By renewals	23 34
6th yr	To int on balance 44 40	2 67		
-	To depr or amort, allowance	11 66	By renewals	21 97
7th yr	To int on balance 36 76	2 21		
	To depr or amort allowance	14 85	By renewals	21 16
8th yr.	To int on balance 32 66	1 96		
-	To depr or amort allowance	17 17	By renewals	20 54
9th yr	To int. on balance 31 25	ı 88		
	To depr or amort allowance	18 58	By renewals	19 73
10th yr	To int. on balance 31 98	1 92		
	To depr. or amort allowance	19 95	By renewals	18 31
	Totals	\$203 04		\$167.50
	Balance			35 54

At the end of the tenth year the amount in the depreciation fund would be \$35.54 for each \$100 of original investment.

b. The Sinking Fund Method accounts, if the replacement requirement or depreciation were estimated during the actual life of each article, would be charged, at the end of each year with \$17.74 and interest on the annual balance. It would be given credit for \$4.00 renewals the first year, \$8.16 the second year; \$12.65 the third year and so on.

	Dr side of Ledger		Cr. side of	Ledger
ıst yr.	To repl allowance	\$17 74	By renewals	\$4 00
2nd yr.	To int. on \$13.74	0 82		
	To repl allowance	17 74	By renewals	8 16
3rd yr.	To int. on \$24.14	I 45		
	To repl. allowance	17 74	By renewals	12 65
4th yr.	To int. on \$30 68	r 84		
	To repl. allowance	17 74	By renewals	17.64
5th yr.	To int. on \$32 62	1 96		
	To repl. allowance	17 74	By renewals	23.34
6th yr.	To int. on \$28.98	I.74		
	To repl. allowance	17 74	By renewals	2T 97
7th yr.	To int on \$26 49	I 59		
	To repl. allowance	17 74	By renewals	,21.16
8th yr.	To int. on \$24.66	1.48		,
	To repl. allowance	17.74	By renewals	20 54
9th yr.	To int. on \$23 34	I 40		
	To repl. allowance	17.74	By renewals	19.73
10th yr.	To int. on \$22.75	I 37		
	To repl. allowance	17 74	By renewals	18.31
	Totals	\$191.05		\$167.50
	Balance			23.55

At the end of the tenth year there would be in the replacement fund \$23.55 for each \$100 of original investment.

c. The amortization and replacement account in the case of the Equal Annual Payment Method, if amortization be allowed during the probable life term of each article, regardless of whether the article fails early or survives, would be about as follows: (Every article which replaces another is here treated as a new article)

/				
	Dr side of Ledger		Cr side of L	edger
ıst yr	To allowance for amort. and repl.	\$17 74	By renewals	\$4 00
2nd yr.	To allowance for amort and repl.	19 51	By renewals	8 16
3rd yr	To allowance for amort, and repl.	22 13	By renewals	12 65
4th yr.	To allowance for amort. and repl.	25 72	By renewals	17 04
5th yr.	To allowance for amort, and repl.	30 40	By renewals	23 34
	Totals	\$115 50		\$55 79
	Balance			\$50 71
	Balance	C		
	Datance	\$59 7I		
6th yr	To allowance for amort. and repl	12 63	By renewals	\$21 97
7th yr	To allowance for amort and repl.	. 16 32	By renewals	21 16
8th yr.	To allowance for amort and repl.	. 19 11	By renewals	20 54
9th yr.	To allowance for amort, and repl	20 98	By renewals	19 73
10th yr.	To allowance for amort, and repl	. 21 46	By renewals	18 31
•	Totals	\$150 21	-	\$101 71
	Balance	-		48 50

The amount available for amortization at the end of the fifth year should be according to this account about \$59.71 for each \$100 of original investment and \$48.50 at the end of the tenth year. In actual bookkeeping the account would have been balanced at the end of each year. The remaining investment, or the present value, is shown by this account to have been \$40.29 on each \$100 at the end of the fifth year and \$51.60 at the end of the tenth year. The large reduction of capital which results from this method of applying the Equal Annual Payment Method shows its undesirability.

d. The amortization and replacement account in the case of the Equal Annual Payment Method, if amortization be estimated during the actual life of each article (plan A, Table 10) and the balance is applied at the end of each year to retire capital, would show:

	Dr. side of Ledger			Cr. side of Led	ger
ıst yr.	To allowance for amort and repl	\$17	74	By renewals	\$4.00
2nd yr.	To allowance for amort, and repl	18	66	By renewals	8 16
3rd yr.	To allowance for amort, and repl.	19	31	By renewals	12 65
4th yr.	To allowance for amort and repl.	20	09	By renewals	17 64
5th yr.	To allowance for amort, and repl.	19	89	By renewals	23 34
	Totals	\$95	69	\$	65 79
	Balance				19 90
	Balance	\$10	00		
6th yr.	To allowance for amort, and repl.	-	64	By renewals	21 97
7th yr.	To allowance for amort, and repl	19	69	By renewals	21 16
8th yr	To allowance for amort and repl.	19	34	By renewals	20 54
9th yr	To allowance for amort. and repl.	20	41	By renewals	19 73
10th yr.	To allowance for amort, and repl.	20	44	By renewals	18 31
-	Totals	\$119	42	\$1	01 71
	Balance				17 71

In actual bookkeeping the account would have been balanced every year. The amount available for amortization at the end of the fifth year should be about \$19.90 and at the end of the tenth year about \$17.71.

e. The amortization and replacement account, in the case of the Equal Annual Payment Method, if amortization be estimated during the actual life of each article (plan B, Table 10) and the balance is applied at the end of each year to retire capital, would show:

	Dr. side of Ledger		Cr. side of	Ledger
ıst yr	To allowance for amort and repl.	\$15 10	By renewals	\$4.00
2nd yr.	To allowance for amort, and repl	17 46	By renewals	8 16
3rd yr.	To allowance for amort, and repl	19 65	By renewals	12.65
4th yr.	To allowance for amort, and repl.	17 30	By renewals	17.64
5th yr.	To allowance for amort. and repl.	19 40	By renewals	23 34
	Totals	\$88 91		\$65 79
	Balance			23.12
	By Balance	\$23 12		
6th yr.	To allowance for amort. and repl.	\$20.30	By renewals	\$21.97
7th yr.	To allowance for amort, and repl.	20 60	By renewals	21.16
8th yr.	To allowance for amort. and repl.	21.10	By renewals	20.54
9th yr.	To allowance for amort, and repl.	19.70	By renewals	19.73
10th yr.	To allowance for amort. and repl.	18 30	By renewals	18.31
	Totals	\$122.82		\$101.71
	Balance			21.11

The amount available for amortization at the end of the fifth year is shown by this account to be \$23.12 for each \$100 of original investment and \$21.11 at the end of the tenth year.

f. The amortization and replacement account in the case of the Straight Line Method (5-year probable life) would show the following:

10110 11 222	Dr side of Ledger		124 A. A. A.	T J
	• •		Cr side of	Leager
ıst yr.	To allowance for amort and rep	ol. \$20 00	By renewals	\$4 00
2nd yr.	To allowance for amort and rep	ol 20 00	By renewals	8 16
3rd yr	To allowance for amort and rep	ol 20 00	By renewals	12 65
4th yr	To allowance for amort, and rep	ol 20 00	By renewals	17.64
5th yr.	To allowance for amort and rep	ol 20 00	By renewals	23 34
	Sub-Totals	\$100 00		\$05 79
	Balance			34 21
	n n 1			
	By Balance	\$34 21		
6th yr	To allowance for amort and rep	ol 20 00	By renewals	\$21 97
7th yr	To allowance for amort and rej	ol 20 00	By renewals	21 16
8th yr	To allowance for amort, and rep	ol 20 00	By renewals	20 54
9th yr	To allowance for amort and rep	ol 20 00	By renewals	19 73
10th yr.	To allowance for amort. and rep	ol. 20 00	By renewals	18 31
	Totals	\$134 21		\$101 71
	Balance			32 50

These figures show that at the end of the fifth year about 34 per cent of the total cost of the depreciating articles would have been returned to the owner and at the end of the tenth year about 32.5 per cent. The remaining investment at 5 years would have been about 66 per cent and at the end of the tenth year about 67.5 per cent.

g. The replacement account in the case of the Unlimited Life Method would show the following:

	Dr. side of Ledger Cr. si		Cr. side of	Ledger
ıst yr.	To allowance for repl	\$4 00	By renewals	\$4 00
2nd yr.	To allowance for repl.	8 16	By renewals	8 16
3rd yr.	To allowance for repl.	12 65	By renewals	12 65
4th yr.	To allowance for repl.	17 64	By renewals	17 64
5th yr.	To allowance for repl.	23 34	By renewals	23 34
6th yr.	To allowance for repl.	21 97	By renewals	21 97
7th yr.	To allowance for repl.	21.16	By renewals	21 16
8th yr.	To allowance for repl.	20, 54	By renewals	20 54
9th yr.	To allowance for repl.	19.73	By renewals	19.73
10th yr.	To allowance for repl.	18 31	By renewals	18.3 1

Theoretically there would be no accumulation in the replacement fund because in the case of numerous articles the annual demand on the fund would be offset by the replacement allowance.

The simplicity of the Unlimited Life Method, as well as its advantage in requiring least earnings in the early years, appears from a comparison of such accounts as above presented.

Before leaving this subject attention may be called to the fact that the balance which these figures show to be probable in a replacement fund under the Sinking Fund Method of procedure and the amount of the amortization under the Equal Annual Payment Method do not agree with what would be found on the impossible hypothesis that all articles serve throughout their probable terms of usefulness and no longer. On this purely hypothetical assumption (see page 94) the fund should contain about 38 per cent of the cost of the depreciating articles after five years of operation (5 years probable life being here under consideration).

If the various methods of procedure and fixed rules of accounting are closely adhered to and a comparison of results be made at the end of the tenth year, numerous articles all having a probable life of 5 years being under consideration, the results in so far as the amount of amortization or the amount in the replacement funds are concerned should be about as follows:

On the hypot	hetical assumption of agreement	
between a	actual and probable life, and an	
amortizati	ion allowance estimated by com-	
pound into	erest methods, the original invest-	
ment shou	ald have been reduced by about	38 per cent
By the Sinkin	ng Fund Method, if the replace-	-
ment allo	wance continues during probable	
life, the a	amount in the replacement fund	
should be	about.	35 per cent
	ng Fund Method, if the replace-	001
	wance continues during the actual	
	amount in the replacement fund	
	about	24 per cent

By the Equal Annual Payment Method, if the amortization allowance be continued during the actual useful life, the original investment should have been reduced (so-called depreciation) as follows:	48 per cent
a. According to the plan A of estimating	
the annual allowance by about	18 per cent
b. According to the plan B of estimating	
the annual allowance by about	21 per cent
By the Straight Line Method, the original in-	
vestment should have been reduced by	
about.	33 per cent
By the Unlimited Life Method there would be	
no reduction of the invested capital and	
there would theoretically be no accumula-	
tion in the replacement fund.	

Rental Value as an Aid in Determining Present Value.—Rental value is a convenient aid in forming a clear conception of present or remaining value of any item of property. Take, for example, a high-duty pump and assume that the same be rented by Smith, the owner of a water-works property, from Jones on such terms that Jones will recover the cost of the pump during its life and interest on his investment. The care and the maintenance of the pump falls upon Smith. The rental value may be so determined that as the business grows the return to Jones of capital will be increased, but ordinarily, the pump being assumed to be in full service all the time, the rental value remains uniform throughout the period that the pump renders efficient service. If the interest rate agreed upon is 6 per cent and if it be further agreed that the probable life of the pump is 25 years, Jones will expect \$7.82 per annum

throughout the probable life term of the pump for each \$100 of its cost. The agreement will provide that he gets this sum annually so long as the pump remains an efficient appliance and that the payment shall cease as soon as the pump becomes useless. The present value of this pump to Jones for each \$100 of its cost, at any time of its life, will be the present value of an annuity of \$7.82 for the expectancy or remaining life of the pump. This will be true if he is under no obligation to replace it and if scrap value be disregarded

When the pump is new, the probable life or expectancy is 25 years and the present value will be 100 per cent.

When the pump is 10 years old, its expectancy may be about 17 years and the remaining value at that time \$82 for each \$100 of first cost or 82 per cent.

When the pump is 25 years old and still in good condition, its expectancy may be about 9 years and the remaining value of each \$100 of original cost at that time would be the present value of an annuity of \$7.82 for 9 years or \$53 or when compared with the total original cost, 53 per cent thereof.

If the pump is defective or is put out of service by some accident before it has served 25 years, Jones will be a loser. If it serves beyond the term assumed as the limit of its usefulness or beyond 25 years, Jones will be the gainer. If Jones is in the business of supplying pumps in large numbers on these terms he will find, if probable life has been correctly determined, that his capital is yielding 6 per cent per annum

Further Comparison of Methods with Tables and Diagrams. — A further comparison of the various methods of procedure which have found favor when rates for the output of public service properties are to be fixed, is given in the following pages with tables and diagrams where practicable. These comparisons are strictly theoretical and are based in each case as noted on the impossible assumption that the actual life of every item which goes to make up the property will coincide with the probable life of that item. The comparison nevertheless shows where these methods, if consistently applied from the beginning, would

lead and are therefore of value to the appraiser and to the property owner, as well as to the rate-regulating authority.

Tables 11 and 12 illustrate forcefully the fact that under the Equal Annual Payment Method the amount to be earned annually from the beginning of operations is uniform and that under the Straight Line Method the amount of earnings should be greatest at the beginning.

TABLE 11 STRAIGHT LINE METHOD (Hypothetical)

Cost of plant \$100 Probable life 10 years. Interest 6 per cent. No distinction is made between probable and actual life. The annual amortization or depreciation increment is \$10

Year	Accrued depreciation or amortization at end of year	Present value, re- maining investment or rate-base at be- ginning of year	Interest on rate- base.	Annual amortiza- tion and interest.	
ı	\$10	\$100	\$6 ∞	\$16 00	
2	20	90	5 40	15 40	
3	30	80	4 85	14 80	
4	40	70	4 20	14 20	
5 6	50 60	60	3 60	13 60	
6	60	50	3 00	13 00	
7	70	40	2 40	12 40	
8	80	30	1 80	11 80	
9	90	20	I 20	11 20	
10	100	. 10	o 6o	10.60	

TABLE 12. EQUAL ANNUAL PAYMENT METHOD (Hypothetical)

Cost of plant \$100.. Probable life 10 years Interest 6 per cent. No distinction 1s made between probable and actual life.

Year.	Annual depreciation or amortization increment.	Accrued depreciation or amortization at end of year.	Present value, remaining investment or ratebase at beginning of year.	Interest on rate-base.	Annual amor- tization and interest.
ı	\$7 59	\$7 59	\$100.00	\$6,00	\$13 59
2	8 04	15 63	92 41	5.55	13 59
3	8 53	24 15	84 37	5 06	13.59
	0 55		77 97		
4	9.04	33.19	75 85	4.55	13.59
5	9.58	32 77	66.81	4.01	13.59
6	10.15	52.92	57.24	3 · 43	13.59
7	10.76	63 68	47.08	2.83	13.59
8	11.41	75.09	36.32	2,18	13.59
9	12.00	87.18	24.91	1.50	13.59
10	12.82	100,00	12.82	0.77	13.59

The Sinking Fund Method, if correctly applied, involves, as already stated, only a one time calculation of the annual depreciation or replacement increment and its result agrees with that of the Equal Annual Payment Method.

The Unlimited Life Method is the most flexible. It may be so applied as to give identical results with the Equal Annual Payment and the Sinking Fund Method, or it may be applied to vary somewhat therefrom so as to make the earnings requirements least in the early years

Under the Sinking Fund Method, on the above assumption with reference to absolute agreement of actual with probable life, the annual requirement for \$100 of cost would be interest of \$6 and the amortization or depreciation increment of \$7.59 for each of the 10 years of life making a total of \$13.59 for each year.

Under the Unlimited Life Method, the replacement increment may be estimated as in the case of the Sinking Fund Method, in which event the required annual earnings would be \$13.59 or they may be graded from a smaller amount in the earlier years to a larger amount in the later years, in which event the required annual earnings would appear on an increasing scale.

The extreme case has been assumed for the illustration of the Unlimited Life Method in Table 13, that no provision whatever is made for replacement until the article to be replaced fails. This will account for the sudden increase noted for the eleventh and twenty-first years, in each of which the replacement requirement is increased by \$100. The first of the original articles fails in the tenth year (actual life being assumed the same as the probable life), and thereafter \$100 of the original investment goes out of use and has to be replaced annually until and after the twenty-first year when the annual replacement requirement is \$200.

TABLE 13 COMPARISON OF METHODS OF PROCEDURE (Hypothetical)

Annual investment \$100 pears Interest 6 per cent actual life Probable life of all parts of the property 10 No distinction is made between probable and

	Straight Line Method		Equal Annual Pay- ment Method		Sinking Fund Method		Unlimited Life Method	
Year	Rate- base be- ginning of year	Required earnings	Rate-base beginning of year	Required carnings	Rate-base beginning of year	Required carning.		Required earnings
					-			
1	\$100	\$16 00	\$100 00	\$13 59	\$100 00	\$13 59	8100	86,00
2	190	31 40	192 41	27 17	200 00	27 17	200	12 00
3	270	46 20	276 78	40 76	300 00	40 76	300	18 00
4	340	60 40	352 63	54 35	400 00	54 - 35	400	24 00
4 5 6	400	74 00	419 44	67 94	500 00	67 94	500	30 00
	450	87 00	476 68	81 53	600 00	St 53	600	36 oo
7	490	99 40	523 76	95 12	700 00	95 12	700	12 00
8	520	III 20	560 o8	108 69	800 00	108 69	800	48 00
9	540	122 40	584 99	122 28	900 00	122 28	900	54 00
10	550	133 00	597 81	135 87	1000 00	135 87	1000	ნი იი
II	650	149 00	697 81	149 46	1100 00	149 46	1100	160 00
12	740	164 40	790 22	163 05	1200 00	163 05	1 200	172 00
13	820	179 20	874 59	176 63	1300 00	176 63	1300	178 00
14	890	193 40	950 44	190 22	1400 00	190 22	1400	184 00
15	950	207 00	1017 25	203 81	1500 00	203 81	1500	190 00
16	1000	220 00	1074 49	217 40	1600 00	217 40	1600	196 00
17	1040	232 40	1111 57	230 99	1700 00	230 99	1700	202 00
18	1070	244 20	1147 89	244 56	1800 00	244 56	1800	708 00
19	1090	255 40	1172 80	258 15	1900 00	258 15	1900	214 00
20	1100	266 00	1185 62	271 74	2000 00	271 74	2000	220 00
21	1190	282 00	1278 03	285 33	2100 00	285 33	2100	326 00

One more comparison may be of interest as shown in Table 14.

TABLE 14 COMPARISON OF METHODS OF PROCEDURE (Hypothetical)

The property is made up of numerous items of all possible ages, the combined cost of which is \$100 The probable life of each item is 10 years. All considerations are theoretical and no distinction is made between actual and probable life. Interest 6 per cent. The property is supposed to have reached its full growth and to be more than 10 years old

Method of procedure	Rate-base beginning of year.	Interest on rate-base.	Annual de- preciation or replace- ment incre- ment.	Required earnings.
Straight line amortization method Equal annual payment amortiza-	\$55 00	\$3.30	\$10 00	\$13.30
tion method	59 78	3.59	10 00	13.59
			7 59	13 59
Equal annual payment amortiza-			10 00	13.59 13.59

The total required earnings appear largest for the Unlimited Life Method for the reason that the required earnings in the early years of the plant's life are supposed to have been low.

When rates are to be fixed for a public service property which is long established, concerning which past records are unreliable, but which is legitimate, rendering a necessary service and entitled to fair income, the question as to which method of procedure should be adopted presents itself.

Let it be supposed that the records indicate a fairly normal development and growth with some years of inadequate earnings and no conclusive evidence that any part of the invested capital has been repaid, but that worn-out parts have been renewed as necessary and that the service rendered has been satisfactory.

In this assumed case the application of the various methods of procedure will theoretically produce results as shown in Table 14. If any of the methods except the Unlimited Life Method be adopted, recourse will probably be had to an addition of large values for intangible elements in order that no injustice may be done to the owner of the property. In most cases the Unlimited Life Method will be indicated as most nearly equitable. This will be yet more apparent when the effect of the non-agreement of actual with probable life upon the application of the various methods of procedure is taken into account.

The comparison of results by the methods of procedure which have been under discussion has been visualized, for the special case of a single item which cost \$100, in a diagram, Fig. 2. It has been assumed in the preparation of this diagram that in the application of the Unlimited Life Method to a single article the replacement requirement will be anticipated and will be estimated by the compound interest Sinking Fund Method. In the practical application of this method a close approximation of the replacement requirements is not essential because no part of the replacement increment goes to the repayment of capital. This entire increment remains in the replacement account and will be subject to being decreased or increased from time to time as this account shows to be necessary.

Interest 6% 1 Stranght I me Method or Replacement Requirements shown thus 11 Annual Amortization Depreciation Required Earnings ----Interest shown thus.... - Requirements Fund-and-Unhunted-Life-Methods-Method Sinking-Fundand-Uhlmited-Life-Methode Fourt Ameril's yrent Method. COMPARISON OF METHODS OF PROCEEDURE Actual Life assumed to coincide with Probable Life. Interest - Unlimited Life and Sinking Fund Stethods Probable Life 10 Years Annual Amortization-Depreciation-or Replacement 5 Years Required Earlings-Straight L Equal-Annual Payment, Sinking Equal Annual Posment Method. Investment \$100 9,0 4.00 3,00 10 00 00 6 8,00 8 6.00 2,00 8 17.00 16,00 15.08 14,00 13 00 12,00 11,00 20 00 19,00 18.00

The Obligation to Replace. — In further substantiation of the advantage which results from use of a rate-base computed from the investment without any deduction of depreciation, attention may be called to the obligation to replace worn-out or discarded essential parts of every public utility plant which goes with the ownership thereof. It does not matter, in the case of any individualized article, such as a steamboat, whether a replacement fund is being set apart to be kept inviolate and is accumulating interest at a rate which will bring it to the value of the steamboat in its life or whether there is no such fund. The owner of the steamboat is burdened with the obligation to replace and this obligation is as real and as binding as though it were represented by an accumulated fund. It is optional with him whether to set apart a fund if none exists and let its earnings go toward a new steamboat or to simply let the obligation stand and to provide funds for the new steamboat when the old one goes out of use. To the extent of this obligation, that is, to the amount which should be in a replacement fund, any capital which he commands is available for no other use than the replacement when the time comes. The interest on this fund, real or imaginary, is available for this use only and as the fund, together with the remaining physical value of the steamboat, is equal to the amount originally invested in the steamboat, it is plain that there is no need of annually drawing the dividing line between the remaining physical value and the amount which should be in the replacement fund (the so-called accrued depreciation). The remaining physical value plus the obligation to replace is the invested capital on which the interest is to be computed whenever earned depreciation or replacement allowances are not regarded as amortizing capital.

Period and Rate of Amortization. — When a municipality constructs improvements under a bond issue or otherwise, suitable provision is made for the replacement of any of the worn-out parts of these improvements at the time that these parts go out of use. This is in strict conformity with the procedure under the Unlimited Life Method. But in the case of the municipal-

ity the bond issue, if any, is also to be taken care of. The cost of the improvement is, in other words, to be distributed fairly to those that will benefit thereby. This is usually done by so fixing the term of the bonds that the cost will be distributed over a sufficiently long period of time. The determination of this time period need not be in any definite relation to the life of the parts of the improvement. The improvement itself will usually be one that may be regarded as having unlimited life, such as parks, playgrounds, streets, and the like. When the term of the bonds of longest life has been fixed on the basis of the probable life of the main elements of the improvement, or in some other way, then a determination must be reached as to the best and most equitable rate of amortization.

This amortization may take place at a uniform rate per year, bearing heavily on present day property owners — Straight Line Method.

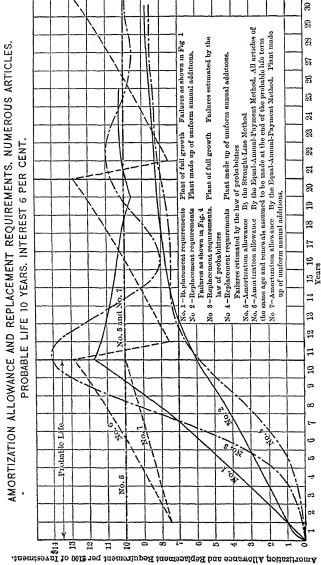
It may take place at an increasing rate per year:

- (a) According to the scheme outlined under the Equal Annual Payment Method.
- (b) According to any arbitrary scheme that will approximate the compound interest Sinking Fund Method of estimating the annual amortization increment.

Or, it may be deferred for a time and then take place according to either of these methods.

In the case of a public service property constructed by a municipality the amortization of capital usually begins at or soon after the acquisition of the property and in the case of a utility constructed by a private owner, the amortization of capital should begin under the Straight Line Method and the Equal Annual Payment Method at the beginning of operation, and, theoretically under the Sinking Fund Method or the Unlimited Life Method as herein fully explained, not at all in so far as rate-base determination is concerned, during the continuance of full private ownership.

In Fig. 3 a comparison is made between the annual amortization allowance computed by the Straight Line Method and the



÷ Fig.

Equal Annual Payment Method and the probable annual replacement requirements as these would be estimated by the hypotheses of failures which are referred to in Chapter VI.

These curves show the wide departure of the results by the Straight Line Method and by the Equal Annual Payment Method from the actual maintenance requirement and demonstrate the desirability of proceeding under the Unlimited Life Method. The replacement requirements line represents approximately the expenditures which are necessary from time to time for renewals to keep the plant in an efficient condition. The wide departure of these from the amortization lines determined by the common methods of estimating depreciation are made apparent by the diagram. Further comment is hardly necessary, except to say that under the Unlimited Life Method the actual replacement requirements may be assumed to approximate lines 1 and 3 for a number of articles of the same probable life in a plant of full growth and to approximate lines 2 and 4 for articles distributed in equal amounts to all possible ages (plant made up of uniform annual additions).

Advantage of the Unlimited Life Method. — The comparison of methods of procedure when valuations are to be used as the basis of fixing rates may be summed up in the broad statement that a close approximation of the rate-base and of the necessary annual earnings is not possible by any method of appraisal which makes the ascertainment of accrued depreciation necessary; that the use of any such method requires trained experts and involves cumbersome calculations and that the uncertainties of the determination of depreciation affect not alone the valuation but also the required annual earnings, while, on the other hand, the methods which make use of the amount of capital reasonably and properly invested as a rate-base are simpler and free from uncertainties except in the matter of the provision which should be made either for current depreciation under the Sinking Fund Method or for replacements under the Unlimited Life Method. The last-named method has the unique advantage of easy adaptation to any situation that may develop, particularly in the matter of adjusting the amount which annually goes into the replacement fund to the amount found, by actual experience, to be necessary for any special plant. This method, when selected for a new plant, also has the advantage over other methods that the required earnings in the early years are less than those estimated by any of the others. It will therefore show for the same amount of earnings a smaller annual loss or a larger annual profit in the early years than the other methods.

Careful investigation might show that the Unlimited Life Method is but an old method under a new name. Industrial establishments and public utilities could without doubt be found which had been operating in substantial conformity with this procedure before they came under control of public service commissions. As the method is theoretically sound and has weighty advantages in its favor both from the standpoint of the rate-payer who wants the burden light in the early years of the utility's life and of the owner who wants his investment protected without the uncertainty and confusion of present value considerations, it seems probable that it may come into general favor despite the present-day leaning of the public service commissions and the courts in another direction.

A good illustration of the difficulty of dealing with depreciating property and of the undesirability of using depreciated or remaining value of physical property in the rate-base will be found in the general principles enunciated by the Engineering Board, Division of Valuation, Interstate Commerce Commission, as submitted to the Commission in November, 1915. The Board lays down the following general rules:

- "When depreciating property under the several accounts, the following general rules shall apply:
- "1. Ordinarily, service condition per cent shall be the ratio between the remaining service life and the total service life. When the depreciation of an item of property is based upon the weighted average of the parts, the service condition of each part shall be determined by this ratio.

"2. When a normal life for a particular item of property has been prescribed for use in determining depreciation, that life must not be departed from unless an investigation of the records of the carrier or actual inspection in the field, or the two combined, warrant such departure. In no case shall a remaining service life of an item of property be taken at more than the prescribed normal life.

"3. When no normal life is prescribed, the total service life and remaining service life shall be determined from observation of actual conditions and the examination of records and data

from reliable sources.

"4 Salvage and scrap will be allowed in cases where such values actually exist. Whether allowance shall be made in a given case for salvage or scrap shall be left to the discretion of the member of the Engineering Board. If an allowance is made this fact shall in all cases be stated upon the pricing sheet even though at the time the amount of the allowance cannot be given."

The complex operations involved in applying these rules when rates are to be fixed, is in strong contrast with the simple operations involved in proceeding under the Unlimited Life Method which requires no estimate of *accrued* depreciation.

The Use of Cost Records and Cost of Reproduction New. — The purpose of the valuation of the public utility may be as already stated:

- a. To fix a selling price
- b. To establish a basis for an issue of securities.
- c. To establish a rate-base.
- d. To serve as a basis for taxation.

The ascertainment of the rate-base is important. By adding to or subtracting from the same, various facts relating to values can be ascertained. It may be possible to determine cost of construction from the cost records. When this can be done, there should be a check by means of estimates of the cost of reconstruction to make certain that the actual reported cost is legitimate and that it does not include too much overhead expense nor too large expenditures for abandoned or discarded items, *i.e.*, for items that were intended for temporary use

only or that failed for any reason to fulfill their intended purpose.

The cost records are not always dependable and they may have to be either entirely disregarded or largely supplemented by cost estimates. Recourse may be had in such cases to the cost of reproduction as a means of approximating the amount that may reasonably be assumed to be properly invested in the property. In making the estimate of the cost of reproduction it is not advisable to use the prices of materials and labor momentarily prevailing but rather the average for a considerable time period, preferably about 5 years.

Tabulation of Field Results. — The results of the field examination, including a classified enumeration of the physical items that go to make up the property, should be tabulated in convenient form on sheets which will show in appropriate columns information about as follows:

- 1. The year of installation.
- 2. The age.
- 3. The probable life new.
- 4. The expectancy or estimated remaining years of service.
- 5. The cost to reconstruct, itemized.
- 6. The sub-totals of (5).
- 7. The contractor's profit.
- 8. The totals of (6) and (7).
- 9. The allowance for overhead expenses.
- 10. The total investment, estimated as the sum of (8) and (9).
- 11. The residual or scrap value.
- 12. The present value, in per cent, computed from (3) and (4) (sometimes called condition per cent).
- 13. The remaining or present value in dollars (computed from (10), (11), and (12)).
 - 14. The accrued depreciation (10) (13).
- 15. The current rate of depreciation (computed from (3), (4), (10) and (11)).

It will be noted that according to this tabulation the cost to reconstruct and the residual or scrap value are considered when estimating the remaining value from the remaining years of service. Theoretically this cost of effecting the replacement should be taken into account, but it is sometimes convenient to let the original cost less residual value take its place.

According to the selected method of procedure, the rate-base is computed from the tabulated information by adding the cost of establishing the business and of any other definitely ascertainable items of intangible character, such as the cost of the franchise, or the cost of the water-right, to the sum of items in column (10) for the Sinking Fund Method or for the Unlimited Life Method of procedure; or by adding these items of intangible character to the sum of the items in column (13) when the Equal Annual Payment Method or the Straight Line Method of procedure is to be adopted.

The Effect of Method of Procedure on Market Value. — When a market value of a successfully operating plant is to be fixed on the assumption that there is accrued depreciation but that there is no deferred maintenance, special consideration must be given to the method of procedure followed by the rate-fixing authority:

If the Unlimited Life Method has been the procedure, the owner may not have recovered any part of the accrued depreciation in the earnings but only enough to meet replacement requirements. The market value in that event should not be less than the rate-base in column (10) determined from the investment without deduction of depreciation, plus some addition for value due to net earnings in excess of interest on the rate-base.

But if, under the Unlimited Life Method or under the Sinking Fund Method of procedure, the owner has received some annual amount to forestall the replacement requirement and retains possession of whatever amount there may be in the replacement fund, the market value will be less than the amount determined on the other assumption by the amount which should theoretically be in the replacement fund.

If the Equal Annual Payment or the Straight Line Method of procedure has prevailed from the beginning and a continuance thereof is a certainty, the market value will be ascertained from the rate-base, column (13), determined from the investment less depreciation to which may be added something for value due to net earnings in excess of interest on the rate-base.

Rate-base Determination. — When a rate-base is to be established, the totals of columns (10) or (13) will be used in combination with all or a part of any allowance for establishing the business and the cost of the ascertainable items of intangible character such as franchises and water-rights. The sum of these items with the total of column (10) will be the rate-base for the Unlimited Life Method and the Sinking Fund Method of procedure, and the sum of these items with the total of column (13) will be the rate-base for the Equal Annual Payment Method and the Straight Line Method of procedure. This is subject to the proviso that past history will show that accruing depreciation has actually been offset by earnings Unless there has been a surplus in the earnings over a fair net return which is to be allowed on the investment, there will have been no amortization, despite the fact that depreciation is obvious. The accrued depreciation as ascertained for any particular time does not always measure the amount of accomplished amortization. This is only the case when the allowance for depreciation, under these two methods of procedure, has actually been earned and has been collected.

CHAPTER X

NOTES ON THE DETERMINATION OF THE VALUE OF REAL ESTATE IN EMINENT DOMAIN PROCEED-INGS AND FOR RATE-FIXING PURPOSES

Market Value of Real Estate and the Rate-base. — The market value of real estate is not to be confounded with the amount at which it is carried in the rate-base. While consideration must sometimes be given to market value in fixing the rate-base this is not always necessary. In fact the determination of the rate-base should ordinarily be independent of market value because it is not "value" but "investment" which should be made the guide and control when rates are to be fixed. When there is uncertainty about the reasonable cost or when the first dependable valuation for rate-fixing purposes is made long after the acquisition of the property, circumstances may, however, point to market value at some agreed time as the best starting point.

Market Value Defined in Court Decisions. — The U.S. Supreme Court in Boom Co. vs. Patterson (98 U.S. 403, 408; 25 L. Ed., 206) says in discussing market value:

"The inquiry in such cases must be what is the property worth in the market, viewed not merely with reference to the uses to which it is at the time applied but with reference to the uses to which it is plainly adapted; that is to say, what is it worth from its availability for valuable uses. . . . Its capability of being made thus available gives it a market value which can be readily estimated."

In this case the Boom Company had brought suit in Minnesota against Patterson, a citizen of Illinois, to condemn three islands which were desired for use in connection with a series of log booms. A verdict was returned by the jury for \$9358.33,

the value of the islands having been found to be \$300 aside from any consideration of their value for boom purposes and \$9058.33 due to their adaptability for this special use.

The Court granted a motion for a new trial, unless the owner would consent to accept \$5500. This amount was acceptable to the owner and judgment for this amount was entered in his favor. The Boom Company then appealed.

The Court is perhaps a little over-confident in saying that the market value can be readily estimated. In all other respects the statement is clear and logical.

The proposition is being generally recognized by the courts that, when the right of eminent domain is exercised, the question to be considered is "What is the value of the property for the most advantageous uses to which it may be applied?"

See Goodin vs. Cincinnati and Whitewater Canal Co. (18 Ohio St. 169).

Young vs. Harrison (17 Ga. 30)

U. S. vs. Chandler-Dunbar Water Power Co. (229 U. S. 53, 76).

In this last-named case referring to the award of special value for canal and lock purposes it was said:

"The exception taken to the inclusion as an element of value of the availability of these parcels of land for lock and canal purposes must be overruled. That this land had a prospective value for the purpose of constructing a canal and lock parallel with those in use had passed beyond the region of the purely conjectural or speculative. That one or more additional parallel canals and locks would be needed to meet the increasing demands of lake traffic was an immediate probability. This land was the only land available for the purpose."

See also Shoemaker vs. U.S. (147 U.S. 282).

In the case of "San Diego Land and Town Co. vs. George Neale et al" (78 Cal. 63, 68; 30 Pac. Rep. 372; 3 L. R. A. 372) the majority of the Court says:

"The consensus of the best-considered cases is that for the purposes in hand the value to be taken is the market value, by

which is undoubtedly meant, not what the owner would realize at a forced sale, but 'the price that he could obtain after reasonable and ample time, such as would ordinarily be taken by an owner to make sale of like property.' . . . But in many instances, as in the case before us, there is no actual demand or current rate of price, either because there have been no sales of similar property, or because the particular piece is the only thing of its kind in the neighborhood, and no one has been able to use it for the purpose for which it is suitable and for which it may be highly profitable to use it. . . . From the necessity of the case the value must be arrived at from the opinions of well-informed persons, based upon the purposes for which the property is suitable. . . . What is done is merely to take into consideration the purposes for which the property is suitable, as a means of ascertaining what reasonable purchasers would in all probability be willing to give for it, which, in a general sense, may be said to be the market value. And in such an inquiry it is manifest that the fact that the property has not previously been used for the purposes in question is irrelevant. The current of authority sustains these views."

What a Purchaser can Afford to Pay is not Market Value.— The value to the person who desires to acquire the property, the amount, in other words, which such person can afford to pay for it, is not its market value. In the Chandler-Dunbar Water Power Co. case above referred to the U.S. Supreme Court says (229 U.S. 80):

"In a condemnation proceeding the value of the property to the Government for its particular use is not a criterion. The owner must be compensated for what is taken from him, but that is done when he is paid its fair market value for all available uses and purposes."

Reference may also be had to the "Minnesota Rate Cases" (230 U. S. 352, 451).

And also to "Five Tracts of Land in Cumberland Tp., Adams Co. Pa. vs. U. S." (101 Fed. 661, 664).

U. S. vs. Honolulu Plantation Co. (122 Fed. 581, 584; 58 Circuit Ct. Appeals 279).

The proposition that the necessity of the party desiring to

acquire property by eminent domain cannot be made the measure or market value is further made clear in:

Tidewater Canal Co. vs. Archer (9 Gill and J. Md. 481; 22 Md. 307).

Gardner vs. Inhabitants of Brookline (127 Mass. 358).

Burt vs Wigglesworth (117 Mass. 302).

Reading and Pottsville R. R. Co. vs. Balthasser (126 Pa. St. 1). Dorlan vs. East Brandywine and W. R. 46 Pa. 520 The Stockton and Copperopolis Railroad Co. vs. Vincent Galgiani (49 Cal. 139).

Admissibility of Evidence Relating to Value for a Special Purpose. — The question relating to the admissibility of evidence bearing directly upon the value of a tract of land for reservoir purposes is quite fully discussed in Spring Valley Water Works vs. Drinkhouse (92 Cal. 528, 532; 28 Pac. Rep. 681). The view expressed in that case that value of the land for reservoir purposes might be shown appears however to be controverted in the more recent case of "Sacramento Southern Railroad Co. vs. Heilbron" (156 Cal. 408) which involves the condemnation of a strip of land for railroad purposes. It was contended in this case that the rule as laid down in California permits evidence of value for the use of the land for a particular purpose in terms of money In reference to this contention the Court says:

"It is seen, therefore, that this Court by its latest utterances has definitely aligned itself with the great majority of the courts in holding that damages must be measured by the market value of the land at the time it is taken, that the test is not the value for a special purpose, but the fair market value of the land in view of all the purposes to which it is naturally adapted; that therefore while evidence that it is 'valuable' for this or that or another purpose may always be given and should be freely received, the value in terms of money, the price, which one or another witness may think the land would bring for this or that or the other specific purpose is not admissible as an element in determining that market value."

Apparently, if this rule is strictly adhered to, it will debar

from consideration the economic features which determine whether and to what extent the enterprise which involves the acquisition of the land can be made profitable. It may make it impracticable to get before the court that information which a prudent purchaser would seek when making up his mind relating to the price which he would be justified in paying.

The presentation, then, in court of the evidence on which the valuation of real estate is to be based is not always a simple matter. The information wanted is the market value. The expert who testifies to value must inform himself what the value is, all purposes for which the property is suitable being taken into account. Recent court rulings appear, as above shown, to be against allowing evidence which will show the value in money for any special purpose.

The local dealer in real estate who knows what sales have been made in recent years and who knows or is supposed to know the effect that the adaptability of any particular tract of land to a particular purpose has upon the market value of that land is, according to such rules of procedure, the proper value expert. His opinion relating to the money value of an island for boom purposes, or of a tract suitable for a dam site or for a reservoir site or for some other public use, is allowed to go before the judge or jury that fixes the value while the trained engineering expert who may have made a careful analysis of all economic features involved but who may have no knowledge of the land value for other than one special purpose, is not allowed to testify to the value which the same property would have for the special use to which it is about to be put for the benefit of the public. He is restricted to a statement of the nature of such use and may present facts relating thereto but he must not express an opinion relating to value in terms of money unless he is in a position to say that he has given consideration to all possible uses.

This rule of the courts is intended to stop the introduction of evidence too speculative and remote in character. The possibility of growing a special crop at an estimated annual profit

per acre and a determination of the land value from the net return resulting from growing such a crop is held to be too speculative and no doubt with reason. But the court should not be denied such information as would be collected by a prudent purchaser who neglects no source of information when he makes up his mind what would be a reasonable price to pay for the property sought to be acquired, and there should be no testimony excluded which will throw light on the circumstances which affect or fix the market value. The favorably disposed purchaser will not restrict himself to the questioning of those only who are supposed to have general knowledge of the market value resulting from every possible use. He will rather make inquiry along every possible line and will not neglect special adaptability to a particular purpose. He will want to know the strategic value of the property when made use of for the special purpose to which it is supposed to be best adapted. If the value for such a special purpose can be ascertained with due consideration of all elements involved - adaptability to special uses with due allowance for risk, the cost of development, the cost of operation, the immediate or deferred market for the output or service and the prospective net return that will result from the use of the property, - the knowledge so obtained will aid the purchaser in reaching his conclusion concerning what a well-informed public would consider such a property worth.

The amount which a particular municipality or a person who is seeking to condemn a property can afford to pay cannot, however, be made the measure of the market value. This may define an upper limit. More cannot be paid by that municipality or person. The purchase at the full amount determined by such necessity would deprive the person, who wishes to utilize the property for the special purpose to which it is adapted, of a margin of profit to which such person is entitled.

In the light of the foregoing it would seem reasonable to exclude from the consideration of the courts in condemnation proceedings evidence relating to the value expressed in money for such special uses as can be exercised only by the person who seeks to condemn the property, as in the case of land required by the Government for fortification purposes, or for a light house, but in all cases where the adaptability to the special use is recognized and the property can be applied to that use by any one suitably circumstanced, the evidence relating to the effect of such adaptability upon market value should be admissible. It should be admitted even though somewhat speculative. The court must determine what weight to give to evidence of value for special uses which only remotely affect the market value.

In the case of a mine on land whose surface has value for no other purpose than grazing, it will, unquestionably, be proper to ascertain the value of the land for the special purpose of mining. If the land has value for mining and at the same time for reservoir purposes or as a dam site, there should be no objection to a consideration of these values separately for each specific purpose, just as the same would be considered by the prudent purchaser when he makes a study of market value. The consideration of the effect of availability for these purposes upon market value is not to be classed as too speculative. If use for any such purpose lies in the future, due allowance must be made for the lapse of time before the ultimate value resulting from that use can be realized. If the time is uncertain and remote, the effect of the special adaptability upon market value will be small when compared with what this effect might be if immediate use were a certainty.

The Value Multiple. — Other considerations, too, may limit this market value. It has, for example, become a practice, almost standard, to value rights of way for railroads and highways at from 1.5 to 3.0 times the market value of adjacent lands of similar character. Custom has given land required for such uses such values. There are no frequent transfers of such property as in the case of town lots or farms. Consequently the determination of the value multiple which may be regarded as generally customary in the region where a right of way is to be valued may be the best guide in fixing this value.

In the case of a storage reservoir from which water may be required for an immediate high use, such as a municipal supply. and where the most profitable other use to which the reservoir land can be put is grazing, there may be a wide difference between values if determined for these two purposes. And yet in such a case the fact that this particular property will sooner or later be used for water development purposes may have given it a market value that cannot be ignored when the right of eminent domain is exercised. If the public has correctly determined this market value, it will lie somewhere between the value of the land for grazing and the amount which a person can afford to pay who wishes to use it for the storage of water This does not mean that the original owner is to share the increment of special value equally with the party who is going to put the land to a special use but merely that he should share, to some extent, in the same.

When the value of land for a special purpose is but little in excess of the value of the land for ordinary uses, the division of the excess might, perhaps, equitably be on the basis of an equal division between the original owner and the person who desires to acquire the land. When, however, this excess is relatively large, then it may be proper and fair to assume that the larger portion of the excess value created by the enterprise should go to the person who is ready to take the risk of a successful carrying out of the enterprise and this fact should not be lost sight of in estimating market value.

When lands are to be taken for fortification and other similar purposes where consideration of the value expressed in money for the special purpose is not admissible, consideration should be given to the fact that the owner is forced to part with property for the good of the public and is entitled to receive for it somewhat more than the price which would obtain between a willing seller and a buyer, and also to the fact that whenever the adaptability of a piece of property for a special use is generally recognized, such property must have acquired greater value than other similar property not available for such use.

The owner who is forced to part with his property in such cases should obtain a reasonable reward for his foresight in acquiring property of special adaptability to some important use. This reward should, whenever practicable, bear some reasonable relation to the value for other uses to which the property can be put. There may be cases in which a 25 or 50 per cent allowance will be adequate, and there may be others in which this allowance may exceed the value for other uses two-, five- or even ten-fold, and there may also be occasional cases where a reasonable allowance not capable of demonstration, but yet fair when all circumstances are considered may be out of all relation to value for other uses.

The value multiple, as applied in any locality to railroad rights of way, and the unit price per pole of a power line are the outcome of numerous adjustments resulting from a regard to considerations as above set forth, and where such multiples are recognized, they, in turn, as already stated, have an effect upon market value of other similar easements.

The Value Multiple in the Minnesota Rate-Cases. — In connection with the valuation of rights of way and other lands as a part of the appraisal to be taken into account when the rates of public utilities are to be fixed, the decision of the United States Supreme Court in the Minnesota Rate-Cases is of interest (230 U. S. 352; 33 Sup. Ct. 729, June 9, 1913). Justice Hughes, in writing the decision of the court, says:

"The increase sought for 'railway value' in these cases is an increment over all outlays of the carrier and over the values of similar land in the vicinity. It is an increment which can not be referred to any known criterion, but must rest on a mere expression of judgment which finds no proper test or standard in the transactions of the business world. It is an increment which in the last analysis must rest on an estimate of the value of the railroad use as compared with other business uses, it involves an appreciation of the return from rates (when rates themselves are in dispute) and a sweeping generalization embracing substantially all the activities of the community. For an allowance of this character there is no warrant.

"Assuming that the company is entitled to a reasonable share in the general prosperity of the communities which it serves, and thus to attribute to its property an increase in value, still the increase so allowed, apart from any improvements it may make, cannot properly extend beyond the fair average of the normal market value of land in the vicinity having a similar character. Otherwise we enter into the realm of mere conjecture. We, therefore, hold that it was error to base the estimates of the value of the right-of-way, yards and terminals upon the so-called 'railway value' of the property. The company would certainly have no ground of complaint if it were allowed a value for these lands equal to the fair average market value of similar land in the vicinity, without additions by the use of multipliers, or otherwise, to cover hypothetical outlays."

The Court in this decision may be correct in stating that market value should be determined without the use of multipliers. Nevertheless the fact that rights of way are actually costing from 25 to 200 per cent more than lands of similar character in the same vicinity will have an unquestioned effect upon the market value of lands required for other rights of way and this effect cannot well be ignored when such lands are to be valued.

Right-of-Way Value in the Georgia Railway Case. — Special Master Thorington, in the Georgia Railway Case (Central of Georgia Railway Company vs. Railroad Commission of Alabama, U. S. Dist. Court, Middle Dist. of Ala., Northern Division, Report of Wm. S. Thorington, Special Master, Jan. 8, 1912), after stating that the fact that the railroad company is compelled to pay in addition to its market value a further sum due to damages or because it is a railroad company making the purchase adds nothing whatever to the actual acreage value, says:

"It is, however, proper to add that right-of-way values, including estimates for damages to property not taken, or excess cost that railroads are compelled to pay in order to acquire right-of-way property needed by them for railroad use, have been recognized by some courts, and some railroad commissions, and such excess cost was held to properly constitute part of the

right-of-way valuation for rate purposes. In Shephard vs. Northern Pacific Railway Co. et al 184 Fed. 765, it is said the evidence was conclusive 'that every railroad company is compelled to pay more than the normal market value of property in sales between private parties for the irregular tracts it needs and acquires for rights-of-way, yards and station grounds. . . . The measure of the value of real estate is its market value for its most available use.'"

California Railroad Commission on the Right-of-Way Value.

— In discussing the valuation of a railroad right of way, the California Railroad Commission says (Stockton Terminal and Eastern Railroad. Decision No. 618):

"After ascertaining the market value of the property at the time of its acquisition, the department (engineering department) also ascertained the market value as of June 30, 1912, and then multiplied that value by 1.5. This multiple was applied for the reason that the investigations of the department throughout the State show that on an average it costs one and one-half times the normal market value of abutting property to acquire rights of way in country districts by purchase or condemnation for railroad purposes. In the absence of more definite information as affecting this particular railroad, this average multiple was used."

The same commission again refers to and approves the use of this multiple in the matter of ascertaining the value of the property of the Nevada County Narrow Gauge R. R. Co. (Decision No. 1384).

CHAPTER XI

THE VALUE OF A WATER-RIGHT AND OF RESERVOIR AND WATERSHED LANDS

Value of Irrigation Water. — When water is used for irrigation. it makes the intense cultivation of the soil possible. producing crops which can be marketed at prices not subject to regulation except by the law of supply and demand. The availability and use of the irrigation water modify the character and increase the amount and consequently the value of the These elements may thus add an increment of value to the irrigated land. Under such use the value of the water at the field and, by comparison with the cost of development, its value at its source can be determined Water and water-rights in districts where water is used for irrigation acquire, in consequence, a recognized market value depending upon the appreciation of the land that results from irrigation, upon the value of the crops harvested and upon the cost of developing and making available the irrigation water.

Payment for Water-Rights. — Ordinarily when water is to be taken from a stream for uses which decrease or otherwise modify the natural flow of the stream below the point of diversion, the riparian rights of lower land owners are thereby affected. The diversion cannot be made in such cases without making compensation to the riparian owners for the damage to their property which results from the taking, except, of course, when such owners sleep upon their rights, virtually admitting too small a damage to make it worth while to attempt to recover compensation.

To the extent of the cost of securing the riparian rights and possibly of securing other water-rights whose use is secondary or which for any reason should be merged in one holding, there is then — a public utility being under consideration — an investment to be assumed in that intangible element, the water-right.

Sometimes by reason of local development and high values of riparian lands and an already established use of the stream flow for power, the cost of settling with the riparian owners and of quieting title to adverse users of the water may be large. At other times the situation is such that equally good rights to use water may be secured without any cost except the cost incident to the construction of the project and the acquisition of the necessary lands and rights of way

Water-Rights have Value. — The fact that in the first case it will have to be conceded that the owner of the public utility is entitled to have the cost of the water-right which he holds made a part of the rate-base and that at least to the extent of cost (reasonable and actual proper cost being assumed), this waterright has or should be made to have value, justifies the public in concluding that the other water-right which has cost nothing should have a similar value, whether the same be made a part of the rate-base or not. Water-rights, then, are to be regarded as having market value. When the water is developed and is actually being put to use or when the need of putting the water to beneficial use is proximate, the existence of such value is easily recognized. When an investment has been necessary to quiet title to adverse rights and to meet other expense of securing the water-right, the propriety of including its cost in the ratebase is unquestioned.

Water-right Value in the San Joaquin and Kings River Canal Case. — The Supreme Court of the United States in "San Joaquin and Kings River Canal and Irrigation Co. vs. The County of Stanislaus" (233 U.S. 458) in reference to the fundamental principle of taking the value of water-rights into account when rates are to be fixed, says, in its decision reversing the decree of the lower court:

"By a statute of March 12, 1885, the boards (of County Supervisors) are authorized to fix these rates for their several counties, but so that the returns to the parties furnishing the water shall not be less than 6 per cent upon the value of the 'canals, ditches, flumes, chutes, and all other property actually used and useful to the appropriation and furnishing of such water.' The rates, when fixed are binding for one year and until established anew or abrogated. . . . The question before the court has been narrowed to a single issue. If the plaintiff is entitled to 6 per cent upon its tangible property alone, it is agreed that the order must stand. But if the plaintiff has water-rights that are to be taken into account, the rates fixed will fall short of giving it what it is entitled to and must be set aside. . . .

"It is not disputed that the plaintiff has a right as against riparian proprietors to withdraw the water that it distributes through its canals. Whether the right was paid for, as the plaintiff says, or not, it has been confirmed by prescription and is now beyond attack. It is not disputed either that if the plaintiff were the owner of riparian lands to which its water was distributed it would have a property in the water that could not be taken without compensation. But it is said that as the plaintiff appropriates this water to distribution and sale it thereby dedicates it to public use under California law and so loses its private right in the same. . . .

"It seems unreasonable to suppose that the Constitution meant that if a party instead of using the water on his own land, as he may, sees fit to distribute it to others, he loses the rights that he has bought or lawfully acquired. Recurring to the fact that in every instance only a few specified individuals get the right to a supply, and that it clearly appears from the latest statement of the Supreme Court of California (Palmer vs. Railroad Commission, Jan. 20, 1914 (47 Cal. 201)), that the water when appropriated is private property, it is unreasonable to suppose that the constitutional declaration meant to compel a gift from the former owner to the users and that in dealing with water 'appropriated for sale' it means that there should be nothing to sell. (See San Diego Water Co. vs. San Diego, 118 Cal. 556, 567; 50 Pac. Rep. 633; 38 L. R. A. 460; 62 Am. St. Rep. 261; Fresno Canal and Irrigation Co. vs. Park, 129 Cal. 437, 443; 62 Pac. Rep. 87; Stanislaus Water Co. vs. Bachman, 152 Cal. 716; Leavitt vs. Lassen Irrigation Co., 157 Cal. 82.)"

According to this decision the water-right must receive the same consideration as other property when rates are to be fixed.

But the court does not attempt to settle the question relating to how a water-right is to be valued.

California Railroad Commission on Water-right Values. — In the matter of valuing water-rights Commissioner Thelen, in writing the decision of the Railroad Commission of California in the San Diego case (Decision No. 1465), sounds a note of warning, when he says:

"This case illustrates clearly the tremendous importance to the people of this State of the claim made by certain water companies and other utilities that the value which adheres to the water which they convey to their customers belongs to the utility, and that the utility is entitled to capitalize the full value of that water, entirely irrespective of its cost to the utility, and to collect a charge for water high enough to yield a return on such amount as the experts for the utility estimate to be the value of the water or of the water-right. I do not deem it necessary at this point to discuss the authorities both in the State and federal courts bearing on this question, for the reason that, according to press despatches this question has now been decided by the Supreme Court of the United States in the case of San Joaquin and Kings River Canal and Irrigation Co. vs. County of Stanislaus. The question of the amount of value to be allowed is, of course, a question of fact, the determination of which still rests in this Commission. I desire, at this time, to draw attention to the grave consequences which may follow if the theories of value of water-rights urged by various public utilities before this Commission are adopted. If it is true that the entire value of the water which a public utility secured by appropriation or otherwise belongs to the utility and that the public must pay rates on such value, it follows that, where there is only one source of water supply for a municipality, the water utility has the right to capitalize the entire life of the municipality. And, in any case, the utility will have the right to take for itself the entire increased value of land due to the placing thereon of this water, entirely irrespective of the fact that the people of this State have given to the utility the right to appropriate the water and that the actual price paid for the water may have been absolutely insignificant as against the amount claimed by the utility."

The Free Grant of Water-Rights in Western States. — The right to appropriate flowing water and to put the same to beneficial use is given by law, in most of the Western states, to any one who will construct proper works for the development of the water and for its transmission to places of use. The water of the stream belongs to the public. The grant of the right to put it to some beneficial use is on a par with a franchise to construct a highway or to build a railroad. This right has value as a franchise has value when the earnings are sufficient to create a value. As in the case of a franchise, so in the case of the water-right, the cost thereof becomes a part of the ratebase in the event that investment and not value be made the starting point.

Determination of the Water-right Value. — The courts and the rate-fixing authorities accepting the view of the public are showing a tendency to allow earnings which will give the water-right value. But neither the courts, nor public service commissions, nor experts have yet agreed upon any method of determining the water-right value. The method of ascertaining the water-right value, in the case of water used to supply the needs of an urban population, by comparison with the ordinary cost of developing water in the same region in like amount, of like quality and under similar conditions of delivery has occasionally been applied but not with entirely satisfactory results.

To illustrate, let it be assumed that the average ordinary cost of making water for domestic use available for distribution, in the region in which a water-right is to be valued, is 10 cents per 1000 gallons. This cost is here supposed to include interest on the investment and the outlay of whatever nature connected with operation. Let it be further assumed that the cost of making available the water which is to be valued has been found to be 9 cents per 1000 gallons. It will readily be seen that under such circumstances a rise of 1 cent per 1000 gallons in the average regional cost of water production would have the absurd effect of doubling the value of the water-right. Furthermore, the water-right of any supply whose cost of development exceeds

the average or ordinary cost would, under strict application of this test of value, prove to be a liability and not an asset. This, too, is an absurdity and condemns the method.

Neither this method nor a comparison with the cost of developing the next most available supply can be used as a dependable method for determining water-right values.

In some sections of the country, as, for example, in portions of California, the demand for irrigation water has nearly, if not quite, reached the limit of supply. In such regions the depressing effect upon the value of water which results from large undeveloped available sources is no longer felt. The earnings that result from the use of the water have become the measure of its value and this value is consequently relatively high.

The recognized value of water-rights in such sections has an effect upon the value of water-rights elsewhere and for uses other than irrigation. The value of water used for domestic purposes, similar general conditions being assumed, should not be less than that of water used for irrigation, and, if for this purpose it has a high value in one part of the state,—the question is asked, why not in another? Such considerations as these are not without effect upon the market value of water-rights.

The fact should not be overlooked that the inclusion of a water-right value in the rate-base of a public service property, to the extent that this value exceeds cost, would be in the nature of an allowance to be regarded as part compensation for having undertaken the water development, and this compensation increment might reasonably be brought into some definite relation to the general cost of developing water in any region.

Water-right Values in Relation to Cost of Works.—It has above been stated that the allowance of a fair reward for the successful development and beneficial use of water is legitimate. The making of a reasonable allowance, preferably based upon some definite percentage allowance on the ordinary regional cost of developing water or of developing hydro-electric power as the case may be, should be encouraged. If this principle were generally recognized, it would result in fixing with some

definiteness the value of water at its source and would remove much of the uncertainty that now obtains in relation to the value of water-rights. If thus determined, the value of the water-right will not be subject to unreasonable fluctuation nor to too wide a range. Where the average regional cost of development, including everything necessary to make water available for distribution, is 10 cents per 1000 gallons and the allowance for water-rights is to be about 10 per cent of this amount, or I cent per 1000 gallons, a change of I per cent in the cost of water development would only modify the value of the water at the source by 0 oi cent per 1000 gallons. A 10 per cent increase or decrease in the regional cost of development would be necessary to affect this value by o.r cent. In other words, when, in a certain region, an amount has been agreed upon and generally accepted as a proper allowance to be made for the value of developed water, or rather, when such value is to be created by a suitable allowance of earnings, this value will be fairly stable and will thereafter pass as the market value whether or not the cost of development is below or above the average.

Strategic Value of Water-Rights. — In addition to the basic value at its source, a water supply may have additional value, due to an inherent advantage of quality and location and other circumstances that determine its development cost in comparison with the development cost of competing supplies. Such value is properly termed "strategic value."

To illustrate, a riparian ownership which controls a water-power may be cited. The case may readily be conceived of a water-power, limited in amount, but completely controlled by the riparian owner. When such a source of power is to be valued in a region where the market for power is good, where, for example, the water-power will be delivered to a market in which it displaces a like amount of power generated by steam, the cost of the latter in comparison with the cost of the former affords a legitimate means of determining value, or, better stated, an upper limit of value. The valuation becomes a simple matter when, under such circumstances, the power is

already fully developed and is in use or is being supplied to a market which takes it all. But when the power is undeveloped, some consideration must be given to the uncertainty of achieving the expected results and due allowance must be made for the time that will have to elapse before a return from the sale of power can be realized.

There will, of course, be cases in which an analysis of the cost of generating and delivering power will show the advantage to be with the power developed by steam. In such cases the hydro-electric enterprise may nevertheless be a legitimate one. It may have been initiated when the price of fuel for generating steam was such that the advantage of cost was temporarily with the water-power; or the margin in favor of the steam-power may be so small that the recognized advantage and economic value to society in conserving the energy which annually reappears in the water of the stream outweighs any financial disadvantage that may appear from a comparison with steam as a source of power, and justifies earnings that might not under other circumstances be considered reasonable.

Furthermore, if the market for the output of a hydro-electric installation has been established, there is no certainty that the market of a competing plant, at a different cost and sale price of power, would be the same. This is a circumstance which should be duly weighed in making the comparison.

In any event, the owner of such utility should be recognized as engaged in a meritorious enterprise, deserving not only adequate protection, but also such reward for having developed the water-power and having made an investment for the benefit of the public as the circumstances may justify. Under this view, even when the water-right which makes the development of power possible would appear to be without market value at current fuel prices, it will be reasonable to allow to the owner, not alone a rate-base increment equal to the cost of securing the water-right, if there has been any such cost, but also, if this cost has been legitimate and reasonable, some excess allowance in the earnings, perhaps proportional to the amount of power

developed rather than to the actual investment in works for developing and marketing this power. But this can only be done within limits or so long as the rates for the service remain reasonable. When it would require excessive rates, the owner must suffer the penalty of having made an untimely if not an unwise investment.

Illustration of Strategic Value. — As an illustration of special water-right value the case of a water supply for general and domestic use which affords water of prime quality in limited amount may be taken, but which, when compared with other sources in use in the same community, has the advantage of proximity, elevation and reliability of service.

Let it be assumed, for example, that such a supply was the first to come into use, that its water was distributed as required throughout the built-up section of a growing town, but that at length a time came when additional water had to be brought in by a second system from some remote source, and that at the time of the valuation the distributing pipes of each of the two systems cover practically the entire built-up territory. The original water-works may now be supplying only a small fraction of the aggregate amount of water being used. Undoubtedly under such circumstances, the charge for water by the two concerns would be the same or very nearly the same. The water from the newer works could not be supplied at a low enough rate to drive the earlier concern out of business. Without any reduction of rates, this original utility should hold its customers. There need be no falling off in the amount of water which it supplies, assumed to be the limit of its capacity. But, if, as assumed, the rates charged by the two concerns are the same, the relative amount of net earnings will be greater for the original than for the new water-works. If it costs the original concern 17.5 cents per thousand gallons to develop and market its water crop (interest on the investment included) and it is costing the new concern 20 cents to do the same, and if this larger cost has been taken into account in fixing the water rates, then the water-right and other intangible elements of value of

the original concern may reasonably be valued at (\$200-\$175) \$25 per day per million gallons of daily delivery more than the water-right and other intangible elements of value of the new concern. This is interest on about \$150,000, if 6 per cent per annum be made the basis of the calculation.

If, in other words, rates are allowed which in the case of the new or main water-works system will create a water-right value of \$50,000 per million gallons of daily delivery for the new waterworks, then the value of the water-rights controlled by the original system may be about \$200,000 per million gallons per day.

In the case of a water-power, too, there may be a pronounced and easily recognized strategic value. The usual distinction is to be made, however, between the power development with an established market and that in which the power output is not yet in full demand.

When there is no question about the market for the power, the problem will have to be solved on the basis of a comparison of the cost of utilizing the water-power as compared with power from other sources and this comparison may show more or less strategic value. Where there is no such strategic value, the water-right should be considered as a privilege similar to a franchise and should be treated accordingly.

It must be remembered in this connection, however, and in any analysis of this character, that the advantage that one concern may have over another in the amount that net earnings exceed interest on the investment, is to be applied to all elements of value in excess of the capital actually invested and can not always be assigned to water-rights alone.

The Time Element in Valuing Water-Rights. — The water-right as thus far discussed is the right to put water to a continuing beneficial use without limit as to the time during which such right may be exercised. There will be cases of reversion of the right within a fixed time to the public which has made the grant thereof and there will be other cases in which a superior supply of water, later to be developed, may at some time throw the original source out of use altogether or leave it available for

only inferior uses. Where a water-right thus limited in life is to be valued, the question not only arises as to its strategic value, but cognizance must be taken of the fact that the life of the right is limited and that it will not be a source of revenue for all time.

The value of any water-right, in excess of cost, like the value of a franchise, results from earnings in excess of a fair interest on the investment. This value is, therefore, directly dependent upon the rates established by the rate-fixing bodies or, in the case of the restricted franchise, upon the rates allowed to be charged under such franchise. Large power is in the hands of the rate-fixing authorities to make or to destroy the value of water at its source and until a definite policy has been adopted by such authorities, there will continue to be more or less uncertainty relating to such value. The real value of property of this character will, for the present, remain somewhat speculative, particularly in cases where the development of the water or of a water-power lies in an uncertain future. This can hardly be otherwise because it is not yet certain that the tendency of today to allow something for the water-right, practically as compensation for making the development, will be adhered to. When it is fully understood that such an allowance will be made and when a definite limit is set to the amount of such an allowance, the valuation experts will be relieved of much embarrassment.

In the case of the established utility much of the difficulty ordinarily encountered when water-rights or franchises are to be valued as a basis for fixing rates will fall away if the method of procedure which the author recommends be followed and the invested capital and not present value be made the rate-base.

Views of the Wisconsin Railroad Commission on Waterpower Value. — The attitude of the Wisconsin Railroad Commission toward the determination of the value of a water-power by a comparison with the cost of steam power appears in the following quotation from the Commission's decision in the case of the City of Beloit vs. Beloit Water Gas and Electric Co. (Wis. R. C. R., Vol. 7, p. 247).

"It seems clear from the expression of opinions thus made and from the general practice of engineers and other men in valuing water powers that the saving effected by the use of the water power over steam power, especially, measures the values of the water power. Other methods of appraisal are used and have been mentioned by the witnesses in these proceedings, namely, rental value and market value. These latter methods, however, are quite often open to objections which destroy their reliability and it appears that it is almost always necessary to fall back upon the method of calculating the saving over steam power and then by capitalizing this saving, arrive at the total value of the water power. The Commission has commented upon this and other methods of determining the value of water power in earlier decisions:

"'From a purely commercial point of view this method of estimating the value of water-power rights may, in the main, be sound. But it is not so clear that this can be said for it when the question is regarded from the point of view of public policy.

. . . it appears to deprive a locality of the natural advantages it might otherwise derive from being located near such water powers. If water-rights are private property under the law, then all the benefits which accrue from these rights would probably go to their private owners. If, on the other hand, water-power rights are public rights rather than private rights, then it would also seem that the public ought to share in any benefits that may be derived therefrom.' Ross et al vs. Burkhardt Milling and Electric Power Co. (Wis. R. C. R., Vol. 5, p. 139, 147)."

On the subject of water-power value the Commission says further in the case of City of Rhinelander vs. Rhinelander Lighting Co. (Wis. R. C. R., Vol. 9, p. 424):

"While calculations of the saving produced by the use of water-power instead of steam-power are of much importance in private and public undertakings in showing the financial feasibility of hydraulic construction, the title of the owners in utility business to the entire savings so produced has not been clearly demonstrated. Indeed, the respondent's claims seem to go so far as to preclude the public from any share in economical methods of service and seem to place upon users of utility service the burden of maximum costs of operation."

Value. — Passing now to the consideration of the value of reservoir sites it may be broadly stated that, in some measure, any value thus or otherwise ascertained as appertaining to waterrights may serve as an aid in determining the value of watershed and reservoir lands or of other lands whose use is necessary to make the development of the water possible.

The value of such lands should not, however, be measured by the necessity of the community which needs the water. Their value is not what the community can afford to pay for them; but they have at least the value which would be determined by the market for similar lands devoted to other uses. They also have an additional value due to special adaptability for use in developing a water supply.

It is owing to the desirability of bringing this excess value of reservoir lands into some relation to the value of the water whose development their ownership makes possible that it may sometimes be found desirable to make the value of the waterright a measure of the excess of value (sometimes perhaps only of the upper limit of the excess) of an assembled reservoir property over the value of the land for other purposes. While this is not an established practice, it is one which appears to have some merit. It is to be understood that the excess of the value of reservoir lands over similar lands not available for reservoir use as here considered is apart from and in addition to the value of the water-right.

Where the water development requires only a few acres of ground, as in the case of artesian supplies, particularly if the land remains available for other uses, or when the topographic situation is unusually favorable, a small fraction of the value of the water-right might prove to be an adequate allowance for special adaptability.

Value Multiple Applied to Reservoir Lands. — In such special cases some use might be made of a value multiple such as has become customary when rights of way are to be secured for railroads and canals which are acquired usually at some increase

over ordinary values, i.e., over the value of grazing and farm lands crossed by the railroads and canals. In the case of railroads the excess is generally 50 per cent to 200 per cent. This would probably also be a fair assumption in the case of canal rights of way. In the case of reservoirs the value of the reservoir site as compared with that of other land of similar location and quality may go to much higher limits, but not enough data are at hand to justify an attempt to give reliable averages. Where the site is large and the use one that may not prove highly remunerative, due to remoteness of location, high cost of construction, scant rainfall on the watershed or other modifying causes, such as the availability of alternative reservoir sites, a bare allowance to the owner for forced abandonment of his holdings may represent the limit of what any prudent purchaser would pay. In other cases a multiple of 5 or even more may not be unreasonable.

When there is also a strategic value due to relatively large earnings that will result, perhaps, from rates that have been or that must be so fixed that they will yield a fair return on some other less favorably situated property, then there will be an excess of value determinable from the large present or prospective earnings, and a part of this excess would naturally be allotted to the person who owns the reservoir site, the rest thereof going to the party who actually makes the water development.

Certain Increments of Reservoir Land Value. — It is generally recognized that the problem of valuing reservoir lands is one of the most difficult that can be presented to the engineer. None of the facts relating to the availability of a reservoir site taken by itself determines its market value. But every such fact must have some effect upon the minds of an intelligent public and therefore influences the market value.

Tracts separately owned which must all be combined under one ownership to be available for use as a reservoir will have less value in separate ownership than after being assembled in one holding. The value of reservoir lands will be less if there are other similar reservoir properties available for alternative The time when the utilization of the property is a necessity, if this time can be definitely or even approximately fixed, will affect its market value. The value will be higher after actual construction has demonstrated that a reservoir will hold water than before. If there have been any judicial determinations of the market value or of the value for rate-fixing purposes of the property to be valued or of similarly situated or otherwise comparable property or if valuations thereof have been made by authorities charged with the regulation of rates or by other public authorities, all such determinations will have an effect upon the market and would be given consideration by a prudent purchaser. They are also, therefore, proper elements for consideration by an appraiser. The cost of the lands in a site already acquired is a factor that should be given due weight with proper allowance for the circumstances attending the purchases. Appreciation or depreciation that may have taken place subsequent to such purchase may also have to be given consideration.

The value of a parcel of land which is required in connection with others to make a reservoir site available is, as above stated, worth less by itself than it will be when united with other tracts into the one holding which makes the development possible. While the individual tract is reservoir land and due to this fact may have acquired a market value in excess of the value which it would have if not located in a reservoir site, this excess value would be estimated by a prudent purchaser at less than the excess value which it acquires when brought into the same ownership as all the rest of the lands which make the storage of water possible. At how much less it should be valued cannot be stated with any degree of confidence for general guidance. And yet it may tentatively be suggested that the party who assembles the property should get the benefit of at least one half and, in many cases, much more than this proportion of the excess of its value, in the constructed reservoir, over its value for ordinary uses.

Effect of Various Factors on Reservoir Value. - Too much weight is sometimes attached to the influence of the cost of developing a water supply upon the market value of the land which is to be acquired to make the water development possible. Whether this cost be great or small, the custom prevails of allowing the owner of the water-works to recover, in the earnings, the cost of operation including replacement requirements and interest on the cost of structures. Whenever, therefore, the necessity of using the land is unquestioned, the cost of developing the water should have but little effect on the market value of the reservoir land. The amount of the water made marketable is, on the other hand, a circumstance which will have greater or less effect on this value, depending upon the value which will be allowed in the rate-base for the developed water-right. There are cases, too, in which the nearness of a storage site to the place of use may give it special value, due to the fact that it adds to the reliability of the service and, finally, there will be cases in which the property to be used for water storage or already in use as a reservoir has acquired greater value for residential or other purposes than, by any fair line of reasoning, could be determined for its use in developing or storing water. When this is the case, the market value of the reservoir land (always to be distinguished from the amount at which it is included in the rate-base) is determined by the other uses to which it might be put. If such a reservoir is a necessity and no other equivalent structure can be substituted for it, the appraisal of market value would be fixed by these other uses. If on the other hand it is possible to substitute for the reservoir some other structure located elsewhere, and costing less though of equivalent service value, then in the case of the constructed reservoir the time will have come when its use should be discontinued or in the case of the reservoir site, not yet in use, the project plans should be so modified as to eliminate the reservoir.

It may be well to repeat that when dealing with water not yet developed, proper allowance must be made in all appraisals of water-right and of reservoir and watershed values for the probable lapse of time before the water will actually be in use.

Watershed Land Value. — When storage reservoirs for water for domestic use are involved and the value of the watershed lands is to be determined, it would be legitimate to give consideration to the modification of the cost of operation which might result from the ownership of these lands. When such ownership would result in safeguarding the quality of water so that filtration and other treatment to make the water attractive and wholesome would become unnecessary, the limiting value of such ownership can be measured by the added cost that would be incurred for a purification of the supply if the water were in danger of pollution due to the use of lands in the watershed for human habitation or for other purposes that would detract from the wholesomeness of the runoff waters. Consideration would certainly be given by a prudent purchaser who is weighing the desirability of acquiring watershed lands to this matter and, therefore, this is properly an element to be considered in determining the value of watershed lands whether already in use or required for early use.

In this connection consideration would be given: first, to the degree of protection which will result from such ownership, because, after all, the ownership may not be an absolute insurance against the necessity of some treatment, such as filtration, sooner or later; second, to the sentimental value attaching to any drinking water not subject to the danger of pollution in comparison with a water known to have been polluted but made wholesome by suitable treatment; third, to the time in the future when the expenditures for filtration and other treatment will begin; fourth, also to any protection which the ownership of the watershed would give against an adverse use of the waters originating therein, and against their diversion from the watershed and, finally, to any other benefits that might result from such ownership, such as uses not incompatible with the development of the water supply.

Frequently, of course, the value of watershed lands, owing to

their use for farming purposes and owing to the inclusion of densely populated areas, is so high that their ownership for the protection of the quality of the runoff waters is entirely out of the question and they do not then come under consideration in the determination of the value of the opportunity to make a water supply available.

Notes relating to Some Water-right Values in California.— The following relating to the cost and value of certain water-rights in California is from the testimony of engineers who appeared before the Master in Chancery in the proceeding entitled "Spring Valley Water Company vs. The City and County of San Francisco" which was on trial from July, 1915, to May, 1916.

Mr. G. G. Anderson in his testimony referred to the value of water-rights in southern portions of California as determined from the value of the shares of stock in the various mutual irrigation water companies, and he says "care was exercised in the analysis of individual cases to limit the values of these water-rights to terms of the right to divert and use only, excluding all interest in attached lands or ditch systems or any assets other than actual rights of diversion and use." He cites:

	Per million gals. per day
Duarte Mutual Co. near San Gabriel	\$154,800
Covina Co. near San Gabriel	117,957
Del Monte Co. near Pomona	98,220
Canyon Water Co. near Pomona	104,490
San Antonio Co near Ontario	144,738
Bear Valley Co. near Redlands.	90,248
Mill Creek Co. near Redlands	92,880
Gage Canal Co near Riverside	77,400
Temescal Co. near Corona	96,750

The foregoing values Mr. Anderson states "attach to the service for the particular purpose (irrigation) which entails delivery of water during the irrigation season or ordinarily 240 days per annum, rarely does the season extend to 270 days, and the service during the limited period does not, in all cases, yield full efficiency on the water-right."

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For the irrigation plants in the Santa Clara Valley Mr. Anderson develops values ranging from \$31,442 to \$67,724 per million gallons of daily supply from gravity systems, over periods up to a maximum of 115 days per annum; and \$167,709 to \$242,786 for pumping systems.

He also finds that the original cost to the Spring Valley Water Company of acquiring water-rights on both sides of San Francisco bay (about 1865 to 1913) averaged nearly \$38,000 per million gallons per day, of water delivered to the inhabitants of San Francisco on the date of valuation, Dec. 31, 1913. By a different analysis and with some allowance for uncertainties Mr. Chas. H. Lee testifying in the same case finds this cost to have been about \$48,000 per million gallons per day.

Mr. F. C. Herrmann calls attention to the purchase by the Pacific Gas and Electric Company from the Livermore Water and Power Company of rights to about one million gallons of water per day at about \$100,000 in 1913. The water-rights involved in this transaction are for the Mocho and Positas Creeks near Livermore, Cal.

He also refers to a number of sales of springs and wells with small yield and to the following sales in southern portions of California:

- In condemnation proceedings by the City of Sierra Madre the value of water for domestic use was placed by the court at \$270,760 per million gallons per day.
- Near Alhambra, in 1892, Richard Garvey bought from De Barth Shorb about 390,000 gallons of water per day at \$64,595 per million gallons per day.
- At Montecito, Mr. Knapp bought 12,000 gallons of water per day a one-third right in the Warm Springs tunnel, at \$386,000 per million gallons per day.
- The value of water fixed by the California Railroad Commission in the Glendale case, in addition to the allowance for structures was \$154,720 per million gallons per day.

While each specific instance of a sale of water-rights, as cited by Mr. Herrmann, should be considered in the light of all cir-

Per million

Per million

cumstances attending each such sale and of the specific needs of the purchaser in each case, these sales, nevertheless, have some effect upon the market value of water-rights showing as they do what a purchaser may, under certain circumstances, be willing to pay for water.

Both Mr Herrmann and Mr. Anderson were witnesses for the plaintiff in the Spring Valley case. Mr. Chas. H. Lee appearing for the defendant, that is for San Francisco, added the following estimates of net water-right values:

1. Culture exclusively citrus (Southern Cal.):

			gals per day
Lugonia Water Co near Redlands .			\$154,000
San Antonio Water Co near Ontario			142,800
Del Monte Irrig Co. near Pomona			90,900
Temescal Water Conear Corona			69,600
Gage Canal Co. near Riverside .			73,500
Redlands Water Co near Redlands			52,400
San Dimas Irrig Co. near San Dimas			51,500

2. Citrus and diversified crop (Southern Cal.):

Santa Ana Irrig Co. near Redlands	gals per day \$43,500
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Alta Mutual Water Co near Riverside	. 42,500
Thermal Belt Water Co. in Santa Clara River Valley	. 42,500
Riverside Water Co. at Riverside	14,700
Los Nietos Ditch Co. near Whittier	. 13,200
South Side Improvement Co. in Santa Clara River Valley	. 2,860

3. Diversified crops, no citrus (Southern Cal.):

Banning Water Co. near Banning	gals per day
• • •	\$37,400
Moneta Water Co. near Redondo	23,200
McKenzie Ditch Co. near San Bernardino	13,200
Stout Ditch Co near Redlands	12,100
Puente Water Co. near Puente	2,400
Arroyo Ditch Co. near Downey	2,300
Little Lake Irrig Co. near Norwalk	1,000

For more northerly portions of the State of California where water is more abundant and where the demand does not yet approach the limit of possible development, Mr. Lee lists the following:

Culture exclusively citrus:	Per million
Lemon Cove Ditch Co. on Kaweah River Rosedale Water Co. on Tule River	\$25,400 23,200
Citrus and diversified crops:	Per million
South Tule Independent Ditch on Tule River	gals per day 5,070
Diversified crops, no citrus:	Per million gals per day
Bishop Creek Ditch in Owens Valley	3,100
Clark Colony Water Co. in Owens Valley	3,100
McNally Ditch Co in Owens Valley	3,100
Owens River Canal Co. in Owens Valley	3,100
Roberts Ditch Co. near Colusa	1,160
Watson Ditch Co. near Visalia	2,560
Evans Ditch Co. near Visalia	2,012
Murphy Slough Assoc. on lower Kings River	1,940
Rawson Ditch in Owens Valley	1,940
Consolidated Peoples Ditch on lower Kings River	870
Oakes Ditch Co. near Visalia	580
Poplar Ditch Co on Tule River	217

Mr. Lee cites also a few canal companies for whose waterrights no value is demonstrable from the market value of stock in the canal company. These were omitted from the above enumeration

The above figures are not presented to show the value of water in regions where society already demands full utilization, but rather to show that with the demand for a higher use of water which must come as the pioneer region gradually changes to a densely populated territory, the value of the water-right in any such country as that under consideration, where nature has set a limit to the available supply, must go up.

CHAPTER XII

THE ACCOUNTING SYSTEM

Purpose of the Accounts. — It is not proposed to take up the matter of accounting in connection with public utility enterprises any further than to indicate its purpose and to refer briefly to certain instructions in relation thereto which have been issued by public service commissions.

The accounts should show clearly and in sufficient detail the facts relating to the investment, to operating expenses and to income. The natural division is therefore into two groups, the one relating to the investment, the other to income and operating expenses.

The accounts relating to investment are designed to show the investment in the property which is devoted to public use. These accounts should be kept in such form that the additions and betterments from year to year will be clearly apparent. They will include not alone the actual cost of all physical properties but also any amount paid for franchises or for waterrights and rights of way.

The income accounts are those which are designed to show, for each year, the amount of money earned for services rendered or for commodity delivered and the cost of rendering the service or furnishing the commodity. They will include the returns from outside investments and other sources on the one hand and on the other every expenditure necessary to render the service or to supply the commodity including taxes, insurance, rents and the like. The net balance of the operating and income accounts is profit or loss.

Construction Account, Interstate Commerce Commission.— In the general instructions of the Interstate Commerce Commission relating to the accounting system of railroads, construction is defined as including all processes connected with the acquisition of the original road and equipment, road extensions, additions and betterments. The following is from these instructions:

"Costs shall be actual money costs to the carrier. When a portion of the funds expended by the carrier has been obtained through donations by the states, municipalities, individuals or others, no deduction on account of such donations shall be made in stating the costs. Contributions for joint expenditures should not be considered as donations. The carrier's proportion only of the cost of joint projects, such as construction of jointly owned tracks and elimination of highway crossings at joint

expense, shall be included in these accounts.

"The charges to the accounts of this classification shall be based upon the cost of the property acquired. When the consideration given for the purchase or the improvement of property the cost of which is chargeable to the accounts of this classification is other than money, the money value of the consideration at the time of the transaction shall be charged to these accounts, and the actual consideration shall be described in the record in sufficient detail to identify it. The carrier shall be prepared to furnish the Commission, upon demand, the particulars of its determination of the actual cash value of the consideration, if other than money.

"It is intended that the accounts for fixed improvements and equipment shall include the cost of construction of such property. The cost of construction shall include the cost of labor, materials and supplies, work-train service, special machine service, transportation contract work, protection from casualties, inquiries and damages, privileges, and other analogous elements

in connection with such work.'

Treatment of Depreciation and Replacement, Interstate Commerce Commission. — The attitude of the Interstate Commerce Commission with reference to the depreciation and replacement of property appears from the following instructions:

"When a unit of property other than land or equipment — such as a section of road, side or yard track, shop or power plant machine, building, or other structure — is retired from service and replaced with property of like purpose, the ledger value of the retired property shall be credited to the appropriate

accounts of this classification at the time that the property is retired from service. The amount of this credit shall be charged concurrently as follows:

"An amount equal to the credit balance in the accrued depreciation balance-sheet account with respect to the property thus retired shall be charged to that account and the remainder (less salvage and insurance recovered, if any), together with the cost of demolishing the property, if demolished by or for the carrier, shall be charged to the accounts in Operating Expenses appropriate for the cost of repairs of the property before retirement. The accounting for the salvage shall be in accordance with the disposition made of the material recovered.

"If, however, the property retired and replaced with property of like purpose is of minor importance, such as a small roadway building or other small structure, and is replaced in kind without betterment, the cost of the replacement shall be charged to operating expense accounts, and no adjustment made in the road

and equipment accounts.

"If so authorized by the Interstate Commerce Commission, the carrier may charge to Profit and Loss any extraordinarily large item representing the cost of property retired and replaced, instead of charging such item to Operating Expenses. The carrier shall file with the Commission a statement of the cost and a description of the property retired and the reasons which, in its judgment, indicate the propriety of charging the cost of such property to Profit and Loss.

"The provisions of this section are applicable in accounting (at the time of retirement) for the cost of property abandoned, even though the new property has been actually installed previously to the date of the demolishment of the abandoned prop-

erty.

"When the renewals to be made to an important building or other structure will constitute the major portion of its value when renewed, the property, when taken out of service, shall be considered as retired and accounted for as provided above, and for the purposes of this classification the renewed property shall be considered as an addition, and the appraised cost thereof shall be included in the accounts of this classification, consideration being given to the second-hand portions remaining therein. In no case shall the charge for the renewed property exceed the cost (at current market prices of labor and material) of new property of equal capacity and equal expectation of life in serv-

ice, less a suitable allowance on account of the second-hand

parts remaining therein.

"When a unit of property other than land or equipment—such as a section of road, side or yard track, shop or power plant machine, building, or other structure—is retired from service and not replaced, the ledger value shall be credited to the appropriate property accounts at the time that the property is retired from service. The amount of this credit shall be concurrently charged as follows:

"An amount equal to the credit balance in the accrued depreciation balance-sheet account with respect to the property thus retired shall be charged to that account, and the remainder (less salvage and insurance recovered, if any), together with the cost of demolishing the property if demolished by or for the account of the carrier, shall be charged to the appropriate profit and loss account. The accounting for the salvage shall be in accordance with the disposition made of the material recovered."

Retirement of Land, Interstate Commerce Commission. — In the matter of land retired from use the Commission says:

"When any land, the cost of which is included in the accounts of this classification, is retired, the ledger value shall be credited to account 'Land for transportation purposes.' If the land is retained by the carrier, its estimated value shall be charged to balance-sheet account 'Miscellaneous physical property,' the necessary adjustment of the difference between the ledger value and the estimated value on account of the loss in the property due to its retirement from transportation service shall be made through Profit and Loss. If sold, the difference between the ledger value . . . and the amount received for the land shall be adjusted in Profit and Loss."

Engineering Account, Interstate Commerce Commission. — Relating to engineering on the construction of a railroad, the Commission says:

"This account shall include the pay and expenses of engineers, assistants, and clerks engaged in the survey and construction of new lines and extensions, or in making additions to and betterments of the carrier's road, including wharves and docks.

"When employees . . . are engaged in the maintenance of the road, their pay and expenses while thus employed shall be

charged to Operating Expenses.

"Expenditures for tentative or preliminary surveys shall be carried in a suspense account until it is determined whether or not to continue the work. If the project is continued, expenditures for all surveys in connection therewith shall then be transferred to this account, and, if abandoned, to Operating Expenses, Income, or Profit and Loss, as may be appropriate"

Interest during Construction, Interstate Commerce Commission. — The Interstate Commerce Commission in the instructions relating to "interest during construction" says:

"When any bonds, notes, or other evidences of indebtedness are sold, or any interest-bearing debt is incurred for acquisition and construction of original road and equipment, extensions, additions, and betterments, the interest, accruing on the part of the debt representing the cost of property chargeable to road and equipment accounts (less interest, if any, allowed by depositaries on unexpended balances) after such funds become available for use and before the receipt or the completion or coming into service of the property so acquired shall be charged to this account.

"When such securities are sold at a premium the proportion of such premium assignable to the time between the date of the actual issuance of the securities and the time when the property acquired or the improvement made becomes available for service shall be credited to this account.

"This account shall also include such proportion of the discount and expense on funded debt issued for the acquisition of original road, original equipment, road extensions, additions, and betterments, as is equitably assignable to the period between the date of the actual issuance of securities and the time when the property acquired or the improvement made becomes available for the service for which it is intended. The proportion of discount and expense thus chargeable shall be determined by the ratio between the period prior to the completion or coming into service of the facilities or improvements acquired and the period of the entire life of the securities issued.

"This account shall also include reasonable charges for interest, during the construction period before the property becomes available for service, on the carrier's own funds expended for construction purposes."

Application of Accounting Principles to the Unlimited Life Method. — Under the Unlimited Life Method of procedure,

which is a new suggestion and which is not contemplated in the foregoing instructions, but which is the one that should find preference for all complex public utility properties made up of a large number of individual items, there will be no depreciation to write off. It will be unnecessary to estimate accrued depreciation except when the transfer of the property is involved or a valuation is to be applied to individual articles. The current replacement requirement, however, will have to be estimated; but the estimate need be only approximate. The replacement fund will not be available for the retirement of capital. Neither will it be available for betterments or additions to the property. If it grows too rapidly, if the accumulation in it becomes unnecessarily large, the current replacement requirement may have been over-estimated; if it is depleted there may have been an underestimate of the replacement requirement. Excessive accumulation in the replacement fund should be checked by reducing the amount annually set apart for the fund. In the accounting system, the cost of each article as it goes out of use is written off the books and each article which replaces a discarded article is entered as a renewal and is treated practically as though it were new construction. Its cost is offset by the amount written off for the discarded article and the replacement fund is depleted by this cost. The capital investment account remains unaffected unless the cost of the new article is greater or less than that of the discarded article. The excess cost, if any, should not come out of the replacement fund but from new capital. The deficiency, if any, represents a reduction of the invested capital.

The accounting system under the Unlimited Life Method of procedure, as will be seen from the above statements, is much simpler than that for procedures which take cognizance of the accrued depreciation and of the constantly changing present value of the physical elements.

The replacement account, if the Unlimited Life Method be adopted, will be credited with all expenditures that are made for the renewal of items which can be conveniently individualized and which serve or are expected to serve for a number of years, and it will be charged with the amount annually set apart out of earnings for replacements. This account should be kept separate from repair accounts although there is no fundamental difference between the replacement of a broken spoke in a wheel and the replacement of a discarded generator. It is fundamentally immaterial whether the replacement relates to single lengths of pipe, to broken window panes and the like or whether they are large as in the case of the dismantled steamboat. The only difference lies in the classification of the accounts. either event funds for the replacement must be made available at the proper time. It is in the making of suitable provision for the replacement of the individualized items that expert advice should be sought in order that a proper distinction may be made between the apparent and the real profits of the business. Without a proper analysis, an apparent profit has, too frequently, been found to be in fact a loss.

Application of Accounting Principles to Other Methods.— Under any method of procedure, other than the Unlimited Life Method, the difficulty is presented of dealing with articles which do not serve throughout their full probable life terms and with articles which remain in service beyond these terms. In the one case the cost of an article may have to be carried for complete amortization long after the article has gone out of use. In the other case an article with many more years of usefulness should be rated as of no value. Accountants have found methods to overcome these difficulties but the book-keeping which is involved becomes complicated.

When a public utility is established and its existence is justified by all attendant circumstances, the owner has reason to assume that the prospective business will prove profitable. There will usually, however, be a certain time during which his operating expenditures will exceed the revenue produced by the property. This time may extend over a period of years. Due consideration must be given to such facts as this as well as to the estimates of the prospective business when the cost of a

commodity or of a service is to be ascertained and its sale price is to be fixed. The cost records should be so kept that they will give all needed information relating to past financial history as well as to the current cost of operation.

Any reduction in the value of an operative property such as may result from an advance in the art of the manufacture of its output or due to decrease in value of real estate and like causes, may result in an operating loss. In the case of a public service corporation where the investment is of real concern, such possible reduction of value is one of the elements covered by the hazards of the business and should be forestalled or should in the course of time be amortized by the excess of earnings over and above earnings on ordinary safe investments.

Any increase in value, no matter what it may result from, belongs among the earnings. Despite the rulings which have been had on this point by competent authority, it would seem preferable not to include appreciation in the rate-base but as elsewhere suggested to permit earnings that will give the owner a limited share in the general prosperity of the community.

CHAPTER XIII

THE VALUATION OF MINES AND OIL PROPERTIES

General Statement

Purposes of the Valuation. — Valuations of mines and oil properties as in case of other properties are needed for many purposes, the most general being, (1) the purchase or sale of such properties or portions thereof, (2) the information of owners or others interested in the property as a guide to the proper operation thereof or to the financial operations connected with such operation and (3) the determining of a basis on which taxation can be figured. The methods used to accomplish purposes (1) and (2) are closely associated and in many cases identical, but the proper method or methods for determining what the basis for taxation shall be, as shown later, must differ from valuations for other purposes.

Limitations upon Accuracy. — Because of the impossibility of fixing definitely the economic value of the mineral deposit, the extent of which may be indeterminable, valuations of mining and oil properties must be less definite than the valuations of industrial and public utility properties, the value of whose physical elements, at least, can usually be determined with considerable accuracy.

In underground metal mining, the development is in many instances kept just a short distance in advance of productive operations. This is sometimes due to the irregular nature of the ore bodies, but generally can be explained as a result of the method of financing mining properties. This is particularly true in small operations where the money earned is used to carry the expense of the development and the necessity to raise additional funds for extensive development is avoided.

This is not always true of extensive mining properties representing large investments. The Utah Copper Company in the United States and the Braden and Chicaquamata copper properties in Chile have ore bodies blocked out sufficient for 45 to 70 years of operation. This extensive development facilitates the valuation of these ore bodies. It should be noted, however, that two mines having equal ore reserves might be valued at very different figures because one may promise a future beyond its reserves while for the other there may be no hope of further production when the reserves in sight are exhausted.

The difficulty that is met in estimating the value of the mineral deposit is apparent also at oil properties when only a number of wells sufficient to develop enough oil to supply the existing market and to provide for a limited period in advance are drilled. Because of the usual great depth of oil wells, the sinking of new bores becomes an item of large expense and the development to a considerable extent of the holdings of an oil producing company might financially embarrass an otherwise profitable concern.

Placer gold deposits (hydraulic and dredging) in most cases are thoroughly prospected by pits or drill-holes and the value ascertained before mining operations are commenced. The drift placer mines of the Western United States form an exception and it is in most cases extremely difficult if not impossible to determine the value of such deposits.

Methods Commonly Employed in Valuation

Enumeration of Methods of Valuation. — Numerous methods of valuation have been employed and these have been modified by such variations as have suited the views of the valuating engineer or valuating body. That this has resulted in confusion is shown by recent court decisions. The engineer or valuer who does not definitely state the methods used by him in obtaining his valuation of the property seems to be the most likely to have his estimate supported by the findings of the court when appeal has been taken.

Among methods that have been used the following are the most important:

- (1) Valuation by empirical methods.
- (2) Valuation based on market value.
- (3) Valuation based on royalty value.
- (4) Valuation by capitalizing profits.
- (5) Valuation by estimation method.
- (1) Valuation by Empirical Methods.—An important example of an empirical method is the so-called "foot-acre" valuation of coal measures. A certain value per foot-acre of coal has been established in a certain district and the value of any property is based on this unit disregarding the fact that thin beds of coal are not worth as much per acre-foot as thick beds.

Another empirical method is that under which a value is placed on a mineral property of so many times the annual proceeds or the annual profits regardless of the actual expectation of life of the mine. It is needless to say that this method is used only for taxation purposes.

- (2) Valuation based on Market Value. A method of valuation of coal lands based on the use of the results of sales of neighboring properties has been found acceptable to the courts. The attempt is made to fix the actual sale value of a coal property by examining the price at which other property similar to that under consideration has been sold. This method is derived from existing methods of real-estate valuation and from an engineer's standpoint is acceptable only when checked or supplemented by a valuation on some other basis. Such checks may demonstrate that recent sales were not made at the actual value of the properties sold. Further than this no recent sales of similar properties may be available for purposes of comparison.
- (3) Valuation Based on Royalty Value. The royalty value method of appraising the value of mining properties makes use of the established royalty value per ton. This method of

valuation has been applied to some extent in coal regions. Because coal becomes more valuable from year to year the value of a leasehold will increase. The annual profits exceed the royalties by continuously increasing amounts and a valuation based on the average royalty payments in a district cannot be a gauge of the actual values, particularly districts where the leases have been in effect for some length of time. In addition to the value of the mine or colliery determined from royalty values, the leasehold itself has a value which increases as the value of the mineral product goes up and the actual value of the property must be the summation of these two values. As the leasehold value is determined by subtracting the valuation based on royalty values from a valuation of the property, itself, by some other method, it is evident that the royalty value method cannot be independently used to determine the value of the property.

(4) Valuation by Capitalizing Profits. — By this method the valuation is based on a capitalization of the net annual profits. It requires an estimate of the remaining life of the property but does not take into account the fact that incompetent management may so deplete the profits that a valuation with these as a basis may not approximate the actual value of the property in competent hands.

This method should only be adopted when estimated profits instead of actual profits are used as a basis. It is then applicable in the case of oil properties and other deposits the extent of which is indeterminable. If the life of adjoining or neighboring properties has been proven, an approximate basis may be had for estimating the life of the property which is to be valued. The depths to which mineralization extends in neighboring mines often is used to determine the expectation of a prospect when such information supplements the geological features of the deposits.

(5) Valuation by Estimation Method. — This method is adopted by mining engineers in the valuation of mineral properties when these are to be bought, sold or financed. Deter-

mination of all the elements that have a bearing on the values is necessary. The estimated profits capitalized furnish the foundation for a valuation of the ore bodies or deposits. These profits are dependent on the sales value of the product, the geological structure of the deposit, the cost of mining (present and future), the cost of surface treatment, and other factors.

In fixing the value of coal lands use can be made of the "footacre" method of valuing coal lands provided account is taken of the fact that a foot-acre of coal in a thin vein is not so valuable as it is in a thick vein. The sales of adjoining properties will be of assistance to the engineer in fixing a value, particularly in case of coal lands which have not been explored but which contain the same beds and show geological conditions similar to properties that have recently changed hands. The "royalty value" should not be ignored and when properly determined furnishes a valuable check on the valuation.

When account has been taken of the risks of operation, the geological features of the deposit and the economics of the necessary surface plant, the total prospective estimated profit can be suitably discounted and a certain definite valuation placed on the property. In determining the present value of a property for which the profits have been estimated, it is necessary to remember that certain fixed charges such as replacements, sinking fund payments (paid or not paid), etc., are to be deducted before the profits are capitalized.

A mining property to be a good investment must promise to an investor the return of his capital in addition to interest on the capital at a rate commensurate with the risk he takes in investing in such a property. What the risk will be should be determined by the investor before he makes the investment. Comparison of the proposed investment with certain standard investments such as government bonds, railroad bonds and realestate mortgages should be made.

Proper Rate of Interest. — The proper rate of interest on mining investments has been and still remains the subject of much discussion among engineers. A reasonable solution of

the problem has not been advanced and it is the belief of the author that it is not possible to determine a rate of interest that would apply equally well to all mining investments. The risk incurred by investing in a property that has been slightly developed is much greater than the risk in the case of a wellprospected ore body even when the mines contain similar ores and will be operated under similar conditions. Then again the average risks in copper mining differ from the average risks in gold mining and so on.

Mr. H. C. Hoover has tabulated the risks of mining as follows:*

- "I. The risk of continuity in metal contents beyond the sample faces.
- "2. The risk of continuity in volume through the blocks estimated.
 - "3. The risk of successful metallurgical treatment.
 - "4. The risk of metal prices, in all but gold. 5. The risk of properly estimating costs.

 - "6. The risk of extension of the ore beyond exposures.
 - "7. The risk of management."

Several of these risks are found in industrial enterprises (Risks 4, 5 and 7). The risks of continuity of ore body and of ore values are peculiar to the mining industry. The limited market for the mineral products and the effect of the volume of the output on the prices that can be obtained increases the risk that capital must take. The problem of obtaining proper metallurgical treatment is an important one particularly when starting operations at new properties. The fact that the mineral constituents of the ore may change as greater depths are reached and that the previously satisfactory flow sheet may no longer realize the percentage of extraction on which the profits were based cannot be ignored by the investor. The interest return that might be attractive to an investor in a proven district might not be sufficient to attract capital in a district where the mines are still prospects and where the depth of the mineralization has not been tested.

^{*} H. C. Hoover, "Principles of Mining," 1909.

It is a fact that industrial enterprises because of additional risks demand greater interest returns than Government bonds and it seems reasonable that mining investments taken as a class should call for a greater rate of interest than industrial enterprises.

This claim for a higher rate of interest is opposed by such an authority on mine valuation as Mr. J. R. Finlay* who can be quoted as follows:

"I have generally assumed that 5 per cent was a normal interest—or discount rate. If that is so, it is a fair figure to use in a mine valuation, which should be nothing but a candid inquiry into the present value of expected profits."

If Mr. Finlay's statement is correctly understood, he is willing to ignore the risk that these prospective profits may be diminished or may entirely be cut off before the estimated life of the property has been accomplished. If it is true that at any mining property risks exist over and above the risks existing in so-called "safe" investments, then a 5 per cent discount or interest rate is not sufficient to induce sane investment. Mr. Finlay's 5 per cent interest rate, as applied to the iron mines of Michigan in his appraisal made for the State Tax Commission in 1911, was changed by that Commission to a 6 per cent basis in 1913.

Other authorities have gone on record as advocating higher interest rates and in this connection the following will be found of interest:

Mr. J. H. Curle † states that a suitable mining investment must fulfill the following requirements:

- "rst. The development in the bottom must be good;
- "2nd. The mine must pay 10 per cent per annum;
- "3rd. There must be 60 per cent of the price of the shares in sight."

Mr. Hoover in his admirable work on mine valuation says: ‡

- * J. R. Finlay, "Valuation of Iron Mines," Trans. A.I.M.E., Vol. 45, p. 295.
- † J. H. Curle, "The Economist," London, Sept. 15, 1903. ‡ H. C. Hoover, "Principles of Mining," 1909.

"What rate of excess return the mine must yield is a matter of the risks in the venture and the demand of the investor. Mining business is one where 7 per cent above provision for capital return is an absolute minimum demanded by the risks inherent in mines, even where the profit in sight gives warranty to the return of capital."

Mr. G. A. Denny, an English engineer, says:

"A normal mining risk stated in terms of interest may be taken at 10 per cent per annum on the capital expended plus a rate for the redemption of capital."

John Hays Hammond† expressed his views on this question as follows:

"In many mines persistency of the ore deposits and, therefore, the reliability of the mines as dividend payers, justified the investment upon a basis in some instances as low as 8 per cent, dividends to which, of course, must be added a certain percentage to provide for the amortization of the capital. Generally speaking, however, investments in mining securities are not to be regarded as attractive unless they return from 10 per cent to 15 per cent in dividends, in addition to the profits to be set aside for amortization."

Price of the Mineral Product. — Because of wide fluctuations in the prices that minerals bring in the market, the use of the current market price of such a product in measuring the value of a mineral deposit is not proper for valuation purposes in general. No hardship would be forced on the owner or operator if, for taxation purposes, valuations were made annually based on the current price of the mineral produced. This method, however, results in a valuation fluctuating in amount from year to year and because of the constant changes required, does not seem to be practical.

The "normal" price of a mineral may be defined as the average market price over a certain definite period of years. Periods of time five or ten years in length are in general use. The

^{*} G. A. Denny, Mexican Mining Journal, July, 1919.

[†] John Hays Hammond, Engineering and Mining Journal, Jan. 1, 1910.

"basic" price represents the point at which the mineral production falls off to such an extent that a rise in price results.

Mr. Hoover in his "Principles of Mining" states that safety lies somewhere between the "basic" and "normal" prices. No such limitation should be placed on the exercise of the judgment of the valuating engineer, as this assumption cannot hold good during the years following protracted periods of financial depression such as the hard times of 1893 to 1898. The "normal" price of many minerals estimated on a 10-year or 20-year average basis was for a considerable period of time following this depression less than the actual value of the mineral.

For illustrative purposes certain tables of statistics giving the production and average prices of four important metals (copper, lead, silver and zinc) from 1880 to 1914 are given at the close of this Chapter. Curves have been plotted which illustrate the relative increase in the production of these metals from year to year and also the changes in the average annual price. (Fig. 4 to Fig. 7.) For comparison normal curves of production and price (based on ten-year averages) have been plotted assuming that the average figure for production and price are applicable midway of the ten-year period. By continuing these normal curves to date on the assumption of a five-year future history for the metal, a present "normal" price can be obtained that will be as close an approximation as can be made.

Valuations for Purchase and Sale. — When a valuation of a mining property is needed because of a contemplated change in ownership, the most satisfactory method of valuation is the straightforward one involving the determination of the value of the ore body on the basis of its production past, present and expected.

Such a valuation includes an estimate of the probable life or expectancy of the property under the actual or an assumed rate of production, an estimate of the price that the mineral product will bring in the markets of the world and an estimate of the cost of producing the mineral. This method requires the use of all available information as to geology, actual operating costs,

adjacent developed properties, recent sales of same, determination of the "normal" price of the mineral, recent sales of metal or of ore at the property, determination of the proper rate of interest to be used as a basis for capitalization, and the like.

The rate of production that is properly brought under consideration must be determined from consideration of the fact that the operating costs per ton decrease as the capacity increases, while the fixed charges increase when the cost of the installation is increased. It may be found that the rate of production actually in use is not that which brings out the full value of the property and an assumption of a proper rate should be made in determining the profits that can be expected. In the case of certain minerals having a limited market, the output is controlled by market conditions which must be considered in determining the rate of production.

Valuations for Owner's Information. — Valuations made for the purpose of informing the owner as to the condition of the property, although made in a similar way to those for purchase and sale purposes, may be limited by the special purpose for which they are required.

During the operation of a property it may be desirable to determine from time to time the present value of the property, taking into account the new underground developments, improved methods of ore treatment and changed market conditions. A desire on the part of the owner to ascertain on what basis he might, with advantage to himself, join in coöperative operations with adjacent properties, may lead to a valuation. This often occurs in the case of oil lands in small holdings that for reasons of economy can only market their product by cooperative means. The rental of coal lands and other deposits that contain large mineral bodies to other operators requires a determination of the "royalty" that should be charged and this should be based on an actual valuation and not on the average royalty in current use in the district although this value may be used for a check.

Valuation for Taxation Purposes. — Whether it is desirable from an economic standpoint to tax the ore bodies in the possession of mining companies is an open question that has been hotly debated by engineers and economists. Different methods of taxation have been suggested:

- (1) Taxation based on surface improvements only.
- (2) Taxation based on estimated value of the ore body and surface improvements.
- (3) Taxation based on "foot-acre" or "royalty" value and on empirical methods of valuation.
- (4) Taxation based on annual production or gross proceeds.
- (5) Taxation based on annual profits or net proceeds.

Probably the earliest popular method of taxing mining properties was to tax only their surface improvements. It was not considered advisable to tax an industry that was of such economic value to a community. Also because of the risks of the business it was deemed necessary to make mining investments as attractive as possible to the capitalist and not to hamper operations, particularly in the early stages, when an enterprise was just getting on its feet. The history of the mining industry in this respect is similar to that of public utilities, which have anticipated and facilitated the growth of population in the territory served. Just so long as the operations were conducted at a risk the public showed no desire to interfere or to assume responsibility, but once the investments have become firmly established and are earning a reasonable interest on the capital investment regulation of the utility with its accompanying embarrassments has been commenced and the practice of allowing the company a return on its depreciated "fair" value and not on its investment has sprung up.

In the case of the mining properties it became apparent that the large coal, iron and copper operations after a successful struggle for existence had become profitable and represented a very large portion of the business of certain communities. Taxation above a mere tax on surface improvements was proposed and has become an established practice in many states.

Valuation of Mines for Taxation in Various Localities

In a general way some methods of valuing mining property for taxation purposes will now be outlined.

Alabama, California, Iowa, New Jersey, New York, Oregon, Washington, and West Virginia appraise mining property in a similar manner to real estate and other property. They have no laws pertaining to the valuation of mining property. This method requires a perfunctory valuation of the surface improvements and of the ore body, the latter in some cases being entirely ignored.

Michigan, Minnesota and Wisconsin assess their mining properties on a basis of the estimated value of the ore body and the surface improvements giving weight to all the elements that enter into value.

In Michigan the present system was introduced by Mr. T. R. Finlay in 1911. Data are compiled from the reports of mine operators in sworn statements. Then an inspection is made by the State Geologist and his assistants to estimate the reserves and check up other data. The State Tax Commission passes on the figures and after a public hearing the final results are reported to the local assessors. The valuation is based on four elements — ore reserves (developed and prospective), average annual profits (five-year average), life or expectancy, and the rate of interest. The developed reserves consist of ore blocked out above the bottom levels. Prospective reserves consist of the expected extension of the ore bodies downward or laterally beyond existing workings. The average annual profits are estimated from a five-year record when such exists. Undeveloped properties are judged by the operations of other mines. The expectancy of a property is taken at the ratio of the estimated reserves to an average five-year shipment. The life of the undeveloped mines is estimated by comparison with adjacent operations. The interest rate is now 6 per cent for both principal and sinking fund. It was 5 per cent and 4 per cent respectively in 1911 when Mr. Finlay had charge. The criticism of this system is that the values of the ore bodies cannot be accurately determined, that by using annual profits as a basis a premium is placed on inefficient operations, that the life cannot be determined with any degree of certainty and that the rate of interest is too low, making the present estimate of value too high. It has several times been proposed to introduce a tonnage tax in Michigan. Recently the State Grange proposed to invoke the Initiative in order to obtain a tax bill proposing a levy of $\frac{1}{2}$ cent per pound on copper and ro cents per ton on iron ore. This method of taxing would penalize the concern that was mining with only a small margin of profit and would tend to unnecessarily hamper the mining industry in the State.

Minnesota has the engineers of the State Tax Commission measure up the tonnage of ore underground and then an ad valorem tax on 50 per cent of the present indicated value is levied. Mineral lands that are not productive are taxed. A tax of 4 to $4\frac{1}{2}$ cents per ton has been proposed on stock piles.

In Wisconsin the 1913 Legislature passed a law assessing mining properties or mineral lands upon the estimated value of the ore underground. A report based on production profits, etc., was made by W. L. Uglow, engineer for the State Tax Commission, but his recommendations were greatly altered by the Commission. It was found necessary to reduce the figures for developed properties and to entirely cut out the estimates on undeveloped properties. The system he adopted might be called a semi-empirical method which is based on figures from a theoretical zinc mine. His theoretical or "hypothetical" mine was the result of combining the figures from several actual operating properties. The assumed life was taken as four years which is the average as determined from the previous experience of the district. Mr. Uglow's method of ascertaining the value of a mine as applied consisted of multiplying the annual profit by the factor 2.43 which is the figure obtained by using the "hypothetical" mine and assuming that profits do not fluctuate from year to year. He claims as advantages that his method is based on actual profit and average life and that it bases the assessment on a multiple of the profits rather than on the market value of the mine.

Pennsylvania taxes its coal lands under the acts of 1841 and 1842 which require the taxation of "every subject of taxation at the actual value thereof." The foot-acre method of assessment was in general use until 1907 when a big jump in value was made. Appeal to the state courts resulted in decisions which declared the "foot-acre" and "royalty" methods unsatisfactory and dependence is now placed mainly on recent sales in the vicinity to determine the market value of a property. Although fairly satisfactory for real estate appraisals this method cannot be applied with equal success to lands whose coal deposits, of indeterminate extent, in many instances give them value. Further than this recent sales in the vicinity may be scarce and the resulting standard of value because of this fact very unsatisfactory.

Arizona and Colorado tax mining properties on the basis of production.

Arizona divides mining properties into two general classes, productive and non-productive. Productive properties include those which yield net proceeds during the year, after deducting cost of active operation, cost of transportation, cost of marketing, smelting and refining, and expenses for betterments and repairs. The return of the capital cannot be considered as part of the cost nor can the salaries of those not continuously engaged in the enterprise within the state be included as an element of expense. The assessed value is arbitrarily taken as one-eighth of the gross value plus 4 times the net value of the annual output plus the value of the surface improvements. Non-producing mines are taxed in the same way as other real property in the State.

Colorado determines the net proceeds by deducting the actual cost of extraction, after eliminating unnecessary expenditures and the actual cost of transportation to place of sale, from the gross value of the annual production (the sum actually obtained

by the owner). In 1915, the assessed value was fixed at 25 per cent of the gross annual proceeds or the net annual proceeds whenever this exceeds 25 per cent of the gross proceeds. Improvements are assessed at the same rate as other property in the vicinity is assessed. It is claimed in Colorado that this law permits the owner of a large idle group of claims to pay no greater tax than a prospector actually working a single claim.

Idaho, Montana, Nevada, New Mexico and Utah tax the net proceeds of mining properties.

In Idaho the net proceeds are used as a basis for the assessment Improvements are assessed at full cash value. The ground of undeveloped claims is taxed on the prices paid to the U.S. Government when they were purchased.

Montana bases its mine tax on the net proceeds. The surface is assessed at the price paid to the U. S. Government (\$5 per acre for metal mines and \$2 per acre for coal). If the surface is more valuable for other purposes, it is assessed at the market value of such property. Surface improvements are taxed at market or local value.

Nevada assesses all patented claims at \$500 unless it can be proved that over \$100 worth of work has been done during Improvements are assessed at local value. A tax is levied on the net proceeds of all operating mines. By deducting such charges for operation, transportation and reduction or sale of output as the State Tax Commission permits, from gross value of the production, the net proceeds are estimated. No deduction for office expenses other than those of operating offices are permitted. The salaries of mine officials are limited. The investment in surface plant is not depreciated nor is the redemption of any investment in mine ground or prior development permitted. By an agreement between the Tax Commission and operators, future levies were to be made only on 60 per cent of the net profits or the same percentage which is applied in assessment of other property. During 1915, this percentage has been raised to 70 per cent by the Tax Commission.

New Mexico has placed its mine taxation in the hands of a

State Tax Commission. The owners or lessees of mines are ordered to furnish annual statements supplying the Commission with the necessary data to determine the net proceeds. Surface improvements are taxed as other like property. Unproductive patented mining claims are assessed and taxed on a reasonable valuation as undeveloped mineral lands in addition to their surface value whatever it may be. The law for the taxation of mining property was drawn up by the attorney of one of the large mining companies in New Mexico and is believed to be satisfactory to the mining interests.

Utah has a system for assessing on the basis of the net proceeds but the State Board of Equalization does not allow deduction for interest, taxes, insurance, legal expenses, etc., so the mines are really being taxed on more than their net proceeds.

Mexico prior to the Carranza regime placed an annual surface tax of 6 pesos per pertenencia (100 meters square) on all claims of less than 25 pertenencias in size. On each pertenencia in excess of 25 the annual charge is only 3 pesos. A state or federal tax on output could be levied not to exceed $1\frac{1}{2}$ per cent of the assay value. The tax on reduction plants was 0.6 per cent of the valuation but other improvements were not taxed. When ore was exported unrefined it was taxed $3\frac{1}{2}$ per cent of the assay value but when it was refined in Mexico the export tax was only $2\frac{1}{2}$ per cent of the assay value.

Since the de facto government has been controlled by Carranza, an annual tax has been established at the rate of \$6 per hectare on a property of not over 10 hectares, \$7.50 per hectare on the acreage in excess of 10 hectares and below 20 hectares; \$9 per hectare on that from 20 to 50 hectares; and \$12 per hectare on all above 50 hectares.

Recent developments in Mexico are such that it cannot be stated with definiteness what is the method of mine taxation.

German South-West Africa. — British consular reports state that the mines of German South-West Africa were taxed by an empirical method. The tax amounted to the difference between 66 per cent of the gross value of the output and 70 per cent

of the cost of operating. This amounts to no tax when the profits are 6 per cent or less, to 10 per cent tax on gross production when the profits are 20 per cent of the gross, to 31 per cent when the profits are 50 per cent and so on up to the limit of 66 per cent of the gross production when the operating expenses approach zero.

Propriety of Taxing Mining Property. — The question of the propriety of taxing mining properties is an important one. In states such as Michigan and Minnesota where mines make up a large proportion of the industry of the state it is necessary that mining properties bear a large share in the maintenance of the government which exists largely for their protection. In the states just mentioned a closer approximation of the value of the ore underground is possible than in states where the mining interests are largely concerned with precious metals. This should not be construed to mean that the methods of assessment now in vogue in the former should be considered the proper ones.

There is no doubt that taxation of mines in any form must retard to some extent the development of the mining industry which is of such economic value to the country. It also must be granted that in certain communities the mining interests are so large that it is only proper that they pay their share of the governmental burden. Injustice can be avoided only by the exercise of caution when mineral lands are to be taxed. That taxation may run riot has been shown in Minnesota in the case of mines located within the boundaries of municipalities.

In Buhl, Minnesota, the 1913 municipal tax levy was \$125,000 or \$125 per inhabitant. The town had 172 voters and had over 100 persons on the pay roll during the year. Only $1\frac{1}{3}$ per cent of the assessment was paid by the inhabitants and $98\frac{2}{3}$ per cent by the Mining Companies. In Keewatin the levy amounted to \$374.10 per inhabitant. The township of Stutz in 1914 spent more than \$900 for every voter. By court order some of these levies have been modified but they show what extremes are possible. It is understood that the per

capita expenditures of municipalities have now been limited to \$25 per inhabitant by the State Legislature.

The general principles of mine taxation are not comparable with those controlling the taxation of unimproved real estate which is held idle waiting for the unearned increment. Mining properties may be allowed to remain idle because there is no present market for their product at a price that will permit a profit but they cannot be held for any length of time as an investment waiting for a chance to make a large turn because under ordinary circumstances no important permanent increase in value is to be expected. When large increases in the price of minerals take place, new substances at more reasonable prices are apt to be supplied to do the same work. Water power takes the place of mineral fuels. Oil takes the place of coal in certain localities where coal is high and so on.

Improvements in the methods of production as well as increased production have resulted in decreased prices of the output rather than in increased returns. Furthermore as compared with farming and other manufacturing industries mining is not as attractive a field as it formerly was and in another decade will probably not be as attractive as it is at the present time.

Mr. T. A. Rickard, in a recent paper before the International Engineering Congress, points out that a mine is not an investment but is a speculation. He writes:

"The act of mining cannot be applied on scientific principles until two basic ideas are fully comprehended: (1) A mine is a wasting asset and (2) mining is a speculative business. To treat a mine as an investment, and to appraise it on that basis is to ignore the cumulative facts of to-day and of other days. Mining is a speculation that can be wise or foolish according as a man recognizes the inherent risk and takes his chances accordingly."

Taxation of certain deposits such as coal-beds, oil fields, etc., on the basis of the present value of future production tends to increase the current output and might easily lead to the destructive working of such deposits if vigorously enforced, e.g.,

the abandonment of thin beds of coal or the failure to take care of the water in the oil well. The same is true but probably not to the same extent of other minerals.

The unfair taxation of precious metals will result in the hampering of the organization and the financing of promising prospects or the shut down of properties that are running very close to the margin of no profits.

The following quotation from Mr. T. C. Bonney * emphasizes the difference between real estate and mineral lands:

"The mineral store of each district and of the whole earth is practically limited in quantity, be it gold, or any other metal, be it coal or any fuel The formation of a fresh supply is a process so slow that, for all practical purposes, it may be excluded from consideration. . . . Hence the store, sooner or later, must be exhausted, now in this country, now in that. In agriculture, provided manures can be obtained, the land seems never to lose its productive power. The mine or quarry, once worked out, has played its part for good in the economy of the earth."

Conclusion Relating to Taxation of Mines. — In conclusion, it may be stated that the fairest way to tax mining properties of any class would be on their annual profits, modified by the regulating body in the case of wasteful or careless operations, capitalized at a rate of interest commensurate with the average mining risk for that class of property. By repeating these valuations at frequent intervals no hardship will be thrown on the property whose profits are on the downward grade. When it is found impossible to estimate the life of the property involved with definiteness as is usually the case in the mining of precious metals, empirical or semi-empirical methods of valuation must be adopted. Such method as establishing a taxable value equal to a percentage of or a multiple of one year's net or gross proceeds would probably be preferred to the use of an estimated market value by the mining companies. The percentage or multiple used should be fixed so as to distribute the taxes in the proper proportion between mine owners and other taxpayers. The effort to tax non-productive properties cannot be endorsed

^{*} T. G. Bonney, D.Sc., F.R.S., Dictionary of Political Economy, 1910, p. 768.

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except in cases where it can be clearly demonstrated that deposits economically available do exist.

Methods of Valuing Oil Properties

In the Appalachian oil fields, oil properties have been valued on the basis of the daily production of the wells measured after the output has reached a settled stage. The valuation is so fixed as to permit the purchaser to recover his capital in 5 to 10 years and to obtain a reasonable return on his investment. It is stated that the average sale price of producing lands varied from \$800 to \$1000 per barrel-day a few years ago.

In *Illinois*, many properties changed hands during the early stages of the operations at a rate that would insure the return or amortization of capital within 18 months. This was before the life of the wells had been given a practical test.

Two large valuations of oil properties have been made in *California* since 1900 and the methods that were adopted are of particular interest to the valuation engineer.

In 1910 the Kern River oil-field was valued because of the intended consolidation of several separate holdings within the field. Because of the object of the investigation only values bearing comparison with one another were required. The values of all the properties involved were expressed in terms of the value of a certain selected tract of oil land located in the approximate center of the field. Concentric circles drawn about the center of the field were used to express the relative value of individual tracts as affected by their distance from the center of greatest productivity. A measurement of the productivity of all the producing wells was made covering a month's time, thus determining the production at that time and experiments were conducted on samples of the oil-bearing strata to determine the amount of oil that could be recovered from a known volume. The results were used as a measure of the extractable oil which could be pumped from the oil strata, the volume and area of which could be estimated. A safety factor was introduced by assuming that only one-half of the oil measures would be productive. Subsequent experience indicates that this safety factor was not sufficient.

A method of oil land valuation, which has been applied more recently, is that adopted by the engineers in valuing the properties which market their product through the Independent Oil Producers Agency, which valuation had in view the merging of a number of companies in one large organization or corporation. The method used was based on the determination of a production curve for each property. A study of the history of producing wells has given basis for the assumption that the wells in a field constantly decrease in production at a rate the changes of which can be plotted in curves and, using these curves as a basis, old wells, new wells and wells not yet drilled can all be taken into account in estimating the future production of the oil lands. The total operating expenses have been estimated from existing records and the prospective future net receipts have been discounted in obtaining the present value of the prop-Amortization of all capital except the salvage of surerties face improvements has been assumed to take place in ten "History" is also prerequisite to this method and attention should be called to the fact that, because of the inaccessibility of the reservoirs of oil, the more extensive the record of previous operations, the more satisfactory will be the valuation no matter what method may be adopted.

PRODUCTION AND PRICE OF COPPER, SILVER, LEAD AND ZINC IN THE UNITED STATES

Plates and Tables

Accompanying Chapter on Valuation of Mines and Oil Properties

Figures 4 to 7, and Tables 15 to 18 which directly follow this page deal only with four of the principal metals, copper, silver, lead and zinc. The statistics given are based on the best available data and the figures are believed to be sufficiently accurate to justify their use by valuating engineers who have in hand problems in which these metals have a part.

The figures submitted for the United States prices and production are those of the United States Geological Survey.

Other available sources were found to be less complete and no discrepancies sufficiently large to question the reliability of these figures could be located. The world production of these metals has been taken from the annual reports of the U. S. Geological Survey supplemented by the annual statements of the Director of the U. S. Mint, Henry R. Merton & Co. (Limited) of London, and the Metallgesellschaft and Metallurgische Gesellschaft, A.-G., Frankfort-am-Main, Germany.

TABLE 15 COPPER IN THE UNITED STATE
ANNUAL PRODUCTION
(Based on smelter returns and prices at New York City)
(1880-1914)

Year.	Produ	iction	Per cent of world	Average annual	Price received
Pounds	Total value	production	price at N. Y	for Lake copper.	
1880	60,480,000	\$ 11,491,000	17 4	\$0 190	\$0 195
1881	71,680,000	12,176,000	19 5	0 170	0 180
1882	91,646,000	16,038,000	22 7	0 175	0 180
1883	117,152,000	18,065,000	26 I	0 154	0 154
1884	145,222,000	17,790,000	30 0	0 122	0 131
1885	170,963,000	18,293,000	34 I	0 107	0 110
1886	161,235,000	16,528,000	33 4	0 102	0 108
1887	185,227,000	21,116,000	36 9	0 114	0 120
1888	231,271,000	33,834,000	39 7	0 146	0 152
1889	231,246,000	26,908,000	40 I	0 116	0 120
1890	265,115,000	30,849,000	44 0	0 116	0 150
1891	295,812,000	38,455,000	47 2	0 130	0 125
1,892	352,972,000	37,977,000	50 2	0 107	0 115
1893	339,786,000	32,055,000	50 I	0 094	0 105
1894	364,867,000	33,141,000	51 6	0 091	
1895	385,913,000	38,012,000	52 3	0 098	
1896	460,061,000	49,457,000	54 4	0 107	
1897	494,078,000	54,080,000	57 2	0 109	
1898	526,513,000	61,865,000	54 7	0 117	
1899	568,667,000	101,223,000	54 8	0 178	
1900	606,117,000	98,494,000	55 6	0 162	0 166
1901	602,073,000	87,301,000	51 8	0 145	0 150
1902	659,509,000	76,569,000	53 5	0 116	0 119
1903	698,045,000	91,506,000	53 2	0 131	0.133
1904	812,537,000	105,630,000	55 8	0 130	0.132
1905 1906	888,784,000	137,762,000	57 5	0 155	0 156
1900	868,996,000	177,596,000	57.5	0 193	0.191
1907	942,571,000	173,799,000	54 7	0 200	0 184
1900	1,092,952,000	124,419,000 142,084,000	58 4	0 132	0 134
1910	1,080,160,000	137,180,000	56 8	0.130 0.127	0.133
1911	1,097,233,000	137,154,000	56 0	0.125	0.130
1912	1,243,269,000	205,139,000	55 0	0.123	0.128
1913	1,224,484,000	189,795,000	55.7	0.155	0.154
1914	1,150,137,000	152,968,000	33.7	0.133	0.134
J '		. 5-,5,		2.233	0.130

TABLE 15. COPPER IN THE UNITED STATES (Continued)

NORMAL PRODUCTION AND PRICE

(Normals for ten-year periods)

Ten-year	Produ	ction	Normal production,	Normal price	
periods (inclusive)	Pounds	Value	pounds	(ten-year aver age).	
1880–1889	1,466,122,000	\$ 192,239,000	146,612,200	\$0 131	
1881-1890	1,670,757,000	211,597,000	167,075,700	0 127	
1882-1891	1,894,889,000	237,876,000	189,488,900	0 126	
1883-1892	2,156,215,000	259,815,000	215,621,500	0 121	
1884-1893	2,378,849,000	273,805,000	237,884,900	0 115	
1885-1894	2,598,494,000	289,156,000	259,849,400	0 111	
1886-1895	2,813,444,000	308,875,000	281,344,400	0 110	
1887-1896	3,112,270,000	341,804,000	311,227,000	0 110	
1888-1897	3,421,121,000	374,768,000	342,112,100	0 110	
1889-1898	3,716,363,000	402,799,000	371,636,300	0 108	
1890-1899	4,053,784,000	477,114,000	405,378,400	0 118	
1891-1900	4,394,786,000	544,759,000	439,478,600	0 124	
1892-1901	4,701,047,000	593,605,000	470,104,700	0 126	
1893-1902	5,007,584,000	632,197,000	500,758,400	0 126	
1894-1903	5,365,843,000	691,648,000	536,584,300	0 129	
1895-1904	5,813,513,000	764,137,000	581,351,300	0 131	
1896-1905	6,316,384,000	863,887,000	631,638,400	0 137	
1897-1906	6,774,129,000	992,026,000	677,412,900	0 146	
1898-1907	7,149,047,000	1,111,745,000	714,904,700	0.155	
1899-1908	7,565,105,000	1,174,299,000	756,510,500	0 155	
1900-1909	8,089,390,000	1,215,160,000	808,939,000	0 150	
1901-1910	8,563,433,000	1,253,846,000	856,343,300	0 146	
1902-1911	9,058,593,000	1,303,699,000	905,859,300	0 144	
1903-1912	9,642,353,000	1,432,269,000	964,235,300	0.149	
1904-1913	10,168,792,000	1,530,558,000	1,016,879,200	0.151	
1905-1914	10,506,392,000	1,577,896,000	1,050,639,200	0 149	

TABLE 16. SILVER IN THE UNITED STATES

ANNUAL PRODUCTION

Year	Annual pr	roduction	Per cent world	Average commercial
1 Gal	Troy ounces	Total value.	production	value per ounce
1880 1881 1882 1883 1884 1885 1886 1887 1888 1890 1891 1892 1893 1894 1895 1896 1899 1900 1901 1902 1903 1904 1905 1906 1907	30,318,700 32,257,800 36,196,900 35,732,800 37,743,800 39,999,400 41,721,600 45,792,700 50,094,500 54,516,300 58,330,000 63,500,000 60,000,000 49,500,000 55,727,000 58,834,800 53,860,000 54,764,500 57,647,000 55,214,000 55,500,000 55,216,000 55,216,000 55,216,000 55,216,000 55,216,000 55,216,000 56,517,900 56,517,900 56,517,900 56,517,900	\$34,717,000 \$37,657,500 41,105,900 39,618,400 41,921,300 42,503,500 39,482,400 40,887,200 45,838,400 57,630,000 55,662,500 46,800,000 31,422,100 36,445,500 32,316,000 32,318,400 32,858,700 33,128,400 29,415,000 29,422,000 33,456,000 34,222,000 33,456,000 34,222,000 33,456,000 34,222,000 38,256,400 37,299,700	40 6 40 8 41 9 40 1 46 3 43 6 42 5 42 9 42 2 41 7 43 2 42 5 41 5 36 4 30 1 33 2 31 5 33 6 32 2 31 9 34 1 35 1 36 4 37 5 38 6 39 6 31 6 32 6 33 6 31 7 32 8 33 9 34 1 35 1 36 4 37 7	value per ounce \$1 15 1 13 1 14 1 11 1 17 1 07 0 99 0 98 0 94 1 05 0 99 0 87 0 78 0 63 0 65 0 68 0 60 0 59 0 60 0 59 0 60 0 53 0 60 0 58 0 60 0 58 0 60 0 58 0 60 0 58 0 60 0 58 0 60 0 60 0 58 0 60 0 60 0 60 0 60 0 60 0 60
1908 1909 1910 1911 1912	52,440,800 54,721,500 57,137,900 60,399,400 63,766,800 66,801,500	28,050,600 28,455,200 30,854,500 32,615,700 39,197,500 40,348,100	25 8 25 8 25 8 26 7 28 4	0 53 0 52 0 54 0 53 0 615 0 604
1914	72,455,100	40,067,700	·	o 553

TABLE 16. SILVER IN THE UNITED STATES (Continued)

NORMAL PRODUCTION AND PRICE

(Normals Based on ten-year periods)

Ten-year periods	Produ	iction.	Normal produc-	Normal price (ten-year aver-
(inclusive)	Pounds.	Value	tion, pounds.	age).
1880-1889 1881-1890 1882-1891 1883-1892 1884-1893 1885-1894 1886-1895 1887-1896 1888-1897 1889-1899 1891-1900 1892-1901 1893-1902 1894-1903 1895-1904 1896-1905 1898-1907 1899-1908 1900-1909 1901-1910 1902-1911 1903-1912	389,462,200 413,659,800 439,732,000 467,035,100 491,302,300 503,058,500 518,876,100 550,155,300 558,800,600 563,470,600 563,470,600 563,485,300 557,968,100 557,968,100 558,342,700 558,342,700 558,680,500 556,683,300 556,640,300 556,131,200 561,316,600 569,583,400	\$407,776,700 430,301,800 450,274,300 464,830,900 472,012,500 461,513,300 455,455,300 447,056,300 436,129,600 420,149,900 400,648,900 376,147,300 349,899,800 332,421,800 334,455,700 332,232,200 335,817,700 331,749,900 327,346,400 322,459,800 331,729,600	38,946,220 41,365,980 43,973,200 46,703,510 49,130,230 50,305,850 51,887,610 53,801,690 55,015,530 56,347,060 56,660,130 56,347,060 56,3660,130 55,548,530 55,796,810 55,868,050 55,664,030 55,664,030 55,664,030 55,664,030 55,6131,660 56,958,340	\$1 045 1 045 1 025 0 995 0 96 0 915 0 88 0 78 0 75 0 67 0 63 0 605 0 605 0 595 0 605 0 595 0 595 0 595 0 595 0 595 0 595 0 595 0 595 0 605 0 595 0 595
1904–1913 1905–1914	582,084,900 596,857,200	342,755,700 349,367,400	58,208,490 59,685,700	o 59 o 585

TABLE 17. LEAD IN THE UNITED STATES

ANNUAL PRODUCTION

(Based on smelter returns and prices at New York City)

Year.	Annual pr	oduction.	Per cent world	Average annual
Year.	Pounds.	Total value.	production.	price at N Y.
1880	195,650,000	\$ 9,782,500	21 7	\$0 050
1881	234,170,000	11,240,160	24 4	0 048
1882	265,780,000	12,624,550	26 2	0 049
1883	287,914,000	12,322,719	29 3	0 043
1884	279,794,000	10,537,042	29 I	0 037
1885	258,824,000	10,469,431	26 4	0 039
1886	261,258,000	12,200,749	24 3	0 046
1887	291,400,000	13,113,000	24 9	0 045
1888	302,838,000	13,399,256	24 8	0 044
1889	312,794,000	13,794,235	26 0	0 039
1890	287,260,000	12,668,166	21 6	0 045
1801	357,108,000	15,534,198	26 4	0 043
1892	347,308,000	13,892,320	22.7	0 040
1893	327,964,000	11,839,590	23 3	0 037
1894	318,662,000	9,942,254	23 4	0 033
1895	340,000,000	11,220,000	23 8	0 032
1896	376,000,000	10,528,000	²⁵ 3 ₂₆ 8	0 030
1897	424,000,000	14,885,728	26 8	0 036
1898	444,000,000	16,650,000	25 9	0 038
1899	421,000,000	18,945,000	23 3	0 045
1900	541,648,000	23,561,688	29 0	0 044
1901	541,400,000	23,280,200	27 6	0 043
1902	540,000,000	22,140,000	27 3	0 041
1903	564,000,000	23,520,000	28 0	0 042
1904	614,000,000	26,402,000	29 4	0 043
1905	604,000,000	28,690,000	29 0	0.047
1906	700,306,000	39,917,442	33 7	0 057
1907	750,198,000	39,760,494	35 0	0 053
1908	621,524,000	26,104,008	27.0	0 042
1909	726,638,000	31,245,434	32 6	0 043
1910	778,422,000	34,250,568	32 2	0 044
1911	811,726,000	36,527,670	33 I	0 045
1912	830,790,000	37,385,550	32 4	0 045
1913	872,860,000	38,405,840	34 4	0 044
1914	1,025,588,000	39,997,932		0.039

TABLE 17. LEAD IN THE UNITED STATES (Continued)

NORMAL PRODUCTION AND PRICE

(Normals for ten-year periods)

Ten-year periods	Production		Normal produc-	Normal price	
(inclusive).	Pounds	Value	tion, pounds	(ten-year average).	
1880-1889 1881-1890 1882-1891 1883-1892 1884-1893 1885-1894 1886-1895 1887-1896 1889-1899 1891-1900 1892-1901 1893-1904 1896-1905 1896-1905	2,690,602,000 2,782,212,000 2,905,150,000 2,986,678,000 3,026,728,000 3,065,596,000 3,146,772,000 3,261,514,000 3,394,114,000 3,535,276,000 3,43,302,000 4,081,982,000 4,274,674,000 4,510,710,000 4,506,048,000 5,060,048,000 5,394,354,000	\$119,483,500 122,369,200 126,663,200 127,931,000 127,447,990 126,853,200 127,603,800 127,703,800 127,703,800 130,954,500 136,105,300 146,998,800 154,744,800 162,992,500 174,572,900 191,132,600 208,602,600 237,992,000 262,866,800	269,060,200 278,221,200 290,515,000 298,667,800 302,672,800 306,559,600 314,677,200 326,151,400 339,411,400 353,527,600 364,330,200 480,98 200 427,467,400 451,071,000 480,604,800 506,004,800 539,435,400	\$0 044 0 044 0 044 0 043 0 042 0 041 0 039 0 038 0 037 0 038 0 038 0 038 0 038 0 039 0 040 0 041 0 044	
1898–1907 1899–1908 1900–1909 1901–1910	5,720,552,000 5,898,076,000 6,203,714,000 6,440,488,000	202,800,800 272,320,800 284,621,200 295,310,100	572,055,200 589,807,600 620,371,400 644,048,800	0.046 0.046 0.046	
1902-1911 1903-1912 1904-1913 1905-1914	6,710,814,000 7,001,604,000 7,310,464,000 7,722,052,000	308,557,600 323,803,200 338,689,000 352,284,938	671,081,400 700,160,400 731,046,400 772,205,200	o o46 o o46 o o46 o o46	

TABLE 18. ZINC IN THE UNITED STATES

Annual Production

(Based on smelter returns and prices at St Louis)

	Annual pr	oduction.	Per cent world	Average annual
Year -	Pounds.	Total value	production.	price at N.Y.
1880	46,478,000	\$ 2,277,432	9 1	\$0 055
1881	53,600,000	2,680,000	9 3	0 052
1882	67,530,000	3,646,620	III	0 053
1883	73,744,000	3,311,106	II 2	0 045
1884	77,088,000	3,422,707	118	0 044
1885	81,376,000	3,539,856	12 4	0 043
1886	85,282,000	3,752,408	12 7	0 044
1887	100,680,000	4,782,300	14 I	0 046
1888	111,806,000	5,500,855	15 3	0 049
1889	117,720,000	5,791,824	15 7	0 05
1890	127,366,000	6,266,407	16 4	0 055
1891	161,746,000	8,033,700	199	0 05
1892	174,520,000	8,027,920	20 8	0 046
1893	157,664,000	6,306,560	18 7	0 04
1894	150,656,000	5,288,026	178	0 035
1895	179,372,000	6,278,020	19 4	0 036
1896	162,998,000	6,519,920	17 4	0 039
1897	199,960,000	8,498,300	20 4	0 041
1898	230,798,000	10,385,910	22 3	0 046
1899	258,102,000	14,840,865	23 9	0 058
1900	247,772,000	10,654,196	23 5	0 044
1901	281,644,000	11,265,760	25 0	0 041
1902	313,854,000	14,625,596	260	0 048
1903	318,438,000	16,717,995	25 2	0 054
1904	373,404,000	18,670,200	26 9	0 051
1905	407,698,000	24,054,182	28 0	0 059
1906	399,388,000	24,362,668	25 7	0 061
1907	447,490,000	26,401,910	27 5	0 059
1908	381,498,000	17,930,406	23 9	0 047
1909	460,450,000	24,864,300	27 0	0 054
1910	504,958,000	27,267,732	27 6	0 054
1911	543,242,000	30,964,794	27 6	0 057
1912	647,814,000	44,699,166	30.3	0 069
1913	674,504,000	37,772,224	30.8	0 056.
1914	686,836,000	35,028,636		0 051

TABLE 18 ZINC IN THE UNITED STATES (Continued)

NORMAL PRODUCTION AND PRICE

(Normals for ten-year period)

Ten-year periods	Produ	ection	NT. 1	Normal price
(inclusive)	Pounds	Value	Normal production	(ten-year aver- age)
1880—1889 1881—1890 1882—1891 1883—1892 1884—1893 1885—1894 1886—1895 1887—1896	815,304,000 896,192,000 1,004,338,000 1,111,328,000 1,1268,816,000 1,268,816,000 1,366,812,000 1,444,528,000 1,543,808,000	\$ 38,705,100 42,694,100 48,047,800 52,428,100 55,424,600 57,289,900 60,028,000 62,795,500 66,511,500	81,530,400 89,619,200 100,433,800 111,132,800 119,524,800 126,881,600 136,681,200 144,452,800 154,380,800	\$0.047 0.048 0.048 0.047 0.046 0.045 0.044 0.043
1889-1898 1890-1899 1891-1900 1892-1901 1893-1902 1894-1903 1895-1904	1,662,800,000 1,803,182,000 1,923,588,000 2,043,486,000 2,182,820,000 2,343,594,000 2,566,342,000	71,396,500 80,445,600 84,833,400 88,065,500 94,663,200 105,084,600 118,456,800	166,280,000 180,318,200 192,358,800 204,348,600 218,282,000 234,359,400 256,634,200	0 043 0 045 0 044 0 043 0 043 0 045 0 046
1896-1905 1897-1906 1898-1907 1899-1908 1900-1909 1901-1910 1902-1911 1903-1912 1904-1913	2,794,668,000 3,031,058,000 3,278,588,000 3,429,288,000 3,631,636,000 3,888,822,000 4,150,420,000 4,484,380,000 4,840,446,000	136,233,000 154,075,800 171,979,400 179,523,900 189,547,300 206,160,800 225,859,800 255,933,400 276,987,600	279,466,800 303,105,800 327,858,800 342,928,800 363,163,600 388,882,200 415,042,000 448,438,000 484,044,600	0 049 0.051 0 0525 0.052 0 052 0.053 0 054 0 057
1905-1914	5,153,878,000	293,346,000	513,387,800	0.057

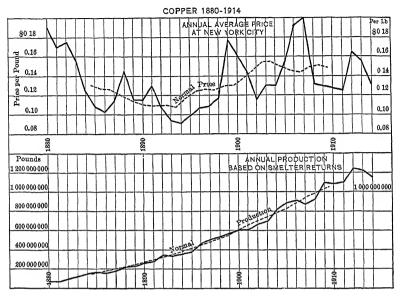


Fig. 4.

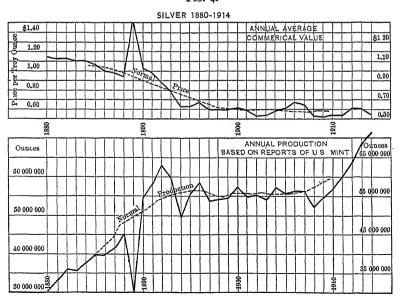


Fig 5



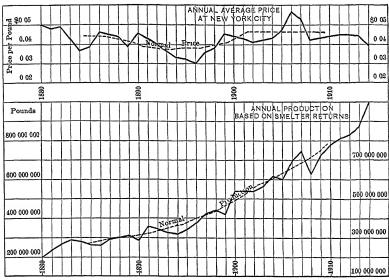


Fig 6.

ZINC 1880-1914

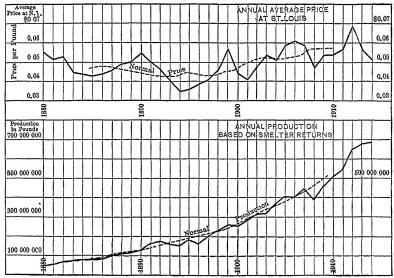


Fig. 7.

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CHAPTER XIV

TABLES

There have been prepared by the author and are here presented a set of tables which are intended to meet the requirements of the valuation engineer and the economist.

Where values are not noted for every year from 1 to 100, as is the case in some of the tables, methods are indicated whereby other tables can be called to aid.

It is believed that all values have been noted with sufficient accuracy to meet every ordinary requirement.

The following tables with explanations preceding each will be found in this chapter:

Table 19. The probable useful life of various articles

Table 20. Expectancy and remaining value.

Table 21. The amount of \$1 at compound interest.

Table 22. The present value of \$1 due at a future date.

Table 23. The amount of an annuity of \$1.

Table 24. The annuity which will amount to \$1 in a given time.

Table 25. The present value of an annuity of \$1.

Table 26. The annuity which \$1 will purchase.

Table 27. Amortization and depreciation.

EXPLANATION OF TABLE 19

The Probable Useful Life of Various Articles of the Character Most Frequently Encountered in Public Utility Valuations

The probable life of any article does not depend merely upon its wearing qualities when in service, but upon a number of other factors as elsewhere explained. Its useful life under averTABLES 271

age conditions is determinable with a greater or less degree of approximation from a study of all available experience.

Without entering upon a full discussion of the basic available data, there is here presented a table in which the probable useful term of life for many articles, appliances, machinery and other things likely to be connected with industrial establishments or with public utilities is noted, due credit being given, so far as this is practicable, for the source of information in each case.

Under ordinary conditions and particularly when used in connection with the Unlimited Life Method of procedure, the life terms here presented may be accepted as covering failure from any cause such as physical deterioration, destruction by accident, inadequacy and obsolescence.

It is to be noted that in some of the sources from which information presented in this table was drawn, the depreciation was expressed in percentage by the Straight Line Method. This percentage has been translated into years of probable life.

Summary of Authorities Quoted in Table 19 and Reference to Publication

Alvord.	John W. Alvord in Proceedings American Water Works Association, 1903.
Arbitrators.	Board of Arbitration in street lighting controversy at Atlanta, Georgia, 1899.
Arnold.	Bion J Arnold. Coney Island and Brooklyn figures adopted by Public Service Commission of New York. Four Chicago appraisals quoted by H. A. Foster.
Bryan.	Wm. H. Bryan. "The Appraisal and Depreciation of Water Works." Journal of Association of Engineer-
	ing Societies, Dec., 1907.
Burdick.	Chas. B. Burdick. Appraisal of Mt. Vernon Water Works, Illinois, 1906.
Cal. R. R. Com.	California Railroad Commission — Reports 1913-4.
	Calistoga Electric Co. vs. Napa Valley Electric Co, May,
	Cuyamaca Water Co. Case, March, 1913.
	San Jose Water Co. Case, May, 1914.
	Stockton Terminal & Eastern R. R, April, 1913.
Chi. Tele. Com.	Chicago Telephone Commission. Published 1908.
Chi. Trac. Com.	Chicago Traction Commission. Quoted by G. W. Cravens,
	Electrical Review, April 23, 1910.

Chi. U. T. Co.	Chicago Union Traction Co. Quoted in Report Wisconsin R R Commission, Vol. 10, p. 220.
Connette.	E. G. Connette. 3rd Ave Case figures adopted by Public Service Commission of New York
Cooley.	Mortimer E. Cooley. Milwaukee 3¢ Fare Case. Report Wisconsin R. R. Com, Vol. 10, p 226.
Dodge.	B. L. Dodge. Mt. Vernon Water Works Appraisal, Illinois,
Floy.	Trans. American Institute of Electrical Engineers, June, 1911. Third Avenue Case figures adopted by Public Service Commission of New York.
Foster.	H. A "Foster "Engineering Valuation of Public Utilities and Factories," Van Nostrand, 1912.
Gillette.	H. P. Gillette. Appraisal of Great Northern & Northern Pacific R R.'s in Washington Direction of Washington Railroad Commission, 1906.
Hammond.	Robt. Hammond Journal of Institute of Electrical Engineers, England, April 25, 1907.
Kiersted.	W Kiersted Galena Appraisal, Kansas, 1905
Metcalf.	Leonard Metcalf. Transactions American Society of Civil
Trictcuii:	Engineers, 1909
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P G. & E. Co.	Pacific Gas & Electric Company Figures used in rate hearings before Master in Chancery, San Francisco,
Parsons.	 1913-4. T. C. Parsons. Discussion of Mr Hammond's paper, Journal Institute Electric Engineers, England, April 25, 1907
Preece.	Sir Wm. Preece. Report of Bristol Corporation, England,
Rosecrans.	W H Rosecrans. Galena Appraisals, Kansas, 1905.
S J. L. & P. Corp.	San Joaquin Light & Power Corporation. Rate hearing before California Railroad Commission, 1915.
Starrett.	Milton G. Starrett. Milwaukee 3¢ Fare Case. Report Wisconsin R R. Commission, Vol. 10, p. 226.
St. Louis P S C.	St Louis Public Service Commission. Case of Union Elec- tric Light & Power Co
Stone & Webster.	Figures for Chicago Union Traction Co. Case. Report Wisconsin Railroad Commission, Vol. 10, p. 220.
Telephone "Data."	Compiled from practice A T & T. Co., Wisconsin R. R. Commission and others Quoted by H. A. Foster.
Trac. Val. Com.	Traction Valuation Commission. Case of the Chicago Consolidated Traction Co.
Wilgus	W. G. Wilgus. Appraisal of Lehigh Valley Railroad, January, 1914.
Williams.	Benezette Williams. Galena Appraisal, Kansas, 1905.

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Wis. R. R. Com.

Wisconsin Railroad Commission. Quoted by Floy and by Foster and in published reports.

Madison Electric Railway & Light Co. Case, March 8, 1910.

Milwaukee Electric Railway & Lighting Co. Case, Nov. 25, 1913.

Milwaukee 3¢ Fare Case. Aug. 23, 1912. Wisconsin Telephone Co. Case, Aug. 3, 1909.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES

Description.	Expect- ancy	Residual value.	Authority	Remarks.
A ccumulators	Years 15 15		Hammond Parsons	Jour Inst Elec Engrs. Jour Inst Elec Engrs
	15	10%	Preece	Report Bristol Corp , 1906.
Air brakes	20		Wis R R Com	Quoted by Henry Floy
Air compressors	20	5%	PG & E Co	S F rate hearing, 1913-14.
Arc lamps	20-25 63		Trac Val Com Arbitrators	Chi Con Trac Co Case Ata St Lgt Contrysy, 1899.
Art iumps	12	5%	Preece	Report Bristol Corp , 1906.
	123	370	St Louis P S C	Union E L & P Co Case.
	15		Wis R R. Com	Quoted by Henry Floy
	15		P. G. & E. Co	S. F rate hearings, 1913-14.
Arc lamp posts	40	5%	Preece	Report Bristol Corp , 1906
Ammonia concentrators and	,			
tanks	15		Wis R R Com	Quoted by H A. Foster.
Belting .	8		PG & E. Co	S F. rate hearings, 1913-14.
	20		Wis R R Com	Quoted by Henry Floy.
	10-20		Wis. R R Com	Mil 3¢ Fare Case
with shafting and ropes	15		Chi U. T. Co	Chi Con Trac Co Case.
with shafting and ropes	20		Stone & Webs	Chi Con Trac Co Case.
(leather)	20-25		Foster	"Eng Valuation," etc.
Benches (gas plant)	25		Wis R R Com	Quoted by H. A. Foster. Quoted by H. A. Foster.
Blowers (gas plant)	15		P G & E Co	S F rate hearings, 1913-14.
Boilers	20	1	Arnold	Coney Isl. & Brooklyn Case.
Doners	15		Chi. U T. Co	Chi Union Trac Co Case.
	15		Alvord	Aver 32-Proc. A W W A.
	15		St Louis P S C	Union Elec L. & P Co Case
	12-16		Metcalf	Trans Am S C E , 1909
	16 7		Dodge	Mt Vernon W W Ill , 1906.
	15-20		Wis R R Com	Milwaukee 3¢ Fare Case.
	20	1	Stone & Webs	Chi Union Trac Co Case
	20	000	P G. & E Co.	S F rate hearings, 1913-14.
	20		Hammond	Jour Inst. Elec Engrs , 1907.
	10-28		Chi Trac Com	Chicago Trac Com , 1908
	25		Williams Burdick	Galena Appraisal, Kan, 1905. Mt Vernon W. W Ill, 1906.
	25 28 57		Arnold	4 Chicago Appraisals.
	25-30	1	Trac Val. Com.	Chicago Con Trac Co Case.
	30		Kiersted	Galena Appraisal, Kan, 1905.
	25-40		Rosecrans	Galena Appraisal, Kan., 1905.
water-tube	20		Floy	3rd Ave Case, N. Y.
water-tube	20		Wis. R. R. Com	Quoted by Henry Floy.
water-tube .	25	5%	Preece	Report Bristol Corp , 1906.
water-tube .	20-30		Foster	"Eng Valuation," etc.
fire-tube	10		Arbitrators	Ata St Lgt Contrvsy, 1899
fire-tube .	10-15		Foster	"Eng Valuation," etc.
fire-tube electric light	16-30		Wis R R Com.	1
fire-tube water works	20-25	~	Wis R R. Com	Quoted by Henry Floy.
Lancashire	22	3%	Preece	Report Bristol Corp., 1906.
boilers and accessories.	20		SJL & P. Cor.	Cal R. R. C. rate hearings.

TABLE 19 THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description	Expect- ancy.	Residual value	Authority.	Remarks.
	Years			
Boilers (continued)				
steel breechings	10		Arnold	4 Chicago Appraisals
breechings and connections	10-30	1	Trac. Val Com	Chi Con. Trac Co Case
Bonds .	20		Trac Val Com	Chi Con Trac Co Case
20125	20		Wis. R R Com	Quoted by Henry Floy
Bridges				, , , , , , ,
Howe truss R R	10		Gillette	Gt Nor & Nor Pac apprais.
trestles and wooden bridges	10		Cal R R Com	Stockton Term & East Case
Buildings	25-50		Wis R R Com	Quoted by Henry Flov
Danamgo	50		St Louis P S C	Union Elec L & P Case
	50		Connette	3rd Ave Case, N. Y City.
railroad station	10		Cal R R Com	Stockton Term & East R R.
Tainford Station	1			Case.
railroad	331		Gillette	Gt Nor & N Pac Appraisal.
street railway	50	1	Cooley	Milwaukee 3¢ Fare Case
street railway	50		Starrett	Milwaukee 3¢ Fare Case.
street railway	50		Stone & Webs.	Chi. Union Trac Co Case.
street railway	50		Chi U.T Co	Chi. Union Trac Co Case.
street railway	35-75	1	Wis. R R Com	Milwaukee 3¢ Fare Case
street railway power plants	20		Chi U T Co	Chi Union Trac. Co. Case.
street railway power plants	25		Cooley	Milwaukee 3¢ Fare Case.
street railway power plants	50		Stone & Webs	Chi Union Trac Co Case.
street railway power plants	60		Wis R. R Com	Milwaukee 3¢ Fare Case
sub-station	50	0%	Preece	Report Bristol Corp , 1906
sub-station	50	1%	Cal R R. Com	Calistoga E Co vs Napa Val E Co
wooden, wood frame	20		PG&ECo	S F rate hearings, 1913-14.
wooden	25		Rosecrans	Galena Appraisal, Kan , 1905.
wooden frame	50	1	Arbitrators	Ata St. Lgt Contrysy, 1899.
frame dwellings .	35		Wis R R Com	Quoted by H A Foster.
frame stables and sheds	20-25		Wis R. R Com	Quoted by H. A Foster.
corrugated iron	20		PG&ECo	Wooden frame and floor.
corrugated iron	25	j	PG&E.Co	Wooden frame, conc floor.
corrugated iron	30		PG&ECo	Steel frame, wood floor
corrugated iron .	36		PG&ECo	Steel frame, conc floor
brick	14		Rosecrans	Galena Appraisal, Kan , 1905.
brick	30		Williams	Galena Appraisal, Kan , 1905.
brick	30	1	Kiersted	Galena Appraisal, Kan , 1905.
brick	30	1	Dodge	Mt Vernon W W , Ill , 1906.
brick .	30		P G. & E Co	Corr roof, steel roof, frame,
brick .	30		PG&ECo	Wood r, wood r-f., wood f.
brick	36		P. G & E Co	Slate r., wood r-f, and floor.
brick	40		PG&ECo.	Corr r, steel r-f, conc. f
brick	50		PG&ECo	Slate r , steel r-f , conc. f
brick .	. 50		P G & E. Co	Rein conc.r, steelr-f, conc f.
brick .	35	1	Burdick	Mt Vernon W W, Ill, 1906.
brick	60		Hammond	Jour. Inst. E. E , 1907.
brick	663	_	Trac Val Com	Chi. Union Trac Co Case.
brick .	80	0%	Preece	Report Bristol Corp , 1906.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description.	Expect- ancy.	Residual value	Authority.	Remarks.
	Years			
Buildings (continued)				
brick gas retort	30		Wis R R Com.	Quoted by H. A Foster.
1st class stone and brick	75		Wis R R Com	Quoted by H. A Foster.
2nd class shops, etc.	50		Wis R R Com	Quoted by H A Foster.
fireproof	40	0%	Chi Tel Com	Chicago Tel Com , 1908.
reinforced concrete	50		PG&ECo.	Steel r-f , conc r , and floor.
foundations	100	0%	Preece	Report Bristol Corp , 1906
Cables				
mains	30	15%	Parsons	Jour. Inst. E. E , 1907
mains	20		Chi Tel. Com.	Chicago Telephone Com.
armored mains	25		Hammond	Jour. Inst E. E , 1907.
solid mains	30		Hammond	Jour. Inst. E. E , 1907
armored	35	15%	Preece	Report Bristol Corp , 1906
cables and feeders	20]	Mil E R & L.	Milwaukee 3¢ Fare Case
cables and feeders	15-25		Wis R. R Com	Milwaukee 3¢ Fare Case.
main lead-covered	20	26%	PG&ECo.	S F. rate hearings, 1913-14.
subsidiary cables.	15	40%	Chi Tel. Com.	Chicago Telephone Com
aerial	15		Chi. Tel Com.	Chicago Telephone Com
aerial lines	20	-	St L P S. C	Union Elec. L & P Case.
aerial exchange	12-15	30%	Tel "Data"	Quoted by H A Foster.
aerial lead-covered	10-15		Foster	"Eng Valuation," etc.
aerial lead-covered	15		Wis R. R Com	Wisconsin Tel Co Case
aerial terminals	12	0%	Chi Tel Com	Chicago Telephone Com.
aerial terminals	10-12		Tel "Data"	Quoted by H A. Foster
underground lead covered	20	1	St L P. S C	Union Elec L & P Co Case.
underground lead covered	25	}	Wis, R R Com	Quoted by Floy.
underground lead covered	20		Wis R. R. Com	Quoted by H A Foster
underg'd main exchange	20-25	40%	Tel. " Data "	Quoted by H A Foster
underground sub-exchange	13-20	40%	Tel "Data"	Quoted by H A Foster
toll	30	40%	Tel. " Data "	Quoted by H A Foster
submarine	10	,	Tel "Data"	Quoted by H. A. Foster.
Coal and ash machinery.	10	1	Wis R. R Com	Quoted by Henry Floy.
	10		Mil E R & L.	Milwaukee 3¢ Fare Case.
	14		Trac Val Com.	Chicago Con Trac Co Case.
	14 2	1	Arnold	4 Chicago Appraisals
	15		Wis. R. R Com	Milwaukee 3¢ Fare Case.
	15	1	Chi. U T Co.	Chi Union Trac Co Case.
	20	1	Stone & Webs.	Chi Union Trac Co Case.
	20	1	Henry Floy	3rd Ave. Case, N. Y. City.
Compressors (air) .	20	5%	P. G & E. Co	S F rate hearings, 1913-14.
•	20-25	1	Trac Val. Com	Chi. Con. Trac. Co. Case.
Compressor station	1			
natural gas	10	50%	Cal. R R Com	Midway Gas Co, et al.
Condensers	10		Arbitrators	Ata. St. Lgt Controversy.
	15		St L. P. S. C.	Union E. L. & P. Co. Case.
	20	1	Wis. R. R. Com	Quoted by Henry Floy.
	20	1	Floy	3rd Ave. Case, N. Y. City.
	20		Arnold	Coney Isl & Brooklyn Case
	20	0%	P G & E. Co.	S. F rate hearings, 1913-14.
	25		SJL & P. Co.	Cal. R. R. Com. rate hear'gs
	1	1		

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description	Expect- ancy	Residual value.	Authority.	Remarks.
	Years			
Condensers (continued)				
old equipment	20 25		S. J L & P Co Trac Val Com	Cal R. R. Com. rate hear'gs. Chi Union Trac Co Case.
Conduits	50		Wis R R. Com	Milwaukee 3¢ Fare Case.
	50		St L P S C	Union Elec. L & P Co , Case
	100		Floy	3rd Ave. Case N Y City.
mains	30	15%	Parsons	Jour Inst. E E, 1907.
ducts	30		Hammond	Jour Inst E E, 1907.
"solid" system	30		Hammond	Jour. Inst E E., 1907.
"ducts" system	30		Hammond	Jour Inst E E , 1907.
"solid" sys in wood	40	1207	Preece	Report Bristol Corp., 1906.
clay in concrete	50	0%	Chi Tel Com.	Chicago Telephone Com.
fibre in concrete	20	0%	Chi Tel Com	Chicago Telephone Com.
subsidiary	20	0%	Chi. Tel Com	Chicago Telephone Com.
main-vitrified clay	50	%	Tel "Data"	Quoted by H. A Foster.
main-concrete	55	000	Tel "Data"	Quoted by H. A Foster.
maın-fibre	20	0%	Tel "Data"	Quoted by H. A Foster.
main-iron	20	0,0	Tel "Data"	Quoted by H. A Foster.
main-creosoted wood	20	0%	Tel "Data"	Quoted by H. A Foster.
subsidiary	15	00%	Tel " Data"	Quoted by H A Foster.
vitrified tile	50		PG&ECo	S F rate hearings, 1913-14
vitrified tile and fibre	40		PG&ECo	S F rate hearings, 1913-14.
fibre duct	30		PG&ECo	S F rate hearings, 1913-14.
wrought iron pipe	20	~	P G.&E Co	S F rate hearings, 1913-14.
Edison tube and fittings	20	1500	PG&ECo	S F rate hearings, 1913-14.
paper conduit	25		PG&ECo.	S F rate hearings, 1913-14.
wood conduit	25			S F. rate hearings, 1913-14.
w. 1. lateral pipes conduit under tracks	15 40		PG&ECo PG&ECo	S F rate hearings, 1913-14.
manholes and paving	40		PG&ECo	S F rate hearings, 1913-14.
service holes and paving	40		PG&ECo	S F rate hearings, 1913-14. S F rate hearings, 1913-14.
Converters	40		I G & E Co	5 1 Tate hearings, 1913-14.
static transformers	15		Hammond	Jour Inst E E , 1907.
rotary.	20		Hammond	Jour. Inst E. E , 1907.
Conveyors .	10		Hammond	Jour Inst. E E., 1907.
Cranes	15		Chi U T. Co	Chi Union Trac Co. Case.
C. 22	20		Stone & Webs	Chi. Union Trac Co Case
	50		Wis R R Com	Milwaukee 3¢ Fare Case.
	50		Arnold	4 Chicago Appraisals.
wooden	15		P G & E. Co	S. F. rate hearings, 1913-14.
steel	same			
	as bldg		P. G. & E Co	S F. rate hearings, 1913-14
Culverts	1			
cast iron	16		Cal R R Com.	Stockton Term & East Case.
log and timber .	163		Gillette	Gt Nor & N. P. R. R. Appraisal.
Dams	1			<u>.</u>
earth .	Ico		Cal R R Com	Cuyumaca Water Co. Case.
earth and loose rock .	50		S J L. & P. Co	Cal R. R Com rate hear'gs.
concrete	50		S J L. & P. Co	Cal R R Com rate hear'gs.
concrete diverting	100		Cal. R. R. Com.	Cuyumaca Water Co. Case.

TABLE 19 THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description	Expect- ancy	Residual value	Authority.	Remarks.
	Years			
Distribution system				
street railway	121		Cooley	Milwaukee 3¢ Fare Case.
street railway	14	_ (4	Starrett	Milwaukee 3¢ Fare Case.
distrib and trans	30 03		Wis R. R Com	Milwaukee 3¢ Fare Case.
Ditches	50		S. J. L & P Co	Cal R R Com rate hearing.
concrete lining	20		SJL & P. Co	Cal R R Com rate hearing.
Economizers	10-20		Floy	4 Chicago Appraisals
	15		Chi U T Co	Chi Union Trac Co. Case.
	20		Stone & Webs	Chi. Union Trac. Co Case.
Electrical machinery	1	i		
dynamos and alternators	25		Hammond	Jour Inst Elec E , 1907
dynamos and alternators	30	8%	Preece	Report Bristol Corp , 1906
generators	Io		Arbitrators	Ata St Lgt Controversy.
generators	15		St L P S C	Union Elec L & P. Co Case.
generators	15		Chi U T Co	Chi Union Trac Co Case
generators	20		Stone & Webs	Chi Union Trac Co Case
generators	15-20		Wis R R Com	Milwaukee 3¢ Fare Case
generators	20	2¢ lb	PG&ECo	S F. rate hearings, 1913-14.
generators	20		Arnold	Coney Isl & Brooklyn Case
generators	20		Floy	3rd Ave Case, N Y City
generators	20		SJL & P Cor	Cal R R Com rate hearing.
generators	13-33		Trac Val Com	Chicago Con Trac Co Case
generators, modern types	20		Wis R R Com	Quoted by Henry Floy
generators, obsolete types	15		is R R Com	Quoted by Henry Floy
generators belted	10-20		Chi Trac Com	Quoted by G W Cravens
generators direct conn	20		Chi Trac Com	Quoted by G W Cravens
generators, telephone plant	20	2000	Wis R R Com	Wisconsin Tel Co Case
motors	10		Arbitrators	Ata St. Lgt Controversy
motors	20		Hammond	Jour Inst Elec Eng, 1907.
motors	25	9%	Preece	Report Bristol Corp , 1906
motors	25	9%	Parsons	Jour. Inst Elec Eng, 1907.
motors	20	2¢ lb	PG&ECo	S F. rate hearings, 1913-14
motors, modern types	20		Foster	"Eng Valuation," etc.
motors, obsolete	15		Foster	"Eng Valuation," etc
motors, street railway	20		Floy	3rd Ave Case, N Y City.
motors, street railway	20	İ	Wis R R. Com	Quoted by Henry Floy.
motors, street railway	30		Trac Val Com	Chi Con Trac Co Case
motors, small, misc	20		P G. & E Co.	S F rate hearings, 1913-14.
Engines ,	15		St L. P. S. C	Union Elec L & P Co Case.
	20		Arbitrators	Ata St Lgt Controversy.
	25		Hammond	Jour Inst E E, 1907.
engines and machinery	25	6%	Ртеесе	Report Bristol Corp , 1906.
	10-33		Chi Trac Com	Quoted by G. W Cravens.
	13~20		Arnold	Coney Isl & Brooklyn Case
	20-33}		Arnold	4 Chicago Appraisals.
11	15~20		Wis R R Com	Milwaukee 3¢ Fare Case.
	20-33		Trac Val Com.	Chicago Con Trac Co. Case.
machinery .	27	6%	Parsons	Jour. Inst Elec. E., 1907.
steam	15		Chi. U. T. Co.	Chi Union Trac Co. Case.
steam	20		Stone & Webs	Chi. Union Trac Co Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description	Expect- ancy	Residual value.	Authority.	Remarks.
- (Years			
Engines (continued)				
steam .	20		Mil E R. & L	Milwaukee 3¢ Fare Case.
steam .	20	5%	PG&ECo.	S. F rate hearings, 1913-14.
steam	20		Floy	3rd Ave Case, N Y City.
steam	20		SJL&P Cor	Cal R R Com rate hearing
steam	15-25		Metcalf	Trans Am. Soc. C E, 1909.
steam, slow speed	20		Wis R R Com	Quoted by Henry Floy.
Corliss slow speed	25-30		Foster	"Eng. Valuation," etc.
steam high speed	15-20		Foster	"Eng Valuation," etc.
steam, high speed	15		Wis R R Com	Quoted by Henry Floy.
gas	10-15		Foster	"Eng. Valuation," etc.
gas ·	15		Wis R R. Com Stone & Webs	Quoted by Henry Floy.
steam turbines steam turbines .	20		Mil E R & L	Chi Union Trac Co Case.
steam turbines .	20		St L P S C	Milwaukee 3¢ Fare Case Union Elec L & P Co Case.
steam turbines .	15		Wis R. R Com	Quoted by Henry Floy.
steam turbines	20		Hammond	Jour Inst E E . 1907
steam turbines .	30	16 lb	P. G & E Co	S F rate hearings, 1913-14
Exhausters	30	16.10	r.Garco	5 F rate nearings, 1913-14
gas plant	25	ré lb	Wis R R. Com	Quoted by H A. Foster.
Fences	25	16 10	WIS IC IC. COM	Quoted by H A. Poster.
wooden	12		PG&ECo	S F rate hearings, 1913-14.
wire mesh	12		P G. & E Co	S F rate hearings, 1913-14.
railroad .	14 3		Gillette	Gt Nor & N P Appraisal
railroad	15		Cal. R. R. Com	Stockton Term. & East Case.
Filter beds	-3		Cuit Iti Iti Com	Diocason I cimi. a 12450 Casc.
water filters	15-20		Bryan	"Appraisal of water-works"
water works	30-50		Foster	"Eng Valuation," etc.
Flumes	J- J-			
wooden	30		Cal R R Com	Cuyumaca Water Co Case.
stecl	25		S J L. & P. Cor	Cal. R. R. Com. rate hearing.
concrete	50		SJL&P Cor	Cal R. R Com. rate hearing
Foundations		as life	Trac. Val Com	Chi Con Trac Co. Case.
2 00000000	of article sup-}		Floy	3rd Ave Case, N. Y. City.
	ported			
Fire protection apparatus	12	1	P. G. & E. Co	S. F rate hearings, 1913-14.
Fuel oil handling apparatus	25		Trac. Val Com.	Chicago Con Trac Co.
Fire hydrants	40		Williams	Galena Appraisal, Kan., 1905.
	40-50		Metcalf	Trans. Am. Soc. C. E , 1909.
Furniture and fixtures	121		Cooley	Milwaukee 3¢ Fare Case.
	20		Starrett	Milwaukee 3¢ Fare Case.
	20		Mil. E R. & L.	Milwaukee 3¢ Fare Case.
	20		Wis. R. R. Com	Mılwaukee 3¢ Fare Case.
Gates and valves				
water works	40		Williams	Galena Appraisal, Kan., 1905
water works	40-50		Metcalf	Trans. Am. Soc. C. E , 1909.
Gas holders	50		Wis. R. R. Com	Quoted by H. A. Foster.
Governors				
gas plant	50		Wis. R. R. Com.	Quoted by H. A. Foster.
consumers	25		Wis. R. R. Com	Quoted by H. A. Foster.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description	Expect- ancy	Residual value	Authority.	Remarks.
	Years			
Head gates				
on ditches	30		SJL&P Cor	Cal R R. Com rate hearing
Heaters (feed water)	15		C U T Co	Chi Union Trac Co Case
	20		Stone & Webs	Chi Union Trac Co Case
	20		PG&ECo	S F rate hearings, 1913-14
	16-25		Trac Val Com	Chicago Con Trac Co Case
	15-30		Wis R R Com	Milwaukee 3¢ Fare Case
	30		Wis R R Com Arnold	Quoted by Henry Floy. 4 Chicago Appraisals.
Lamps	333		Arnoid	4 Cincago Appraisais.
arc	62		Arbitrators	Ata St Lgt Controversy
arc	12	5%	Preece	Report Bristol Corp , 1906
arc	121	0.0	St L P. S C.	Union Elec L & P Co Case
arc	15		PG&ECo	S F rate hearings, 1913-14.
arc	15		Wis R R Com	Quoted by Henry Floy.
Nernst	8-10		Foster	"Eng Valuation," etc
arc lamp posts	40	5%	Preece	Report Bristol Corp , 1906.
Lighting systems				
incandescent street	15		PG&ECo	S F rate hearings, 1913-14
arc (commercial & mun)	15	1	PG&ECo.	S F rate hearings, 1913-14
municipal street	20	5%	Cal R R Com	Calıs. E C vs. Napa Val E C
Lighting protection	10		PG&ECo	S F rate hearings, 1913-14
	15		SJL&P Cor.	
	15-20		Foster	"Eng Valuation," etc
	10-12		Foster	"Eng Valuation," etc.
Machinery	27	6%	Parsons	Jour Inst Elec E, 1907.
engines and machinery	25	6%	Preece	Report Bristol Corp , 1906
Meters			77	T T . 171 17
electric electric	IO	-07	Hammond Preece	Jour. Inst. Elec E , 1907. Report Bristol Corp , 1906
electric	12	2% 5%	Preece	Jour Inst. Elec E, 1907.
electric	12 5	5%	St L. P S C	Union Elec L & P. Co Case
electric	15		PG&ECo	S F rate hearings, 1913-14
electric	23 6	10%	Cal R R Com	Calis E C vs Napa Val E
electric-switchboard	20	10/0	Wis R. R Com	Quoted by Henry Floy.
electric service	15		Wis R. R Com	Quoted by Henry Floy.
gas-station (drums) .	20	1	Wis R R. Com	Quoted by H A. Foster.
gas-station (cases) .	50		Wis R R Com	Quoted by H A Foster
gas-consumers	25	1	Wis R R Com	Quoted by H A Foster
water	20		Williams	Galena Appraisal, Kan., 190
water	20-30		Metcalf	Trans. Am Soc C E., 1909
water	50		Cal R. R. Com	Cuyumaca W Co Case.
Paving	10		Mil. E R. & L.	Milwaukee 3¢ Fare Case.
	IO		Cooley	Milwaukee 3¢ Fare Case.
	12 5		Starrett	Milwaukee 3¢ Fare Case
	10-12 5	i	Wis. R. R. Com	· ·
	10-26	1	Stone & Webs.	Chi Union Trac. Co. Case.
	10-25		Ch. U. T Co.	Chi. Union Trac Co Case.
asphalt	. 10		Chi. U T Co	Chi Union Trac Co Case
asphalt	. 10		Stone & Webs	Chi. Union Trac Co Case

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description.	Expect- ancy.	Residual value.	Authority	Remarks
	Years			
Paving (continued)				
asphalt	12		Wis R R Com	Milwaukee E. R & L Case
brick .	12		Wis R R Com	Milwaukee E R & L Case
cobble	25		Chi U T Co	Chi Union Trac Co Case.
cobble	26		Stone & Webs	Chi. Union Trac. Co. Case.
creosoted blocks	12		Wis. R R Com	Milwaukee E R. & L Case
granite	16		Chi U T Co	Chi Union Trac. Co Case.
granite	16		Stone & Webs	Chi Union Trac. Co Case.
granite	21		Wis R. R Com.	Mil El Ry & Lt. Co Case
Pipe				
water pipe				
cast iron	75		Kiersted	Galena Appraisal, Kan, 1905
cast iron	80		Williams	Galena Appraisal, Kan, 1905
cast iron	100		Burdick	Mt Vernon W W, Ill, 1906
cast iron	100		Dodge	Mt Vernon W W, Ill, 1900
cast iron	100		Alvord	Proc Am W. W. Assoc., 190
cast iron mains	50-75		Metcalf	Trans Am Soc C E, 1909
cast iron mains	100		Cal. R R Com	San Jose Water Co Case, 1914
cast iron, small size	20~40		Metcalf	Trans. Am Soc C E, 1909
wrought iron	20		Kiersted	Galena Appraisal, Kan, 1905
wrought iron	20	1	Dodge	Mt Vernon W. W., Ill , 1906
wrought iron underground	30		Williams	Galena Appraisal, Kan., 1909
wrought iron services	15-30		Metcalf	Trans Am Soc C E., 1909
galv.wrought iron incl fit's	30-50		Foster	"Eng. Valuation," etc
black w iron and services	25-35		Foster	"Eng. Valuation," etc.
steel pipe	25-50		Metcalf	Trans. Am Soc. C E, 1909.
services	20		Dodge	Mt Vernon W W, Ill., 1900
wood-stave	20-30		Metcalf	Trans. Am Soc. C. E., 1909
wood-stave	30		S J. L & P. Cor	Cal R R Com. rate hearing
distributing system	16 5		Cal R R Com	Cuyumaca Water Co Case.
power-plant pen stocks	30		S J L. & P. Cor	Cal R R. Com. rate hearing
gas pipe				
w. i & steel under 3" diam	20		Wis R R. Com	Quoted by H. A. Foster.
w. iron & steel 3" and over	30		Wis R. R. Com	Quoted by H. A. Foster.
c iron mains 3" & 4" diam	50		Wis R R. Com	Quoted by H. A. Foster.
cast iron mains over 6"	75		Wis. R R Com	Quoted by H. A. Foster.
services.	20		Wis R R Com	Quoted by H. A. Foster.
miscellaneous				
pipe and covering	15		Chi U. T. Co.	Chi Union Trac Co Case.
pipe and covering	17		Arnold	Coney Isl & Brooklyn Cas
pipe and covering	20		Stone & Webs	Chi Union Trac. Co Case.
pipe and covering	20		Wis. R. R Com.	Milwaukee 3¢ Fare Case.
pipe and covering	20	1	Floy	3rd Ave Case, N. Y. City.
pipe and covering	28 5		Arnold	4 Chicago Appraisals.
pipe and covering	22-25	1	Trac Val Com.	Chicago Con. Trac. Co. Cas
power stations	20	1	PG&ECo.	S. F. rate hearings, 1913-14
Poles	10	0%	Chi Tel Com.	Chicago Telephone Com
wooden	10		Arbitrators	Ata St Lgt Controversy.
wooden	133		Mil. E R & L.	Milwaukee 3¢ Fare Case.
wooden in earth .	12-18	1	Wis. R. R. Com	Quoted by Henry Floy.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description.	Expect- ancy	Residual value.	Authority.	Remarks.
	Years			
Poles				0 11 77 79
wooden in concrete	20		Wis. R. R Com	Quoted by Henry Floy
cedar	10	1¢-7¢ ft	P G. & E. Co.	S F rate hearing 1913-14.
cedar with cross arms	12	0%	Wis R. R. Com	Wisconsin Tel Co. Case.
cedar under 35' long	10-15	%	Tel "Data"	Quoted by H A Foster.
cedar 35' long and over	15-20	0%	Tel "Data"	Quoted by H A Foster.
cedar in earth	10-18		Foster	"Eng Valuation," etc.
cedar in concrete	12-20		Foster	"Eng. Valuation," etc.
chestnut under 35' long	8-12		Tel "Data"	Quoted by H. A Foster.
cnestnut 35' and over	12-15		Tel "Data"	Quoted by H A Foster.
pine-creosoted	20		Tel. " Data"	Quoted by H. A Foster.
redwood	16	6¢−25¢ ft	P G & E. Co.	S F. rate hearing, 1913-14.
telephone	12-15		Wis. R. R Com	Quoted by H A Foster.
telephone cross arms .	8-12	_	Wis R R. Com	Quoted by H. A. Foster.
average exchange	10	0%	Tel. " Data"	Quoted by H A Foster.
average toll .	15	0%	Tel. " Data"	Quoted by H A Foster.
iron .	20		Chi U T Co.	Chi Union Trac Co. Case.
iron .	40		Wis. R. R. Com	Milwaukee 3¢ Fare Case.
iron .	40		Arnold	4 Chicago Appraisals.
steel	50		Floy	3rd Ave Case, N Y. City
Power plant equipment	121	1	Cooley	Milwaukee 3¢ Fare Case
	20	1	Starrett	Milwaukee 3¢ Fare Case.
	21 01		Wis R R. Com.	Milwaukee 3¢ Fare Case.
Pumps.	. 15	1	St L P S C.	Union Elec L. & P Co. Case
	20		Floy	3rd Ave Case, N Y. City.
	20		Trac. Val. Com.	Chi. Con Trac. Co. Case.
	20		Arnold	Coney Isl & Brooklyn Case
	20	1	Arnold	4 Chicago Appraisals
	20		Arbitrators	Ata St. Ltg Controversy.
	21 3		Alvord	Proc Am W W Assoc., 1903
	22		Rosecrans	Galena Appraisal, Kan , 1905
	25	6%	Hammond	Jour. Inst Elec. E , 1907.
	25	0%	Preece Williams	Report Bristol Corp., 1906.
	25-30	1	Kiersted	Galena Appraisal, Kan , 1905.
	30		Burdick	Galena Appraisal, Kan, 1905
	30	1	Dodge	Mt. Vernon W. W., Ill , 1906
and auxiliary machinery	40		Metcalf	Mt Vernon W. W, Ill, 1906
and condensers .	20-30		Wis. R. R Com	Trans. Am Soc C. E., 1909
smail steam.	15-25		Wis R R Com	Milwaukee 3¢ Fare Case.
general service	15	-07		Quoted by Henry Floy.
centrifugal	16	5% 5%	P. G. & E. Co.	S. F rate hearings, 1913-14.
centrifugal	20-30	570	Foster	S F. rate hearings, 1913-14. "Eng. Valuation," etc.
geared power	20-30	1	Foster	"Eng Valuation," etc.
boiler feed	15-20		Foster	"Eng Valuation," etc.
oil	15-20	5%	P G. & E Co.	
Purifiers	10	570	1 G. & E Co.	S. F. rate hearings, 1913-14.
gas—modern	. 50		Wis R R. Com.	Quoted by H. A. Foster.
Reservoirs	50-10		Metcalf	Trans Am. Soc. C. E., 1909.
116301 ulittà	100	1	Burdick	
	100	1	Durdick	Mt. Vernon W. W., Ill., 1906

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

	AR.		•	
Description.	Expect- ancy.	Residual value	Authority.	Remarks.
	Years			
Reservoirs (continued)				
earthen	100		Cal R. R. Com	Cuyumaca Water Co Case
Retort house				
floors .	15-30		Wis R R Com	Quoted by H A Foster.
Rolling stock				•
street railway	12 5		Cooley	Mılwaukec 3¢ Fare Case.
street railway	16 7		Starrett	Milwaukee 3¢ Fare Case.
street railway .	15-20		Wis R R. Com.	Milwaukee 3¢ Fare Case
railroad	28		Gillette	Gt Nor & N. P. Appraisal.
railroad locomotives	20-35		Wilgus	Lehigh Val R R App.
railroad passenger cars	35-40	30-40%	Wilgus	Lehigh Val R. R. App
railroad freight cars	30-50	25%	Wilgus	Lehigh Val R R App
street railway cars	30		Arnold	Coney Isl. & Brooklyn Case
street railway bodies	15		Wis R R. Com	Milwaukee 3¢ Fare Case.
street railway open bodies	25		Trac. Val. Com	Chi Con. Trac. Co. Case
street ry closed bodies	20		Trac Val Com	Chi Con. Trac Co Case.
street ry bodies and trucks			MER.&LCo	Milwaukee 3¢ Fare Case.
street ry bodies and trucks	20		Chu U T Co	Chi Union Trac Co, Case
street ry bodies and trucks	20		Stone & Webs	Chi Union Trac Co Case.
street railway trucks	15-20		Wis R R Com	Milwaukee 3¢ Fare Case.
street railway trucks	30		Trac. Val. Com	Chi Con. Trac Co Case.
street railway trucks	30		Arnold	Coney Isl & Brooklyn Case
street railway elec equip	10		M E.R.&L Co	Milwaukee 3¢ Fare Case.
street railway elec equip	12-15		Chi U T Co	Chi. Union Trac Co Case.
street railway elec. equip	12-15	1	Stone & Webs	Chi Union Trac Co Case.
street railway elec equip	10-20		Wis R R. Com.	Milwaukee 3¢ Fare Case.
st. ry fenders and registers	1 -	1	Wis R. R. Com	Milwaukee 3¢ Fare Case.
Scrubbers and condensers	30	İ	Wis R R Com	Quoted by H. A Foster.
Services (electric) .	15		P G. & E Co	S. F. rate hearings, 1913-14.
	20		S. J. L & P. Cor.	
	21 6	30%	Cal. R. R. Com.	Calıs E Co. vs. Napa Va E C.
Snowsheds (R. R.) .	25		Gillette	Gt. Nor. & N. P Appraisal.
Stacks				,
brick	33		Trac. Val. Com.	Chi. Con. Trac Co. Case
brick .	33 3		Floy	4 Chicago Appraisals.
steel .	12		P G & E. Co.	S. F rate hearings, 1913-14.
steel	14 2		Floy	4 Chicago Appraisals
Stand pipes .	25	1	Rosecrans	Galena Appraisal, Kan, 1905
	30		Kiersted	Galena Appraisal, Kan, 1909
	40	1	Williams	Galena Appraisal, Kan, 190
		i i		
	25-40		Metcalf	Trans. Am Soc. C E, 190
Station buildings and R. R	25-40			
structures			Metcalf Cal R. R. Com.	
structures Steam vessels	25-40 10	-	Cal R. R. Com.	Stockton Term & East Cas
structures. Steam vessels on Great Lakes	25-40 10 40	15%	Cal R. R. Com.	Stockton Term & East Case Lehigh Val. R. R. Appraisa
structures Steam vessels on Great Lakes . on tide water	25-40 10	15% 15%	Cal R. R. Com.	Stockton Term & East Case Lehigh Val. R. R. Appraisa
structures Steam vessels on Great Lakes on tide water Stokers	25-40 10 40 32		Cal R. R. Com. Wilgus Wilgus	Stockton Term & East Case Lehigh Val. R. R. Appraisa Lehigh Val. R R. Appraisa
structures Steam vessels on Great Lakes . on tide water	25-40 10 40		Cal R. R. Com.	Trans. Am Soc. C E, 1900 Stockton Term & East Case Lehigh Val. R. R. Appraisa Lehigh Val. R R. Appraisa Chi. Con. Trac Co. Case. Chi Con. Trac. Co Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description.	Expect- ancy.	Residual value	Authority.	Remarks.
Storage batteries Sub-station equipment	Years 10 10 12 5 15 20 20 20 20 50	6% 5%	M E R.&L.Co Wis R R.Com P G & E Co Wis R R.Com Chi U.T.Co. Stone & Webs St. Louis P S C Floy S J L & P.Cor Preece Cal R. R.Com	Milwaukee 3¢ Fare Case. Milwaukee 3¢ Fare Case. S F rate hearings, 1913-14. Wisconsin Tel. Co Case. Chi. Union Trac Co Case. Chi. Union Trac. Co. Case. Union Elec L. & P. Co Case 3rd Ave. Case, N. Y City. Cal. R R Com rate hearing Report Bristol Corp, 1906. Calis. E. C. vs. Napa Vai E. C.
Sumps and wells at gas plant tar and ammonia	30 50		PG&ECo Wis RRCom	S F. rate hearings, 1913–14. Quoted by H. A Foster.
telephone, central telephone, central telephone, central telephone, central telephone P B X telephone P B X Systems and plants	30 20 12½ 15 16 7 20 20 20 20 15-20 33 3 50 50 8-10 12 8	20% 15% 20% 20% 10% 15%	S. J L. & P Co P. G & E Co. St L P. S C Chi U T. Co Arnold Stone & Webs. M E R & L Co Floy P G & E Co. Hammond S J L & P Co Wis. R R Com Trac Val Com Arnold Chi Trac. Com Wis R R Com Chi Tel Com. Tel. "Data" Wis R. R Com Chi. Tel Com. Tel "Data" Wis R. R Com Chi. Tel Com. Tel "Data" Wis R. R Com	Cal. R. R. Com rate hearing S F rate hearings, 1913-14. Union Elec. L & P. Co. Case. Chi. Union Trac. Co Case. Coney Isi & Brooklyn Case Chi. Union Trac. Co Case. Milwaukee 36 Fare Case. 3rd Ave. Case, N. Y. City. S. F rate hearings, 1913-14. Jour Inst Elec E, 1907. Cal R R Com rate hearing Milwaukee 36 Fare Case. Chi. Con. Trac. Co Case. 4 Chicago Appraisals. Quoted by G W Cravens. Quoted by G W Cravens. Chicago Telephone Com. Quoted by H A Foster. Wisconsin Tel Co. Case. Chicago Telephone Com. Quoted by H A Foster. Wisconsin Tel Co. Case. Wisconsin Tel Co. Case.
electric light and power	17 46		Wis. R. R. Com	Fon du Lac W. Co. Case aver 6 plants.
electric light and power electric light and power	18		Wis R. R. Com Mass G. & E. Com	Madison Gas & E. Co. Case Mass. Gas & Elec. Com.
electric light and power	20-25		Wis. R. R. Com	Jefferson Mun. E. L. & W
electric light and power electric railway	22 0 18 02		P. G & E. Co. Wis R R. Com	S. F. rate hearing, 1913-14. Fon du Lac W. Co. Case aver
electric railway.	18 6	Ř	Wis R. R. Com	7 plants Milwaukee 3¢ Fare Case.

TABLE 19 THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

			- (
Description.	Expect- ancy	Residual value.	Authority	Remarks.
	Years			
Systems and plants (con)				
electric railway	198		Wis R R Com	Duluth St Ry Co Case.
telephone	10-25		Wis R R Com	Oregon Telephone Co Case.
telephone	14-15		Wis R R Com	Various cases in 1912-13
water supply	50-65		Wis R R Com	Various cases in 1910-11
overhead system	331	35%	Cal R R Com	Calis E C. vs Napa Val. E.C
st ry distrib system	121		Cooley	Milwaukee 3¢ Fare Case
	14		Starrett	Milwaukee 3¢ Fare Case.
st ry distrib & trans sys	30 03		Wis R R Com	Milwaukee 3¢ Fare Case.
Tanks	j			
wooden	12	1 1	PG&ECo	S F rate hearings, 1913-14.
steel	30		PG&ECo	S F. rate hearings, 1913-14.
Tar extractors				
P & A	40		Wis R R Com	Quoted by H. A Foster
Telephone equipment		8		
street railway	10-12	1	Wis. R R Com	Milwaukee 36 Fare Case
street railway	13 3		Mil E R & L	Milwaukee 3¢ Fare Case.
electric power plants	12		PG&ECo	S F rate hearings, 1913-14
elec power plant lines	20		S J L & P Cor	Cal R R Com rate hearing
elec power plantinstrum'ts	20		S J L & P Cor	Cal R. R Com rate hearing
elec power plant equipm't	8-14	ļ	S J L & P Cor	Cal R R Com rate hearing
farmers line	15		Wis R. R. Com	Wautoma & Mt Morris Tel Line
subscribers instruments	8-10	10%	Tel "Data"	Quoted by H A Foster
	10	5%	Chi Tel Com	Chicago Tel Commission.
	10	5%	Wis R R Com.	Wisconsin Tel Co Case
telephone drop wires	8	15%	Chi Tel Com	Chicago Tel Commission.
Testing instruments	1			
electric	10	20%	Cal R R. Com	Calis E C. vs. Napa Val E Co.
Tools and shop machinery	5-25	1	Wis R R. Com	Milwaukee 3¢ Fare Case
	121	1	Cooley	Milwaukee 5¢ Fare Case
	14 2		Starrett	Milwaukee 3¢ Fare Case.
	13 3	1	Mil. E R & L.	Milwaukee 3¢ Fare Case.
	20	1	Chi U T Co	Chi Union Trac. Co Case.
	20	1	Stone & Webs.	Chi Union Trac Co Case.
tools, teams and furniture	4	10%	Chi Tel Com	Chicago Telephone Com.
tools and furniture	7		Wis R R Com	Quoted by H A Foster.
tools and sundries	10	5%	Preece	Report Bristol Corp , 1906.
Track	1		}	
rails, ties and bonding	121		Wis R. R. Com.	0,
rails, ties and bonding	121		Cooley	Milwaukee 3¢ Fare Case.
rails, ties and bonding	122		Starrett	Milwaukee 3¢ Fare Case.
rails, ties and bonding	12 85		Chi U T Co	Chi Union Trac Co Case.
rails, ties and bonding	13 86		Stone & Webs	Chi Union Trac Co Case.
rails, ties and bonding	13 33		Mil E R. & L.	Milwaukee 3¢ Fare Case.
straight .	18		Wis R R. Com.	~
special work	8 3		Mil E. R & L	Milwaukee 3¢ Fare Case
special work special work	10		Arnold Wis R R Com	4 Chicago Appraisals.
special work	12			Quoted by Henry Floy.

TABLE 19 THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Continued)

Description.	Expect- ancy	Residual value.	Authority.	Remarks.
Track (continued) special work special work ties railroad rails, new railroad rails, relay R R rails and track fast railroad track fastenings railroad ties, redwood railroad ties, redwood railroad ties, creosoted trestles and bridges ballast frogs and switches cast-iron culverts Transjormers			Chi. U. T. Co Stone & Webs Trac Val Com. Cal R. R. Com Gallette Cal R. R. Com Gillette Cal R. R. Com O Chanute Cal R. R. Com Cal R. R. Com Cal R. R. Com Cal R. R. Com Cal R. R. Com Cal R. R. Com St. L. P. S. C. Wis R. R. Com	Chi. Umon Trac Co Case. Chi Umon Trac Co Case. Chi Con. Trac. Co Case. Stockton Term. & East Case. Stockton Term. & East Case. Gt Nor & N. P Appraisal Stockton Term. & East Case. Gt Nor and N. P. Appraisal. Stockton Term. & East Case. Houston & Texas Cent. R R. Stockton Term. & East. Case. Union Elec. L & P. Co Case Quoted by Henry Floy
station service station line	20 25 20	10%	Wis R. R. Com S J L & P. Cor Cal R R Com	Quoted by Henry Floy. Cal R R Com. rate hearing. Calis E. C. vs. Napa Val. E. Co.
distribution power static transformers .	20 20 15	\$1 00 kw \$ 50 kw	P G & E Co P G & E Co. Hammond	S F. rate hearings, 1913-14. S F. rate hearings, 1913-14. Jour. Inst. E. E., 1907.
power plants Turbines steam steam steam	50 15 20 20		S J L. & P Cor St L P S. C. Wis. R R. Com Hammond	Cal. R. R. Com rate hearing. Union Elec L. & P Co. Case. Quoted by Henry Floy. Jour. Inst Elec. E, 1907.
steam water water-type prior 1900 water-type after 1900 Washers (gas plant)	30 30 25-40 30-50	ı¢ lb.	P. G & E. Co Wis R R. Com. Foster Foster	S F. rate hearings, 1913-14. Quoted by Henry Floy. "Eng. Valuation," etc. "Eng Valuation," etc.
cast iron Water gas machines Watt meters service	40 30		Wis. R. R. Com Wis. R. R. Com Foster	Quoted by H. A. Foster. Quoted by H. A Foster. "Eng. Valuation," etc.
Wells water water, driven or drilled water, large open masonry water well pumps gas wells gas wells wharves and docks were	30 50-75 75-100 10 10 10 33 ¹ / ₃	0% 25%	S. J. L. & P. Cor Foster Foster S. J. L. & P. Cor Cal. R. R. Com Cal. R. R. Com Gillette	Cal. R. R. Com. rate hearing. "Eng. Valuation," etc. "Eng. Valuation," etc. Cal. R. R. Com. rate hearings. Midway Gas Co. et al. Case. Midway Gas Co. et al. Case. Gt. Nor. & N. P. Appraisal.
aerial copper	20 15		St L P. S C. Chi. Tel. Com.	Union Elec. L. & P. Co. Case. Chicago Telephone Com.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (Concluded)

Description	Expect- ancy.	Residual value.	Authority.	Remarks.
	Years			
Wire (continued)				
bare .	25	35%	P. G. & E Co.	S. F rate hearings, 1913-14.
ınsulated	20	15%	P G & E. Co.	S F. rate hearings, 1913-14.
copper, weatherproof	13		Arbitrators	Ata St. Lgt. Controversy.
copper, weatherproof	16		Wis R. R. Com.	Quoted by Henry Floy.
copper, weatherproof	10-15		Foster	"Eng. Valuation," etc.
2-kv distribution .	15		S J. L & P. Cor.	Cal. R. R. Com. rate hearing.
4-kv. distribution	15		S J. L. & P. Cor	Cal. R. R. Com. rate hearing.
10-kv. distribution .	20		SJL & P Cor	Cal. R. R. Com. rate hearing.
telephone, iron .	10	0%	Wis. R R. Com	Wisconsin Tel Co. Case.
telephone, iron	8-15		Wis R R. Com.	Quoted by H. A. Foster.
telephone, weatherproof,				
iron	15		Wis R. R Com	Quoted by H. A. Foster.
telephone, copper	20	75%	Wis. R. R. Com	Wisconsin Tel Co. Case.
telephone exch , bare copper	10-15	40%	Tel "Data"	Quoted by H. A. Foster.
telephone exch , insul cop.	10	10%	Tel "Data"	Quoted by H. A. Foster.
telephone exch , bare iron	8-10	0%	Tel "Data '	Quoted by H. A. Foster.
telephone toll, bare copper	40	40%	Tel "Data"	Quoted by H. A Foster.
telephone toll, bare iron	15	0%	Tel "Data"	Quoted by H. A. Foster.
telephone, distribution	10	15%	Wis R. R Com.	Wisconsin Tel. Co. Case.

EXPLANATION OF TABLE 20 EXPECTANCY AND REMAINING VALUE (ADDROXIMATE Values Only)

The expectancy of any article in use, which is still in serviceable condition, can be approximated from the following tables. when the probable life new of other articles of the same class is There is noted in these tables also the approximate remaining value, although this may also be obtained from the Amortization and Depreciation Table 27 by entering the same with the years of expectancy as elsewhere explained. The expectancy noted in Table 20 is based on a reasonable assumption of the rate at which a large number of articles of the same probable life would go out of use (see Chapter VI) and is in fair accord with the expectancy that would be found if all the articles were assumed to fail according to the law of probabilities within a period equal to twice the probable life term. The strict application of the law of probability under assumptions, relating not only to the probable life of any article in question, but also to the probability of survivals beyond some period, as, for example, twice the probable life, would be an unnecessary refinement not justified until a vastly greater mass of data than now available, relating to actual life of individual articles in each class, has been assembled.

It may be claimed for this table in its present form that it will afford means of making much closer approximation to the actual remaining worth of any article than is possible by the use of the ordinary amortization tables in which age alone is taken into account and in which no distinction is made between actual and probable life.

In this table the expectancy and remaining value are noted for any single article, whose probable life when new is known. The table has been computed for 4 per cent, 5 per cent, 6 per cent and 7 per cent interest per annum. For other interest rates, values may be interpolated.

In Tables 4 to 7, Chapter VI, the annual replacement requirement is noted for groups of numerous articles, all of the

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same probable life new. These tables cover the two cases of a plant which has attained its full growth and of the plant to which a uniform annual addition is being made. The annual replacement requirement is based on the probable annual number of failures. Similar tables can readily be prepared for other terms of usefulness than 5, 10 and 20 years covered by these tables and for any other hypothesis of the rate at which actual failures will take place.

All figures noted in Tables 4 to 6 and in Table 20 are approximation figures derived from smoothed-out curves. They are, in fact, modifications of the values which were obtained as a result of the assumption already fully discussed that failures among any large group of articles will be greatest in number at or near the end of the probable life term, that practically no articles will survive twice the probable life term, and that there will be a uniform increase in the annual rate of failures from the beginning to the year of maximum number of failures and that the decrease in the number of annual failures will follow a similar law.

To find the expectancy of any perishable article which has a probable life term new of n years, not covered by any of the subdivisions of the table here published, the ten-year life subdivision of Table 20 may be called to aid.

Let e = the expectancy which is to be determined,

m = the age of the article,

n = the probable life new of the article,

m' = the relative age of an article whose probable life new was 10 years such that

$$m' = \frac{\text{io } m}{n} \, \cdot \tag{19}$$

Let e' = the relative expectancy at the age m' of an article whose probable life new was 10 years.

Then the expectancy to be determined will be found from

$$e = \frac{ne'}{10} \cdot \tag{20}$$

Example. — What is the expectancy of an article 8 years old, in fair condition, whose probable life new was 12 years? Here n = 12 and m = 8.

$$m' = \frac{10 \times 8}{12} = 6.67.$$

For the age 6.67 years in the ten-year subdivision of Table 20 (this being in the seventh year) the expectancy 5.17 years is found by interpolation. The required expectancy is, therefore,

$$e = 5.17 \times \frac{12}{10} = 6.2$$
 years,

which would be found in a twelve-year probable life table for the end of the eighth year or for the beginning of the ninth year.

For general use the following tormulæ are recommended which will be found to agree fairly well with the results presented in Table 20. They are applicable to any probable life term and any age to the limit beyond which it has been assumed no article will continue in service.

When the age of an article is less than its probable life term new, that is, when m < n

$$e = n - 0.93 m + 0.30 \frac{m^2}{n}.$$
 (21)

When the age of an article is greater than its probable life term new, that is, when m > n

$$e = 0.72 \, n - 0.35 \, m. \tag{22}$$

Example. — What is the expectancy of an article whose probable term of usefulness when new was 12 years, which is 8 years old and apparently in good condition?

Here n = 12 and m = 8.

By equation (21)

$$e = 12 - 7.44 + 1.60 = 6.16$$
 years.

Example. — What is the expectancy of a similar article 15 years old, which has survived its probable term of usefulness but is still in service and in good condition?

Here n = 12 and m = 15.

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By equation (22)

$$e = 8.64 - 5.25 = 3.39$$
 years.

Example. — What is the expectancy of an article 40 years old, whose probable life new was 60 years and which is in good condition?

Here n = 60 and m = 40. By equation (21)

$$e = 60 - 3720 + 8.00 = 30.80$$
 years.

To find the remaining value of an original investment of \$100 in any article or the accrued depreciation when the expectancy of the article is known and the probable life new is different from any covered by the tables here published, proceed as follows:

Find the annuity which in n years will amount to \$100. From tables, such as Tables 20 and 27 in this volume, or by calculation, find the amount of this annuity for (n-e) years This amount will represent the accrued depreciation and \$100 less this amount will be the remaining value of an original investment.

Example. — What is the remaining value (6 per cent interest) of an article which cost \$100, whose probable life new was 12 years, which is 8 years old and watch is still in fair condition?

The expectancy of this article as shown in a preceding example is about 6.2 years. The annuity which in 12 years at 6 per cent will amount to \$100 is \$5 93; this annuity in (12 - 6.2) years amounts to \$40.44, the accrued depreciation. The present worth will be \$100 - \$40.44 = \$59.56.

By use of the ten- and fifteen-year subdivisions of the table and interpolation the procedure would be as follows:

Eight years in a twelve-year probable life table is fairly comparable with 6.67 years in a ten-year table and with 10 years in a fifteen-year table

It remains to be noted that when remaining value is to be determined age may be entirely disregarded when expectancy can be determined by an inspection of the article in question. The condition of the article and all other circumstances affecting its serviceability being taken into consideration its probable remaining term of service or its expectancy is estimated. In this case it will be more convenient to use Table 27 than Table 20.

Select the subdivision of Table 27, corresponding with the probable life new of the article, and use only the headings of the columns at the bottom of the page. For the expectancy noted in the right-hand column, the remaining value and the accrued depreciation will be found in their respective columns. Interpolation may be resorted to for fractional years if this refinement be thought desirable.

Because expectancy in the case of individualized articles which have been long in use can best be determined by inspection and by a consideration of local circumstances affecting continued serviceability, it has been thought sufficient to note values in the long-term subdivisions of Table 20 and for articles long in service only for every fifth or every tenth year.

To find the current depreciation of any single article in any year by the use of Table 20 subtract from its remaining value at the beginning of that year the remaining value at the beginning of the following year.

Example. — What is the current depreciation in the sixth year, at 6 per cent interest, of an article whose probable life is 15 years?

Remaining value at the beginning of the sixth year	\$76.90
Remaining value at the beginning of the seventh year	72.60
Depreciation in the sixth year	\$ 4.30

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100

All values in this table are based on the assumption that in a group of many articles, no article will survive twice its probable life and that the number of annual failures will increase at a uniform rate to a maximum at the end of the probable life term and will thereupon decrease at a uniform rate.

PROBABLE LIFE 5 YEARS

5 YEARS

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5 YEARS

D	To an and To an an an an an an an an an an an an an	Remaining value					
Beginning	Expectancy	4 per cent per	5 per cent per	6 per cent per	7 per cent per		
of year	years	annum	annum	annum	annum		
1	5 00	\$100 00	\$100 00	\$100 00	\$100 00		
2	4 15	84 30	84 60	84 90	85 20		
3	3 40	70 00	70 50	71 00	71 50		
4	2 75	57 30	57 90	58 40	59 00		
5	2 25	47 40	47 90	48 50	49 10		
6	1.85	39 30	39 80	40 30	40 90		
7	1 55	33 00	33 50	34 00	34 50		
8	1 25	26 80	27 20	27 60	28 00		
9	1 00	21 60	22 00	22 40	22 80		

PROBABLE LIFE 10 YEARS

10 YEARS					10 YEARS
1	10 00	100 00	100 00	100 00	100 00
2	9 10	92 50	92 90	93 20	93 50
3	8 25	85 20	85 80	86 40	87 00
4	7 50	78 50	79 30	80 10	81 00
5	6 80	72 10	73 10	74 00	75 00
6	6 10	65 60	66 70	67 70	68 80
7	5 50	59 90	60 90	62 00	63 10
8	5 00	54 90	56 10	57-20	58 40
9	4 50	49 80	51 00	52.20	53 30
10	4 10	45 90	46 90	48.10	49.20
11	3.70	41 60	42 70	43 90	45 00
12	3.35	37 90	39 00	40 10	41 20
13	3.00	34 20	35 30	36 30	37 40
14	2.65	30 40	31 40	32.30	33.30
15	2 30	26 50	27.40	28.30	29.30
16	2.00	23 30	24 10	24 90	25.70
17	1.65	19 50	20 20	20 90	21 60
18	1.35	15 70	16 30	16 90	17 50
19	1.00	11 90	12 30	12 .80	13 30

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

PROBABLE LIFE 15 YEARS

15 YEARS

	Remaining value.						
of year.	Beginning Expectancy of year. years.	4 per cent per annum.	5 per cent per annum	6 per cent per annum.	4 per cent per annum.		
1 2 3 4	15 00 14 10 13 20 12.40	\$100 00 95 50 90 90 86 60	\$100 00 95 80 91 50 87 40	\$100 00 96 10 92 10 88.30	\$100.00 96.40 92 60 89.00		
5	11 65	82 40	83 50	84.50	85.50		
6 7 8 9	10 90 10 15 9.45 8 85	78 20 73 90 69.60 65 90	79 50 75 30 71.10 67.50	80.70 76 70 72 60 69.10	81.80 78.00 74.00 70.60		
10	8 25	62.10	63 80	65.50	67.10		
11 12 13 14	7 75 7 25 6 75 6 30	58 90 55 60 52 30 49 20	60 60 57 40 54 00 51.00	62 30 59 10 55.80 52.70	64 00 60.80 57.50 54.40		
15	5 90	46 40	48.20	49.90	51 60		
16 17 18 19	5·55 5 20 4·85 4·50	44 00 41 50 38 90 36.40	45.70 43 20 40.60 37.90	47.40 44.80 42.20 39.50	49.10 46.50 43.80 41.10		
20	4.15	33.70	35 30	36.80	38.40		
21 22 23 24	3.80 3 45 3.10 2.75	31 10 28 40 25 70 22.90	32.60 29 80 27.00 24.20	34.00 31.20 28.30 25 40	35.50 32.60 29.60 26.60		
25	2 40	20 20	21.20	22.30	23.40		
26 27 28 29	2 05 1.70 1.35 1.00	17.40 14.50 11.60 8.60	18.30 15.30 12.20 9.20	19.30 16.10 12 90 9 70	20.30 17.00 13.60 10.30		

TABLE 20 EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

PROBABLE LIFE 20 YEARS

20 YEARS

Reginning	Expectancy		Remaini	ng value.	
Beginning of year	years	4 per cent per annum	5 per cent per annum.	6 per cent per annum	7 per cent pe
I	20 00	\$100 00	\$100 00	\$100 00	\$100 00
2	19 10	97 00	97 30	97 60	97 80
3 4	18 20 17 35	93 90 90 80	94 40 91 60	95 00 92 40	95 50 93 10
5	16 50	87 60	88 70	89 70	90 70
6	15 75	84 80	86 10	87 30	88 40
7	15 00	8r 8o	83 30	84 70	86 00
8	14 30 13 60	79 00 76 00	80 60 77 80	82 10	83 60 81 10
9				79 50	
10	12 90	73 00	75 00	76 80	78 50
II	12 20	69 90	72 00 69 30	73 90	75 80
12	11 00	67 20	66 70	71 40 68 80	73 30
14	10 50	62 10	64 30	66 50	68 50
15	10 00	59 70	62 00	64 20	66 30
16	9 50	57 20	59 50	61 70	63 90
17	9 00	54 70	57 00	59 30	61 50
18 19	8 60 8 20	52 60 50 60	55 00 52 90	57 20 55 20	59 40
20	7 80	48 50	50 80	53 00	57 40 55 30
21	7 40	46 30	48 60	50 90	1
22	7 00	44 20	46 40	48 70	53 10
23	6 60	41 90	44 20	46 40	48 50
24	6 25	40 00	42 20	44 30	46 50
25	5 90	38 00	40.10	42 30	44.40
26	5 55	36 00	38.00	40.10	42 20
27 28	5 20 4 85	33 90 31 90	35 90 33 90	38 oo 35 8o	40 00
29	4 50	29 70	31 60	33 50	37 70 35 30
30	4 15	27 60	29 40	31 20	33.00
31	3 80	25.40	27 10	28 80	30.50
32	3 45	23 30	24 80	26 40	28.00
33	3 10	21 10	22.50	24 00	25 50
34	2 75	18 80	20 10	21 50	22 90
35	2 40	16.50	17 70	18 90	20 20
36	2 05	14 20	15.30	16 40	17 50
37 38	1.70	9 50	12.70	13.70	14.60
39	1 00	7.10	7 60	8 20	8 80

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

PROBABLE LIFE 25 Years

25 YEARS

D	D		Remaini	ng value.	
Beginning of year	Expectancy years	4 per cent per annum.	5 per cent per annum	6 per cent per annum.	7 per cent per
r	25 00	\$100 00	\$100 00	\$100 00	\$100 00
2	24 10	97 80	98 10	98 40	98 60
3 4	23 20 22 30	95 60 93 30	96 10 94 10	96 60 94 80	97 10
5					95 40
	21 45	91 00	92 00	92 90	93 80
6	20 60 19 80	88 70 86 40	90 00 87 90	91 10 89 20	92 10 90 50
7 8	19 05	84 20	85 90	87 40	88 80
9	18.30	81 90	83 80	85 50	87 20
10	17 60	79 80	81 8o	83 60	85 30
ıı	16 90	77 60	79 70	81 70	83 50
12	16 20	75 20	77 50	79 60	81 60
13 14	15 55 14 90	73 00 70.80	75 45 73 30	77 70 75 60	79 80 77.80
15	14 30	68 70	71.20	73 70	76.00
16	13 75	66 70	69 30	71 90	74 20
17	13.20	64 70	67 30	69 90	72 40
18 19	I2.70 I2 20	62 80 61 80	65 50 63 60	68 10	70 60 68 90
20	11 70	58 90	61 70	64 40	67 00
21	11 25	57 10	59 90	62 70	65 30
22	10 80	55 20	58 10	60 90	63.50
23	10.40	53 60	56 40	59 20	61 90
24	10 00	51 90	54 80	57 60	60 30
25	9.60	50 20	53 00	55 80	58.50
26 27	9 25 8 90	48.70	51 50	54.30	57.00
28	8 50	47 10 45 30	49 90 48.10	52.70 50 90	55 40 53 60
29	8.15	43 80	46 50	49.30	51.90
30	7 80	42 20	44 90	47 60	50.10
31	7 40	40 30	43.00	45.60	48.20
36	5 60	31.50	33 90	36.30	38.60
41 46	3 80 2 05	22 10	24 00	25 90	27.80
49	1 00	12 40 6 20	13,50 6.80	7 40	8 00

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

PROBABLE LIFE 30 YEARS

30 YEARS

Domming	Expectancy		Remain	ing value.	
Beginning of year	years	4 per cent per annum	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
1 2 3 4	30 00 29 10 28 20 27 30	\$100 00 98 40 96 70 95 00	\$100 00 98 60 97 20 95 80	\$100 00 98 90 97 70 96 40	\$100 00 99 00 98 00 97 00
5	26 45	93 30	94 30	95 20	95 90
6 7 8 9	25 60 24 80 24 00 23 30	91 60 89 90 88 20 86 60	92 80 91 30 89 80 88 40	93 80 92 50 91 20 89 90	94 70 93 60 92 40 91 30
10	22 55	84 90	86 80	88.50	90 10
11 12 13 14	21 80 21 05 20 30 19 60	83 10 81 30 79 40 77 50	85 20 83 50 81 80 80 10	87 10 85 60 84 00 82 40	88 80 87 40 86 00 84 50
15	18 90	75 70	78 40	80 80	83.10
16 17 18 19	18 25 17 65 17 05 16 50	73 90 72 20 70 50 68 90	76 70 75 10 73 50 71 90	79 30 77 80 76 40 74 80	81 60 80 20 78 80 77 40
20	15 95	67 20	70 40	73 30	76.00
21 22 23 24	15 40 14 90 14 40 13 95	65 50 64 00 62 40 60 90	68 70 67 20 65 70 64 20	71 70 70 30 68 70 67 40	74 50 73.10 71.70 70 30
25	13.50	59 40	62 70	65 90	68.90
26 27 28 29	13.05 12 60 12 20 11 80	57 60 56 50 55 00 53 60	60 90 59 70 58 40 56.90	64 50 62 90 61 60 60 20	67.50 66.00 64.70 63.30
30	11 45	52 30	55 70	58 90	62.00
31 32 33 34	11 10 10 75 10 40 10 00	51 00 49 70 48 40 46 90	54 40 53 10 51 70 50 20	57 70 56 30 55 00 53 50	60.80 59 50 58 10 56 60

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

PROBABLE LIFE 30 YEARS

30 YEARS

_	-		Remaini	ng value.	
Beginning	Expectancy	4 per cent per	5 per cent per	6 per cent per	7 per cent per annum.
year	years	annum.	annum	annum	
35	9 65	\$45 50	\$48 80	\$52 10	\$55 20
36	9 30	44 20	47.40	50 60	53 70
37	8 95	42 80	46 00	49 20	52 30
38	8 55	41 20	44 30	47 50	50 50
39	8 20	39 70	42 90	46 00	49 00
40	7 85	38 30	41 40	44 40	47 40
41	7 50	36 80	39 80	42 80	45 80
46	5 70	28 90	31 60	34 10	36 80
51	3 90	20 50	22 60	24 60	26 70
56	2.10	11 40	12 70	13 90	15 20
59	1 00	5 60	6 20	6 90	7 50

		Probable	Life 40 Year	RS	
40 YEARS					40 YEARS
I	40 00	100 00	100 00	100 00	100 00
2	39 10		99 30	99 40	99 60
3	38 20	98.10	98 50	98 80	99.10
4	37 30	97 10	97 70	98 20	98.60
5	36 40	96 00	96 80	97 50	98.00
6	35 50	94 90	95 90	96 80	97 50
7	34 70	93 90	95 10	96.10	96.90
8	33 90	92 90	94 30	95 40	96 40
9	33 10	91 80	93 40	94 70	95.80
10	32 30	90 70	92 50	93 90	95 10
11	31 50	89 60	91 50	93 10	94 40
12	30 75	88 50	90 60	92 30	93.80
13	30 00	87 40	89 60	91.50	93 10
14	29 30	86 30	88.60	90.70	92 40
15	28.55	85.10	87 60	89 80	91 60
16	27 80	83.80	86 50	88.80	90 80
17	27 10	82 70	85 50	87.90	90.00
18	26 40	81 50	84 40	87 00	89.20
19	25 70	80 20	83 30	86 00	88.30
20	25 00	78 90	82.10	85.00	87.40
21	24,40	77 80	81 10	84.00	86.60
22	23,80	76 60	80 10	83.10	85.70
23	23.20	75 50	79 00	82.10	84.90
24	22.60	74 20	77.90	81.10	83.90

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

Probable Life	40	YEARS
---------------	----	-------

40 YEARS			40 IEM		40 Years
	T74		Remain	ing value.	
Beginning of year	Expectancy years	4 per cent per annum	5 per cent per annum.	6 per cent per annum.	7 per cent per annum
25	22 00	\$73 00	\$76 70	\$80 00	\$83.00
26	21 45	71 80	75 60	79 00	82 00
27	20 95	70 80	74 60	78 10	81 20
28	20 45	69 70	73 60	77 10	80 30
29	19 95	68 60	72 50	76 10	79.40
30	19 45	67 40	71 40	75 10	78 40
31	19 00	66 40	70 40	74 20	77 50
36	r6 80	60 90	65 20	69 10	72 80
41	14 80	55 60	59 90	64 00	67 80
41 46	12 85	50 00	54 30	58 40	62 20
51 56 61	10 95	44 10	48 20	52 30	56 10
56	9 10	37 90	41 80	45 60	49 30
61	7 30	31 40	34 90	38 40	41 70
66	5 55	24 70	27 60	30 60	33 50
71	3 80	17 50	19 70	22 00	24 30
76	2 05	9 70	II IO	12 50	13 90
79	1 00	4 90	5 60	6.30	7.00

	1 ROBABLE 1	LIFE 50 YEAR	.5	50 YEARS
50 00 49 05 48 15 47 25	99 40 98 70 98 10	100 00 99 50 99 10 98 60	100 00 99 70 99 40 99 00	100 00 99 80 99 50 99 30
45 45 44 60 43 75 42 90	96 80 96 00 95 .40 94 70	97 70 97 00 97 60 96 60 96 00	98 30 97 80 97 50 97 10	99 00 98.70 98.40 98.10 97 80
42 IO 41 30	94.10 93 30	95 50 94.90	96 60 96 20	97.50 97 20 96 80
39.75 39.00 38.25	91 90 91 .20 90 .40	93.80 93.20 92.60	95.30 94.80 94.40	96.50 96.10 95.70
	49 05 48 15 47 25 46 35 45 45 44 60 43 75 42 90 42 10 41 30 40 50 39 75 39 00	50 00 100 00 49 05 99 40 48 15 98 70 47 25 98.10 46.35 97 50 45 45 96 80 44 60 96 00 43 75 95.40 42 90 94 70 42 10 94.10 41 30 93 30 40 50 92 60 39.75 91 90 91.20	50 00 100 00 100 00 49 05 99 40 99 50 48 15 98 70 99 10 47 25 98 10 98 60 46 35 97 50 98 10 45 45 96 80 97 70 44 60 96 00 97 00 43 75 95 40 96 60 42 90 94 70 96 00 42 10 94 10 95 50 41 30 93 30 94 90 40 50 92 60 94 40 39 75 91 90 93 80 39 00 91 20 93 20	50 00 100 00 100 00 100 00 49 05 99 40 99 50 99 70 48 15 98 70 99 10 99 40 47 25 98.10 98 60 99 00 46.35 97 50 98.10 98 60 45 45 96 80 97 70 98 30 44 60 96 00 97 00 97 80 43 75 95.40 96 60 97 50 42 90 94 70 96 00 97 10 42 10 94.10 95 50 96 60 41 30 93 30 94.90 96 20 40 50 92 60 94.40 95.80 39.75 91 90 93.80 95.30 39.00 91.20 93 20 94 80

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Continued)

50 YEARS

PROBABLE LIFE 50 YEARS

	77		Remain	ng value.	
Beginning of year.	Expectancy years	4 per cent per annum.	5 per cent per annum	6 per cent per	7 per cent per annum.
16 17 18 19 20	37 50 36 75 36 00 35 30 34 60	\$89 60 88 80 88 00 87 20 86 40	\$92 00 91 30 90 60 90 00 89 30	\$93 80 93 30 92 80 92 20 91 70	\$95 30 94 90 94 50 94 00 93 50
21 22 23 24 25	33 90 33 20 32 50 31 80	85 60 84 70 83 80 82 90	88 60 87 90 87 10 86 30 85 60	91 10 90 50 89 80 89 20 88 50	93 10 92 60 92 00 91 50 90 90
26 27 28 29	30 50 29 90 29 30 28 70	81 20 80 30 79 50 78 60	84 80 84 10 83 30 82 50	87 90 87 20 86 60 85 90	90 40 89 80 89 20 88 70
31 36 41 46 51 61 71 81 91	28 10 27 50 24 85 22 45 20 35 18 50 14 80 11 10 7 50 3 90 1 00	77 70 76 80 72 50 68 10 64 00 60 00 51 20 41 10 29 60 16 50 4 50	81 70 80 90 77 00 72 90 69 00 65 10 56 30 45 80 33 50 19 00 5 20	85 20 84 40 80 90 77.10 73 40 69 70 61 10 50 30 37 40 21 50 6 00	88 00 87 40 84 20 80 80 77 40 73.90 65 50 54 60 41 20 24 00 6 80

60 YEARS		Probable 1	Life 60 Year	.s	60 Years
1 2 3 4 5	60 co 59 10 58.15 57.25 56 35	100 00 99.60 99 20 98 80 98.40	100 00 99 70 99 50 99 20 98 90	100 00 99.80 99.60 99.50	100.00 99 90 99 80 99 60
6 ' 7 8 9 10	55.45 54.60 53.75 52.90 52.05	97 90 97 50 97 10 96 60 96 20	98.60 98 30 98 00 97.70 97 30	99.10 98.80 98.60 98.40 98.20	99 40 99 20 99.10 98 90 98 80

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS \$100 (Concluded)

PROBABLE LIFE 60 YEARS

60 YEARS

	-		Remainii	ng value	
Beginning of year	Expectancy years	4 per cent per annum.	5 per cent per annum	6 per cent per annum.	7 per cent per annum
11 12 13 14	\$51 20 50 40 49 50 48 75	\$95 70 95 20 94 60 94.10	\$97 00 96 60 96 20 95 90	\$97 90 97 70 97 40 97 10	\$98.60 98.40 98.20 98 00
15	48 00	93 70	95 50	96 80	97 80
16 17 18 19	47 25 46 50 45 75 45 00	93 20 92 70 92 10 91 60	95 10 94 70 94 30 93 90	96 60 96 30 95 90 95 60	97 60 97 40 97 10 96 90
20	44 25	91 00	93 50	95 30	96 60
21 22 23 24	43 50 42 75 42 00 41 30	90 40 89 80 89 20 88 60	93 00 92 50 92 00 91 60	94 90 94 60 94 20 93 80	96 40 96 10 95 80 95 50
25	40 60	88 00	91 10	93 40	95 20
26 27 28 29	39 90 39 20 38 55 37 90	87 40 86 80 86 10 85 50	90 60 90 10 89 50 89 00	93 00 92 60 92 20 91 80	94 90 94 60 94.30 93 90
30	37 25	84 90	88 50	91 30	93.60
31 36 41 46 51	36 60 33 60 30 80 28 30 26 10	84 20 80 90 77 50 74.10 70 80	87 90 85 10 82 20 79 10 76 10	90 90 88 60 86 10 83 30 80 60	93 20 91 30 89 20 86 80 84 30 81 80
56 61	24 IO 22 20	67 60 64 20	73 10 69 90	77 80 74 80	79 10
71 81	18 50 14 80	57 00 48 70	62 80 54 30	68 oo 59 6o	72 60 64 40
91 101	7 50	39 00 28 10	44 20 32 40	49 10 36 50	53 10
111	3 90 1 00	15.70	18 30	21 00	23 60 6 70

EXPLANATION OF TABLE 21

COMPOUND INTEREST

Amount of one dollar at interest compounded annually

In Table 21 the amount is given of one dollar at interest rates from 2 to 10 per cent, interest being compounded annually for any number of years up to 100.

The formula on which this table is based is the following:

Let A' represent the amount at the end of nth year of one dollar plus interest compounded annually.

Let n represent the number of years.

Let i represent the interest rate expressed decimally as 0.05 for 5 per cent.

Then
$$A' = (\mathbf{I} + i)^n \tag{23}$$

Example. — What is the amount in 7 years of \$400 at 6 per cent interest compounded annually?

From Table 21 the amount of \$1 at 6 per cent interest in 7 years is found to be \$1.503630 and the amount of \$400, therefore

$$$400 \times $1.503630 = $601.45.$$

The section of Table 21 covering interest rates in excess of 19 per cent will be found serviceable for certain special purposes such as the determination of the present value of mining properties. This section of the table has been abbreviated by the omission of certain individual years but it can nevertheless be made to serve in finding the amount of \$1 at compound interest for any year by going into the table with any two years whose sum is equal to the number of years for which the amount is to be determined and obtaining the product of values found in the table for these two years. This product will be the required amount.

Example. — What is the amount of \$1 at 15 per cent compound interest in 46 years?

The amount of \$1 for 16 years at 15 per cent is found to be \$9.357621.

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The amount of \$1 for 30 years at 15 per cent is found to be \$66.211772.

Then
$$9.357621 \times 66.211772 = $619.58$$
,

which is the amount of \$1 at 15 per cent in 46 years.

While occasion may rarely arise when it is necessary to know the amount of \$1 at these high rates of interest for a long term of years this section of the table is valuable for other purposes. It can be used to determine the present value of \$1 due at a future date; the amount of an annuity of \$1, and also the present value of an annuity of \$1.

The method of use for these purposes is explained in connection with the respective tables which follow.

To Illustrate. — What is the present value of a mine yielding \$10,000 net per annum whose estimated life is 10 years and which, owing to the hazards of the enterprise, should yield a net return of 20 per cent per annum?

From Table 21 it is found that \$1 at 20 per cent compound interest will amount to \$6.191736 in 10 years.

According to formula (27) in the explanation of Table 23 the amount of an annuity of \$1 for 10 years will be

$$\frac{6.191736 - 1}{9.29} = $25.95868.$$

And by formula (29) in the explanation of Table 24 the present value of an annuity of \$1 for 10 years

$$\frac{25.95868}{6.191736} = \$4.192149.$$

Consequently the value of the mine equipped to produce the net annual amount of \$10,000:

$$10,000 \times 4.192149 = $41,921.49.$$

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Amounts are noted for the end of each year)

Years.	2 per cent.	2½ per cent	3 per cent.	3½ per cent	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
I	\$1,020,000			\$1 035 000			\$1 o50 ooo
01			90			I 092 025	
~	.06I	1 o76 89I			I 124 864	1 141 166	1 157 625
4	I 082 432	1 103 813	1 125 509	I 147 523	1 169 859	1 192 519	1 215 506
ø	I 104 080	1.131 408	1 159 274	1 187 686	1 216 653	I 246 182	I 276 282
9						1 302 260	I 340 096
7							I 407 100
.∞	1 171 659	1 218 403		1 3.6 809	I 368 569	I 422 IOI	I 477 455
6	1 195 093		I 304 773	I 362 897	I 423 312		I 551 328
10	1 218 994	1 280 085	I 343 916	I 410 599	I 480 244	I 552 969	I 628 895
II		1.312 087			I 539 454	1 622 853	I 710 339
12		I 344 889			I 601 032	I 695 881	I 795 856
1.3		I 378 511		I 563 956	I 665 074	I 772 196	I 885 649
14	1.319 479	I 412 974	I 512 590	I 618 695		I 851 945	т 979 932
15	I 345 868		1 557 967	I 675 349	I 800 944	I 935 282	2 078 928
16		I 484 506	I 604 706		I 872 981	2 022 370	2 182 875
17	1 400 241		I 652 848		I 947 900	2 113 377	292
18			I 702 433		2 025 817	2 208 479	2 406 619
19	1 456 811		I 753 506	I 922 501	2 ro6 849	2 307 860	2 526 950
20	I 485 947		1 806 111	г 989 789	2.191 123	2 411 714	2.653 298
21	1 515 666	I 679 582					2 785 963
22	I.545 980	1 721 571		2 131 512	2 369 919	2 633 652	
23							
24	I.608 437		2 032 794		2 563 304		3 225 100
25	I 640 606	I 853 944	2 093 778	2 363 245	2 665 836	3 005 434	3 336 355

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued)

		7)	Amounts are noted for one	or tot our our	, , ,		
Year.	2 per cent	2½ per cent.	3 per cent	3½ per cent	4 per cent.	4½ per cent	5 per cent
90	\$1 672 ATS	1	951				\$ 3 555 673
0 10	27.073					-	
700	1.700 000		287	620		429	920
0 00	1 775 845	2 046 407	2 356 566	2 711 878	3 118 651	3 584 036	911
6	2 8 T S T			2 806 794	3 243 398	3 745 318	4 321 942
3	202 102						4 538 039
31	I 547 509	2 150 00/	2 500 000	2 000 2			4 764 941
32	1 004 541	2 203 /3/	2 3/3 33				
33	1 922 231	2 250 051	2 731 905	3 220 860	3 794 316	4 466 362	253
٠ 4 و	1.900 %	200 040 0	2 813 862	3 333 500	3 946 089		5 516 o15
90	ogo ggg. 1		200 0-0	390 000	1 100		707
36	2 039 887	2 432 535	2 898 278	3 450 200	4 103 933	2/0/13/0	6 081 407
37	2 080 685		2 985 227	3 571 025	4 200 030		285
38	2 122 299		3 074 783	3 090 011	4 430 013		20,7
39	2 164 745		3 167 027	3 825 372		300	4
40	2 208 040	2 685 064	3 262 038	3 959 260	4 801 021	816	7 039 989
1.7	2,0		2 259 899	4 097 834		101 870 9	7 391 988
4.			3 460 696			351	
4 4		891	3 564 517			637	149
5 V	2 300 053	2 963 808	3 671 452	4 543 342			557
46	2 437 854		3 781 596		5 841 176	7 248 248	8 985 008
ب	186	2 112 841	3 805 044		6 074 823	574	
40	2 400 011		4.011 895			7.915 268	9 905 971
** **	187 070		4 132 252		570	271	401
404	2 638 812	3 353 277	4 256 219	5 396 065	833	_	921
ිස	2 691 588	3 437 109	4 383 906	5 584 927	7 106 683	9 032 636	11 467 400

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued) (Amounts are noted for the end of each year)

12 22 42 42 42 42 42 42 42 42 42 42 42 42	\$2.745 420 2 800 328 2.856 335 2 913 461 2.971 731 3.031 165 3.031 789 3.153 624 3 216 697 3 281 031 3.346 651 3 413 584	\$3 523 036 3 611 112 3 701 390 3 788 773 3 988 773 4 .085 642 4 .187 783 4 .292 478	\$4 515 423 4 650 886 4 790 412 4 934 125 5 082 149 5 234 613 5 234 613 5 234 613	\$ 5 780 399	,	1 6	
**************************************	7,745 420 800 328 856 335 913 461 971 731 0971 731 153 624 216 697 281 031 413 584	523 611 701 793 888 888 888 187 187 292	515 650 790 934 934 082 234 391	5 780		730	
4444 4 6666	860 328 856 335 913 461 971 731 091 789 153 624 216 697 281 031 413 584	611 701 793 888 888 985 085 187 187	650 790 934 082 234 391	082	\$ 7.390 95I	453	\$12 040 770
4446666		701 793 888 888 985 085 187 187			7.686 589		642
4 4 60000		3 793 925 3.888 773 3.985 992 4.085 642 4.187 783 4.292 478		192	7 994 052	307	
4 0000	.971 731 .031 165 .091 789 .153 624 .216 697 .281 031 413 584	3.888 773 3 985 992 4.085 642 4.187 783 4.292 478		408	8 313 814	10 771 587	13 938 696
	031 165 091 789 153 624 281 031 281 031 413 584	3 985 992 4.085 642 4.187 783 4.292 478		6 663 141	8 646 367	11 256 308	14.635 631
	281 031 281 031 281 031 413 584	4.085 642 4.187 783 4.292 478				11 762 842	
	153 624 216 697 281 031 346 651 413 584	4.187 783		7 105 587	351	292	
	216 697 281 031 346 651 413 584	4.292 478			725	845	942
	281 031 346 651 413 584				115	423	789
	.346 651 413 584	4 399 790	5 891 603	7 878 091	10 519 627	14 027 408	981 679 81
	413 584	4.509 784		153	940	658	19 613 145
		4 622 529		8 439 208	11 378 029	15 318 280	20.593 802
	481 856	738	6 437 914		833	007	21 623 493
	551 493	4.856 545	6 631 051	9 040 291		727	22.704 667
	3.622 523	4.977 958	6 829 983	9 356 7or	12 798 735	17 480 702	23 839 9or
	694 974	5.102 407	7 034 882	9 684 185	310	267	25 031 896
	768 873	5.229 967	7 245 929	023	843	989	283
	844 251	5.360 717	7.463 307	373	14 396 836	r9 948 385	27 597 665
	3.921 136	5 494 734	7.687 206	10 737 029	972	846	977
	3.999 558	5.632 103	7 917 822	11 112 825	15.571 618	21 784 136	30 426 426
	079 549	772	8.155.357	501	16 194 483	764	947
	4 161 140	5.917 228	8 400 017	904	842	788	545
	244 363	6.065 159	8 652 or8	12 320 988	17 515 953		35 222 391
74 4.	.329 250	6.216 788	8 911 578	752	216		983
75 4.	4.415 835	6.372 207	9.178 926	13.198 550	18.945 255	27.146 996	38 832 686

TABLE 21, AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent	4½ per cent.	5 per cent.
94	1	531	454	999	703	368	774
77		6 694			20 491 187	645	813
78		862	030	633	310	646	953
.62	4 779 842	7 033 725	330	145	163	373	201
8		7 209 568	10 640 89r	15 675 738	23 049 799	33 830 096	49 561 441
81	4.972 948	7 389 807			1/6	352	939
82	5.072 407		288		930	943	641
83	5.173 855		627		25 927 889	38 605 760	373
84	5 277 332				965	343	242
85	5.382 879	8 156 964	12 335 709	18 617 859	28 043 605	42 158 455	254
98	5.490 536	360	705	269	165	055	417
87	5,600 347	8 569 911	086		33°	038	737
88	5.712 354	784	479	64I	545	109	224
89	5.826 6or	9 003 762	13 883 949	364	807	274	886
8	5.943 133	9 228 856	14.300 467		34 119 333		730
Ιb	966 190 9	450	729	886	484	106	994
05	6 183 236	969	1/1	687	903	371	905
93	6 306 900	9 938 469		516	379	953	455
25	6 433 038	981	095	374	914	651	128
96	6.561 699	10 441 604	19 22 91 PT	26.262 329	511		103 034 676
96		10.702 644		181	43 171 841	68 416 977	186
97			17 587 771	132	898	495	595
86		244	115	111	694	713	275
66		525	658	136	562	075	239
2	7 044 646	11 812 716	10 218 622	21 101 408	50 504 048	81 <88 <18	121 501 258

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
Н	\$1.055 000	\$1 060 000	\$1 065 000	\$1 070 000	\$1 080 000	000 060 I\$	\$ 1 IOO OOO
- 01	1 113 025						I 210 000
~	1.174 241	910 161.1	207			I 295 029	I 331 000
4	1.238 825	I 262 477	т 286 466	1 310 796	т 360 489	1 411 582	1 464 100
2	I.306 960	I 338 226	1 370 087	I 402 552	т 469 328	I 538 624	1 610 510
9	I 378 843	1 418 519	I 459 142		I 586 874	I 677 100	1 771 561
7	I.454 679		I 553 987		1 713 824		
. ∞	I.534 687	I 593 848	I 654 996		I 850 930		2 143 589
6	1.619 o94		I 762 570	I 838 459	r 999 005		
10	1.708 144	1 790 848	1 877 137	1 967 151	2 158 925	2 367 364	2 593 742
н	1,802 092	I 898 299	151 666 I	2 104 852	2 331 639	2 580 426	853
7	1.901 207	2 012 196	2 129 096	2 252 192	2 518 170	2 812 665	
~	2.005 774	2 I32 928	2 267 488	2 409 845	2 719 624	3 065 805	452
4	2,116 091	2 260 904	2 414 874	2 578 534	2 937 194	3 341 727	797
15	2.232 476	2 396 558	2 571 841	2 759 032	3 172 169	3 642 482	4 177 248
9	355	540	739	952	425	3 970 306	
7	2 484 802	2 692 773	2 917 046	3 158 815	3 700 018	4 327 633	5 054 470
. 00	2 621 466	2 854 339	3 106 654	379	966	4 717 120	559
- 6r	2 765 647	3 025 600	3 308 587		4 315 701	5 141 661	115
	2.917 757	3.207 135	3 523 645	3 869 684	4 660 957	5 604 411	6 727 500
	3.078 234	3 399 564	3 752 682	4 140 562	5 033 834	808 801 9	400
64	3 247 537		3 996 606			6 658 600	
53	3 426 152	3 819 750	4 256 386	4 740 530	5 871 464	7 257 874	8 954 302
24	3.614 590		4 533 o51			7 911 083	
10	2.812 202	4 291 871	4 827 699	5 427 433	6 848 475	8 623 o81	IO 834 706

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued) (Amounts are noted for the end of each year)

Years.	5½ per cent.	6 per cent.	6½ per cent	7 per cent.	8 per cent.	9 per cent	10 per cent
\ \ \		077	141		396	399	816
ء د	4 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	822	5 475		988	10 245 082	13 109 994
~~		III	831		627	191	420
	4 477 045	5 418 388	210	7 114 257	9 317 275	172	863
် ဇ္ဇ	4 983 951	743	6 614 366	7 612 255	10 o62 657	13 267 678	17 449 402
		988		145	867	461	194
		453		715	737	15 763 329	21 113 777
1 6				325	949	182	225
2 4	6 174 242	7 251 025		9 978 114		728	547
. 10	6 513 825		9 062 255	10 676 581	14.785 344	20 413 968	
٠	6 872 085			423	996	25I	912
4.0			278	223	17 245 626	24 253 835	903
-00	648		946	620	625	436	404
6	8.069 487		11 658 286	13 994 820	115	815	144
		10 285 718	12 416 075	974	724	31 409 420	45 259 256
-	180	000	223		23 462 483	236	785
	475	557	082	144	339	317	763
. ~	900	250	400	344	366	929	240
o 4	10 546 497		15 972 862	19 628 460	555	336	264
45		13 764 611	OII	005		48 327 286	890
و		100			474		80 I79 532
1	384	465	294	045	232	417	197
-00	90	303	848	728	210	585	Žιο
. 6	13 783 849	17 377 504	884	27 529 930	427	217	718
28	741	420	22 206 679		46 901 613	74 357 520	117 390 853

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent
17	341	525	821	519	653	_	129
22	183	696	435	725		344	142 042 932
23	270	938	153	980	082	295	247
54			983	612	809	196	871
22	19.005 762	24 650 322	31 932 170	41 315 001	913		020
92	ogi	129	200	202			965
57	153	697	218	301	38I	928	, 197
.82	317	358	572	612	811	162	637
59		120	620	155	756	496	Š
9	24 839 770		749	946	257	031	
19	26 205 958	996	593	002	357	874	929
62	27.647 286	964	622	342	106	142	422
63	29.167 886	288	847	986	554	965	265
64	30.772 120	646	282	955	759	482	16/
99	32 464 587		941	272	148 779 847	270 845 963	490 370 725
99	34.250 130	793	837	196	682	222	407
67	36.133 896	601 601	986	049	536	792	348
68	38 121 261	577	405	562	419	753	683
69	40 217 930	732	112	532	413	32I	951
20	42.429 916	075	124		909	730	746
71	44 763 562	620	462	896	236 094 918	235	721
72	225	377	147	206	982	117	593
7.3	822	360	202	641	38I	677	153
74	52.563 226	582	650	416	411	248	268
22	\$5.454 204L	79 o56 921	112 517 632	159 876 019	321 204 530	641 190 893	1271 895 371

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (Continued)

10 per cent.	\$ 1,399 084 909	993	892	182	2,048 400 215		2,478 564 260	470	002	3,298 969 030	865	752	927	070	022	324	757	633	266	8,556 676 047	343		935	12,527 829 400	13,780 612 340
9 per cent	898	298	360	905 093 273	986 551 668	-	122	613		1517 932 029	545	455	765	684	2335 526 582	723	839	574	3296 786 373	3593 497 147	3916 911 890	433	683	-	5529 040 792
8 per cent	900	652	625	436 995 217	.71 954 834		488	527	680	693 456 489	933	847	555	439	1018 915 089	428	462	539	1386 222 730	120	1616 890 192	241	940	815	2199 761 256
7 per cent.	290	042	854	209 564 848	224 234 388	930	725	969	293 925 540	500		07I	276	245		980	\tilde{o} 18	370	578 196 026		_	314	897	949	867 716 326
64 per cent.	831	620	915		154 158 907		850	216	320	211		9	132	271 715 924	377	186	219	7 7	372 274 364		242	688	816	510.048 142	543 201 271
6 per cent.	8	828	158	99 807 541		143	872	- 8	133 565 004	.578	073	0,78	622	740	189 464 511	832	882	655	194		750	884	477	320 096 305	302
5½ per cent.	407	61 721	911	68.698 034	- 2	76.462 620	668	104	785	723	033	420	228	117.346 167	800	600	792	37I	153.366 747		701	080	.004	200.444 205	
Years.	94	77	28	2 62	8	81	82	83	8,	98	98	87	88	8	8	Ιb	92	93	3 2	96	99	07	8	. 6	100

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COM-POUNDED ANNUALLY (Concluded)

Years.	12 per cent.	15 per cent	20 per cent	25 per cent.
1 2 3 4	\$1 12 1 2544 1 404928 1 573519	\$1 15 1 3225 1 520875 1 749006	\$1 20 I 440 I 7280 2 07360	\$1 25 1 5625 1 953125 2 441406
5	1 762342	2 011357	2 488320	3 051758
6 7 8 9	1 973823 2 210681 2 475963 2 773079	2 313061 2 660020 3 059023 3 517876	2 985984 3 583181 4.299817 5 159780	3 814697 4 768372 5 960464 7 450581
10	3 105848	4 045558	6 191736	9 313226
11 12 13	3.478550 3.895976 4.363493 4.887112	4.652391 5 350250 6 152788 7 075706	7 430084 8 916100 10 699321 12 839185	11 641532 14 551915 18 189894 22.737368
15	5 473566	8.137062	15 407022	28 421709
16 17 18	6 130394 6.866041 7.689966 8.612762	9.357621 10 761264 12 375454 14 231772	18 488426 22 186111 26 623333 31 948000	35 527137 44 408921 55 511151 69 388939
20	9.646293	16.366537	38 337600	86 736174
21 22 23 24	10 803848 12 100310 13 552347 15 178629	18 821518 21 644746 24.891458 28.625176	46.005120 55.206144 66.247373	108 420217 135 525271 169 406589 211 758236
25	17 000064	32 918953	95 396217	264.697796
30	29.959922	66 211772	237.376314	807.793567
35	52 799620	133.175523	590 668229	2,465 190326
40	93 050970	267.863546	1,469.771568	7,523.163845
45	163.987604	538.769269	3,657.261988	22,958 874023
50	289 002190	1,083.657442	9,100 438150	70,064 923216
60	897.596933	4,383 998746	56,347 514353	652,530 446800
70	2,787.799828	17,735 720039	348,888 956932	6,077,163 357286
80	8,658.483093	71,750.879401		56,597,994 242667
90	26,891 934202		13,375,565.248934	527,109,897.161526
100	83,522.265659	1,174,313.450700	82,817,974.522015	4,909,093,465.297726

TABLES 313

EXPLANATION OF TABLE 22

THE PRESENT VALUE OF ONE DOLLAR DUE AT A FUTURE DATE

The present value of \$1 due at some future time is the sum which placed at compound interest will amount to \$1 at that time.

The formula on which Table 22 is based is as follows:

Let P represent the present value of \$i due at the end of n years.

Let n represent any number of years.

Let i represent the interest rate expressed decimally as 0.05 for 5 per cent.

Then
$$P = \frac{1}{(1+i)^n}$$
 (24)

Table 22 has been prepared for only a few selected years because the present value of \$1 due at a future time is also readily obtainable from Table 21. According to equation (23), A' may be substituted for $(1+i)^n$, equation (24) may then be written:

$$P = \frac{\mathbf{I}}{A'} \tag{25}$$

That is to say, the present value of \$1 due at any future time is the reciprocal of \$1 at compound interest for the same time.

Example. — What is the present value of \$600 due in 8 years at 4 per cent interest?

From Table 21 the amount of \$1 at 4 per cent compound interest in 8 years is \$1.368569, consequently the present value of \$1 due in 8 years will be

$$1 \div 1.368569 = 0.730690$$

and the present value of \$600 due in 8 years at 4 per cent will be

$$0.730690 \times 600 = $438.41.$$

Note. — To find the present value of r due at the end of any number of years r not noted in this table, select two or more lines from the table the sum of whose years is equal to r

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and multiply the values found in the interest column on these lines.

Example. — What is the present value of \$1 at 5% due in 36 years?

Present value of \$1 due in 30 years at 5% = 0.231377Present value of \$1 due in 6 years at 5% = 0.746215 $0.231377 \times 0.746215 = 0.172657 the required present value.

THE PRESENT VALUE OF ONE DOLLAR DUE AT A FUTURE DATE TARLE "

Years.	2 per cent.	2½ per cent.	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent.
н	í	\$ 975 610			\$ 961 538	\$ 956 938	\$ 952 381
8		176.	.942 596		.924 556		907 029
~		928 599			966 888		
4	.923 845	.905 951	. 888 487	871 442	854 804	.838 561	
10	.905 731	.883 854	.862 609	841 973	821 927	.802 451	783 526
9		.862 297	.837 484		790 315	264 896	746 215
7		.841 265	813 092		816 624.	.734 828	710 681
· ∞	.853 490	.820 747	.789 409	.759 412	730 690	703 185	676 839
6		.800 728	.766 417		.702 587	.672 904	644 600
10	.820 348	861 184	744 094	918 804	.675 564	.643 928	.613 913
9	.743 015	990 466	.641 862	296 891	555 265	.516 720	481 017
8	.672 971	610 271	.553 676	.502 566	.456 387	414 643	376 889
22	.609 531	.539 391	.477 606	.423 147	375 117	332 731	.295 303
<u> </u>	552 071	.476 743	.411 987	.356 278	308 319	000 292.	231 377
200	.500 028	.421 371	355 383	299 977	.253 415	.214 254	181 290
<u></u>	.452 890	.372 431	306 557	252 572	208 289	171 929	.142 046
10	.410 197	.329 174	.264 439		171 198	137 964	111 297
8	.371 528	. 290 942	.228 107	.179 053	.140 713	017 011.	087 204
8	.304 782	.227 284	.169 733	.126 934	.095 060	.071 289	053 536
2	.250 028	.177 554	.126 297	986 680	.064 219	.045 905	032 866
8	205 110	.138 705	093 977	.063 793	.043 384	.029 559	020 177
8	.168 261	108 356	826 990.	.045 224	.029 309	.019 034	012 387
001	.138 033	.084 647	.052 033	032 060	оо8 б10.	012 257	.007 604

Ĥ	TABLE 22. THE	PRESENT	VALUE OF ONE	DOLLAR	DUE AT A FUT	FUTURE DATE	(Continued)
Years.	5½ per cent.	6 per cent.	$6^{1\over 2}_2$ per cent.	7 per cent	8 per cent.	9 per cent.	10 per cent
1		\$ 943 296			\$ 925 926	\$ 917 431	
1 01		889	881	873		841 680	826 446
٠,	851 614	.839 6rg	827 849	816 298	793 832	.772 183	751 315
4		792 094	777 323			.708 425	
2	.765 134	747 258	.729 881	712 986	.680 583	.649 931	620 921
. 9		704 961		.666 342	630 170	296 267	564 474
7	.687 437	.665 057	643 506	622 750	583 490	547 034	.513 158
∞		627 412		582 009	540 269	.501 800	400 507
6		868 165.		543 934	500 249	460 428	424 098
10	585 431	.558 395	532 726	508 349	.463 193	422 411	385 543
15	.447 933	417 265	388 827	362 446	315 242	274 538	239 392
20	.342 729	311 805	283 797	258 419	214 548	.178 431	.148 644
25	.262 234	.232 999	207 138	.184 249	146 018	115 968	.092 296
30	.200 644	174 110	151 186	131 367	099 377	075 371	057 309
35	.153 520	.130 105	110 348	093 663	.067 635	048 986	.035 584
9	.117 463	097 222	o8o 541	086 780	046 031	031 838	022 095
45	.089 875	.072 650	058 785	047 613	.031 328	020 692	013 719
20	794 890.	.054 288	042 906	033 948	021 321	or3 449	618 800
09	.040 258	030 314	022 857	017 257	948 600	005 681	003 284
22	.023 569	.oi6 927	.012 177	008 773	004 574	002 400	oo1 266
80	.013 798	009 452	006 487	004 460	002 119	001 014	000 488 2
8	840 800.	.005 278	.003 456	.002 267	.000 981 4	.000 428 2	000 188 2
90	.004 729	.002 947	.001 804	001 152	000 454 6	000 180 9	000 072 6
-							

EXPLANATION OF TABLE 23

THE AMOUNT OF AN ANNUITY OF ONE DOLLAR

An annuity is a sum uniform in amount due annually.

The amount of an annuity in any term or number of years is the sum of the several annual installments with interest thereon during the term compounded annually.

Table 23 shows the amount of an annuity of one dollar paid at the *end* of each year with the earned interest increments of each year added at the end of the year.

To find the amount of an annuity of one dollar paid at the beginning of each year subtract \$1 from the figures noted in the table and the result will then apply at the beginning of each year, or, which is the same, at the end of the preceding year. Thus for 6 per cent interest at the beginning of the year 1, the amount is zero; at the end of year 11, or beginning of the year 12, it is \$16.87 - 1.00 = \$15.87.

The values given in Table 23 are calculated by the following formula:

Let A'' represent the amount of an annuity of one dollar paid at the end of each year.

Let *i* represent the interest rate expressed in percentage, as 0.05 for 5 per cent.

Let n represent the number of years.

Then

$$A^{\prime\prime} = \frac{(\mathbf{1} + i)^n - \mathbf{1}}{i} \tag{26}$$

and it follows from equation (23) that:

$$A^{\prime\prime} = \frac{A^{\prime} - \mathbf{I}}{i} \tag{27}$$

The amount of an annuity of \$1 paid at the end of each year can be found, in other words, from any table giving the amount of \$1 at compound interest by subtracting \$1 from the amount found in the compound interest table and dividing the remainder by the rate of interest expressed decimally.

Example. — What is the amount of an annuity of \$35 at the end of 17 years at 5 per cent interest?

From Table 23 the amount of an annuity of \$1 for 17 years at 5 per cent is found to be \$25.84037, consequently the amount of an annuity of \$35 will be

$$35 \times 25.84037 = \$904.41$$
.

Or from Table 21 the amount of \$1 at 5 per cent compound interest for 17 years is found to be 2.292018, therefore, according to equation (27),

$$A'' = \frac{2.292018 - 1}{0.05} = 25.84036$$

and the amount of the annuity of \$35 will be

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$$35 \times 25.84036 = $904.41$$
.

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR Amounts are noted for the end of each year)

		TX 7	nounce are noted	Minounity are noted for one on each year	cacar y car,		
Years	2 per cent	2½ per cent.	3 per cent.	3½ per cent.	4 per cent	44 per cent.	5 per cent.
Н	8	8	\$ 1 000 00	\$ I 000 00	\$ I 000 00	\$ I 000 00	
. 01	2 020	2 025	030	2 035 00	2 040 00	2 045 00	
8	990	075		901		3 137 03	3 152 50
4		4 152 52	4 183 63	4 214 94	246	4 278 19	310
ıo	5 204 04	5 256 33	5 309 14	5 362 47	416	5 470 71	5 525 63
پ	308	387		6 550 15		6.716 89	16 108 9
4 0	434	547	662	779	898	019	142
-00		8 736 12	8 892 34	9 051 69		380	9 549 II
6	9 754 63	9 954 52		368	582	802	026
91		11 203 38	11 463 88	11 731 39	12 006 11	12 288 21	12 577 89
II	168	483	807	141	486	13 841 18	206
I I	412	13 795 55	14 192 03	14 601 96	15 025 81	15 464 03	ī2 617 ī3
13	680	140	617	113	626	159	712
. 4	15 973 94			66 949 LI	291	932	598
12	293	17 931 93	18 598 91	19 295 68	20 023 59	20 784 05	21 578 56
91		380	156	1/6	824	719	657
17	012	864	761	705	697	741	840
18	21 412 31		23 414 44	24 499 69		26 855 08	28 132 38
19	840	23 946 or	911	357		063	539
80	24 297 37	25 544 66	26 870 37	28 279 68	29 778 08	31 371 42	33 065 95
21	783	183	9/9	269	696	783	719
22	298	862	536	328	247	303	505
23	28 844 96	30 584 43	32 452 88	34 460 4I	36 617 89	38 937 03	41 430 48
24.			426	999	082	689	202
22	32 030 30	34 157 76	36 459 26	38 949 86	41 645 91	44 565 21	47 727 10

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (Continued)

	5 per cent.	\$ 51 II3 45	699	402	322	66 438 85		298	903	900	320	95 836 32	628	709	095	120.799 77	127 839 76	23I	993	143		168 685 16	119	025	426	209 348 00
	4½ per cent.	\$ 47 570 64	711	.993		61 007 07	64 752 39	999	750	030		86 163 97	041	138	464		112 846 69	924		913	138 849 97	960		587	859	178 503 03
each year)	4 per cent.		084	296	996	56 084 94	59 328 34	701	500	857	73 652 22	77 598 31	702	970	400	95 025 52	99 826 54			412	121 029 39	870		263	145 833 73	152 667 08
Amounts are noted for the end of each year)	3½ per cent	313	759	290	48 910 80	51 622 68	429	334	34I	63 453 15	674	007	457		80 724 91	84 550 28	509	607	848	238	105 781 67		350	388	125 601 85	130 997 91
mounts are note	3 per cent.	553	709	930	45 218 85	575		502	077	730	60 462 08		174	159	234		663	023	483	89 048 41	92.719 86	SOI		408	108 540 65	112 796 87
	2½ per cent	OII	012	850	41 856 30			150	354		928	301	733	227	782	67.402 55	087	839	660	552		554	667	859	94 131 07	97 484 35
	2 per cent	670	344	120	38 792 23	568	42 379 44	44 227 03	46 III 57	48.033 80	49.994 48	51.994 37	54.034.25	56 114 94	58 237 24	60 40I 98	62 610 02	64 862 22	67.150 47	69.502 66	71.892 71	220	817	353	81 940 59	
	Years.	26	2.7	200	20	တ္ထ	31	25	3.3	3 45	32	36	2 6	300	39	9	1,	- 6	43	3 4	45	94	47	- 87	49	26

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (Continued) Amounts are noted for the end of each year)

	ישחווונצי	Amounts are noted for the end	Amounts are not	Amounts are noted for the end of each year)	f each year)		
Year.	2 per cent	2½ per cent	3 per cent	3½ per cent	4 per cent	4½ per cent.	5 per cent.
Şī	270	921	82	\$136 582 84		\$187 535 66	815
22	910	444	969	363	164	974	856
, T.	816	108 055 6I	347	345	174 851 31	838	498
54	95 673 07	757	131 137 49	538	845	146	
92		115 550 92	1/0	160 946 89	191 159 17	227 917 96	712
95	558	•	153	580	199 805 54	174	348
52	88	425	388	445	797	937	
.80	68 ₁	511	780	550	149	229	851
39	834	131 699 11	157 333 43	905	875	074	794
8	114 051 54	166	163 053 44	196 516 88	237 990 69	289 497 95	583
19	332	391	945	349		525	
62		901	013	548	450	184	876
63	092	523	263	988	828	502	469
64	574	261	701	722	199	200	093
99	131 126 16	159 118 33	194 332 76	238 762 88	968	237	798
99	748	960	162	119	194	718	637
67	443	198	197	803	077	985	
89	212	428	443	826	920	075	953
69	050	789	906	200	317	023	520
20	149.977 91	185 284 II	594	288 937 86	364 290 46		588 528 51
71	977	916	238 511 89		862	653	
72	057	689	299	552	026	418	902
73	162 218 16	202 606 35	290	323 456 80	898	207	447
74	462	149 1	719	777	414	990	670
2/2	170 791 77	214 888 30	272 630 86	348 530 01	448 631 37	581 044 36	

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (Continued) Amounts are noted for the end of each year)

Year.	2 per cent.	2å per cent	3 per cent.	3½ per cent	4 per cent	4½ per cent.	5 per cent.
9	1 -	1	\$281 809 8	\$361 728 6	\$ 467.576 6	\$ 608 191 4	\$ 795 486 4
7	8 117 971		291 264 I		487 279 7		836 260 7
			301,0020		507 770 9		879 073 8
6	188 992 I	241 349 0	311 032 1		529 081 7		924 027 5
8	193 772 0	248 382 7	321 363 0	419 306 8	551 245 0		971.2288
H	198 647 4		332 003 9		574 294 8		1020 790 3
7	203 620 3		342 964 0		598 266 6		8 628 2401
3	208.6928		354 252 9		623 197 2		1127 471 3
4	213.8666		365 880 5		649.125 I		1184 844 8
10	219 143 9		377.8570		1 обо 949		1245 087 1
9	224 526 8		390 192 7		704 133 7		1308 341 4
7	230 017 4		402 898 4		733 299 I		1374 758 5
.00	235 617 7		415 985 4		763 631 0		1444 496
6.	241.330 I		429 465 0		795 176 3		1517 721 3
0	247.1567		443 348 9		827 983 3		1594 607
—	253 099 8		457.649 4		862 102 7		1675 337
7	259 161 8		472 378 9		897 586 8		1760 104 (
3	265 345 0		487 550 2		934 490 2		849 rog 8
4	271 651 9		503 176 7				1942 565
ص	278.0850	377.664 2	519 272 0	721 780 8		1432 684 3	2040 693
9	284.646 7		535 850 2		1054 296 0		2143 728
7	291.339 6		552 925 7				2251 914 (
 	298 166 4		570.513 5				2365 510 3
6	305 129 7	421 023 I	588 628 9		•		2484 785 9
100	219 929 2	432.548 7	607 287 7		1237 623 7	1790 846 0	2610 025 2

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (Continued) Amounts are noted for the end of each year)

			Timoring at a magnitude	100 000 000	(mar) ama		
Year.	5½ per cent.	6 per cent	6½ per cent.	7 per cent.	8 per cent	9 per cent	Io per cent
I	8	8	8	\$ 1 000 00		00 000 I \$	\$ I 000 00
1 01	2.055	2 060		2 070 00		960	001
~	3.168 03			214	3 246 40	3 278 10	3 310 00
4	4 342 27	4 374 62	407	4 439 94	206	573	641
2	5 581 09	5 637 09	5 693 64	5 750 74	5 866 60	5 984 71	or Sor 9
9	888	975	063		335	523	715
7	8 266 89			8 654 02	8 922 80	9 200 43	9 487 17
-∞	721				IO 636 63	028	435
6		11 491 32	11 731 85		12 487 56	13 021 04	
10	12 875 35	13 180 79	13 494 42	13 816 45	14 486 56	15 192 93	15 937 42
11	583	170	37I			17 560 29	
12	16.385 59	16 869 94	17 370 71	17 888 45			21 384 28
13	286	882	499	140	495		522
14	292	o ₁₅	167	550	24 214 92	610	974
15	22 408 66	23 275 97	24 182 17	25 129 02	27 152 11	29 360 92	31 772 48
91	641	672		888		003	
17	966	212		30 840 22		36 973 70	40 544 70
18	48I	905		666	450	301	599
61	32 102 67		35 516 72	37.378 96	446	810	159
20	898	36 785 59	38 825 31	40 995 49	45 761 96	51 160 12	57 275 00
21	37 786 08	992	348		422	764	64 002 50
22	864	392	ioi	8	456	873	402
23	III	46 995 83	50 098 24	53 436 14	60 893 30	69 531 94	
24	538	815	354	126	764	789	497
25	\$1 152 59	54 864 51	58 887 68	63 249 04	73 105 94	84 700 90	98 347 06

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23; AMOUNT OF AN ANNUITY OF ONE DOLLAR (Continued) Amounts are noted for the end of each vear)

Year.	5½ per cent.	6 per cent	6½ per cent	7 per cent.	8 per cent.	9 per cent.	Io per cent
90	390	951	71.5	929	954		181
27	180.080	63	68.856	74 483	350	102 723 13	
28	233	228	332	697	338	968	209
29	67 711 35	73.639 80	164	87 346 53	103 965 94	135	630
8		058	86 374 86	94 460 79		136 307 54	164 494 02
31	419		989	073		575	943
32	677	880	33	110 218 15	213	164 036 99	
33	224	343	.535	933	950	800	25I
34	94 077 12	183	115 525 53	258		982	476
35	25I	111 434 78		236	172 316 80	215 710 75	271 024 37
36	765	120	960	148 913 46	102		126
37	637	268	748	337		375	039
38	887	406	026	561	315	629	043
39	128.536 13	058	973	640	941	990	447
40		154 761 97	631	1199 635 11		337 882 44	442 592 56
-	118	047	047	214 609 57	781	291	851
42	001	0.00	271	632	243	528	636
4.2	777	107	353	226	583	845	9
3 4	173 572 67	199 758 03				5^{21}	640
45		743		285.749 31	386 505 62	525 858 73	904
46	245	508	263 335 68	751	426		791 795 32
47	.984	86	452	224	9	862	974
.84	368	564	746	270	132	280	172
49	232 433 63	272 958 40	295	378 999 00		865	189
20	217	290 335 90		406 528 93	573 770 16	815 083 56	1163 908 53

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (Continued) Amounts are noted for the end of each vear)

		7	Amounts are not	Amounts are noted for the end of each year)	each year)		
Year,	5\} per cent.	6 per cent.	6½ per cent.	7 per cent	8 per cent.]	9 per cent.	10 per cent
1	1	756	486	985		\$ 889 441 I	
, S.	657	300 730	201 207	202	671 325		_
22	101.	100	7 7 7	220			
53	77 292 262	340 970 31	806 28	527 216 44	785 114 I	1155 130 1	1708 719 5
54	302	176	960	5			
22	327 377 49	394 172 03	475 879 53	575 928 59	٠	_	
4	282	822	811	243		1374 500 6	2069 650 6
O I	3,5	041	810	450			
, , ,	4 ×	220	027	752			
ရှိ ကို	307 300 21	402 007 72	616 610 18	759 364 84	1159 456 8		
60		128			1253 213 3	1944 792 г	3034 816 4
3	5	1		,66			
19	3		439	204	1334 470 4		2674 227 8
62	496	082	033	409	020		
63	143	147	655		934		
64	541 311 27	677 436 66	850 503 03	799			
. 18			906 785 72	1146 755 16	1847 248 I	2998 288 5	4893 707 3
3 6	,	2	406	820			
oo •	.547	777	2 4	080			485
62	798	6,27	7 7 7	030	2330 247 0	3886 148 6	
8 6	712 052 27	012 200 16	1170 956 49	1507 602 03	2517 666 7		
3 6	5 2	022	990	134	2720 080 I	4619 223 2	
2	7/7	S					
71		1027 008 10	1330 193 13	1/20 125 5/	2930 000 3	1933 933 3	0545 938 2
72	464	628	055	S S			
73	9	900	803	298			10301.334
74	937 513 20	366	005	240	.145	•	
75		1300 948 68	1715 655 88	2269 657 42	4002 556 6	7113 232 I	12708 953 7
2	-						

(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (Concluded) Amounts are noted for the end of each year)

\$30 6 \$1380 co5 6 \$1828 173 5 \$ 2429 533 4 \$ 4323 761 2 \$ 7754 423 0						The state of the s		
\$1045 530 6 \$1380 005 6 \$160 600 8 \$104 034 8 \$105 05 7 \$104 034 8 \$105 05 7 \$105	Year.	5½ per cent	6 per cent.	6½ per cent	7 per cent	8 per cent.	9 per cent.	10 per cent
1104 034 8	94	ı	1	5	1	4323	7754	\$ 13980 849 г
1165 756 7 1552 634 3 2075 624 9 2783 642 8 5045 315 0 9215 120 1230 873 4	11	-				662	-	
1230 873 4 1646 792 4 2211 546 2979 497 8 5449 940 2 10045 588 935 4 10050 574 1239 571 4 1746 599 9 2356 290 7 3189 667 7 588 935 4 10950 574 1372 047 8 1864 539 6 2674 628 938 7419 88 11937 125 1468 589 11937 125 1468 589 11937 125 1468 589 11937 125 1468 688 605 14184 589 1183 1468 589 1468 1468 589 1468 589 1468 589 1468 589 1468 589 1468 589 1468 589 1468 589 1468 589 1688 1468 589 1688 1688 1688 1688 1898 1898 1898 1898 1898 1898 1898	200					315		
1299 571 1746 599 2356 290 7 588 935 10950 574 1372 047 1852 395 2510 449 3413 297 6358 890 11937 125 1372 047 1964 539 628 3413 297 1568 6015 11937 125 1448 510 2684 479 3909 953 8 7419 868 6015 14184 589 14184	20/					940	-	
1372 047 8 1852 395 9 2510 449 6 3413 297 1 6358 890 2 11937 125 1448.510 4 1964 539 6 2674 628 9 -3653 227 9 6868 601 5 13012 467 1529 178 5 2083 412 0 2849 479 8 3909 953 8 7419 089 6 14184 589 1614 283 3 2209 416 7 3035 695 9 4184 650 6 8653 706 1 16854 800 1704 068 9 2342 981 7 3234 016 2 4478 576 1 8653 706 1 16854 800 1704 068 9 2342 981 7 3234 016 2 4478 576 1 8653 706 1 16854 800 1708 723 723 3 2484 560 6 3445 227 3 4795 951 8 100906 943 3 2183 73 215 344 9 2484 563 3 3707 167 0 5789 63 2 100906 943 3 2183 73 2232 731 0 3141 075 2 4436 576 2 5874 939 7 11780 498 7 23796 499 2232 731 0 3141 075 2 4436 576 2 6287 185 4 12723 938 6 25939 184 2232 731 0 3141 075 2 4436 576 2 6287 185 4 1775 393 6 25939 184 22487 140 4 3531 372 1 5034 140 6 7200 268 6	: &					935		20474 002 I
1748 1964 539 6868 601 5 13012 467 1529 178 2084 479 8 3909 953 8 7419 089 14184 589 1614 283 412 2849 479 8655 6 14184 589 1704 068 2342 981 3234 016 4478 576 16853 702 1688 865 702 1688 865 1688	8.					890		-
1529 1785 2683 472 2849 479 8 3909 953 8 7419 089 14184 589 1544 283 4184 650 6 8013 616 8 15462 202 1704 068 9 2342 981 7 3234 016 8 15465 70 16854 80 1798 726 3445 227 4478 576 16854 80 16854 80 1898 726 3445 277 3445 577 1789 9349 165 16854 80 1898 726 364 379 3467 3789 378 378 378 378 378 378 378 378 378 378 378 378 378 378 378 489 378 378 378 378 378 378 378 378 378 378 378 <td< td=""><td>2 %</td><td></td><td></td><td></td><td></td><td>109</td><td></td><td>24775 642 6</td></td<>	2 %					109		24775 642 6
1614 283 2209 416 7 3055 695 4184 650 8013 616 8154 202 1704 068 9 2342 981 7 3234 016 4478 576 1 16854 80 1798 726 3445 227 3445 227 3499 162 16854 80 1898 726 3 360 167 369 172 3489 162 18372 73 2004 156 3 2793 727 3489 1682 188 1698 95 183 183 183 183 188 183 184 183 184 183 184 183 184 183 184 183 184 183 184 183 184 183 184 183 184 183 184 183 184 183 184 184 183 184 184 184<	20.00					680		
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1798 792 7 2484 560 6 3445 227 3 4793 076 5 9349 162 6 18372 732 1898 792 7 2634 634 3 3670 167 0 5129 591 8 10098 095 6 20027 278 2004 156 3 2634 634 3 3909 727 9 5489 663 2 10906 943 3 21830 733 2115 384 9 2962 335 1 4164 860 2 5874 939 7 11780 498 7 23796 499 2232 731 0 3141 075 2 4436 576 2 6287 185 4 12723 938 6 25939 184 2356 531 2 3330 539 7 4725 953 6 6728 288 4 13742 853 7 28274 710 2487 140 4 3531 372 1 5034 140 6 7200 268 6 14843 282 0 30820 434 2487 140 4 3531 372 1 5034 140 6 7705 287 4 1701 744 6 33595 273 2770 304 5 3744 254 4 5362 359 8 7705 287 4 1731 44 6 33591 64 2923 671 2 4209 104 2 6084 187 5 8823 853 5 18701 506 9 39916 635 3085 473 2 4462 650 5 6480 659 7 10104 499 9 21815 517 6 4747 044 3626 258 3 5318 271 8 7831 509 7 11570 712 0	. 78					901		32979 690 3
1898.726 2634 634 3670.167 5129 591 10098 095 20027 278 2004.156 2793 712 3909 727 9 5489 663 10906 943 21830 737 215 324 939 7 1464 860 5874 939 11780 498 7 23796 493 11780 498 7 23796 493 11780 498 7 23796 493 11780 498 7 23796 493 11780 498 7 23796 493 11780 498 1780 499 1780 499 1780 499 18796 499 1879 1876	98	7 207 8071		3445 227 3		162		36278 659 3
2004 156 3 2793 712 3999 727 9 5489 663 10906 943 21830 733 2115 384 9 2962 3351 4164 860 5874 939 11780 498 7 23796 499 7 11780 498 7 23796 499 7 11780 498 7 23796 499 499 499 18742 837 23796 499 2582 18742 837 23796 499 18742 8824 13742 8827 1874 18824 <t< td=""><td>× 50</td><td>1808.726.3</td><td></td><td>3670.1670</td><td></td><td>962</td><td></td><td>39907 525 2</td></t<>	× 50	1808.726.3		3670.1670		962		39907 525 2
215 384 9 2962 335 4164 866 5874 939 7 11786 498 7 23796 499 2232.731 3141 075 4436 576 6287 185 12723 938 6 25939 184 2356.531 3330 539 4725 953 6728 288 13742 853 7 2874 770 364 140 6728 288 13742 853 7 2874 770 364 3620 437 3620 437 770 364 365 7 770 368 14843 282 7 3820 437 7 36820 437 7 36820 437 43620 437 36820 437 7 36820 437 7 36820 437 36820 437 36820 437 36820 437 36820 437 36820 437 36820 437 4482	\& \$,&	2004 156 3		727			733	
2232.731 O 3141 O75 2 4436 576 2 6287 185 4 12723 938 6 25939 184 2356.531 2 3330 539 7 4725 953 6 6728 288 4 13742 853 7 28274 710 2467 140 4 3531 372 1 5034 140 6 7200 268 6 14843 282 0 30820 434 2624.933 2 3744 254 4 5362 359 8 7705 268 6 16031 744 6 33595 273 2770 304 5 3969 909 7 5711 913 1 8245 657 5 17315 284 1 36619 848 2923 671 2 4209 104 2 6084 187 5 8823 853 5 18701 506 9 39916 635 3085 473 2 4462 650 5 6480 659 7 10104 499 9 21815 517 6 47427 044 3256.174 2 4711 409 5 6902 902 6 10104 499 9 21815 517 6 47427 044 3626 258 3 5318 271 8 7831 509 7 11570 712 0 25447 699 7 51696 478 3826 702 5 5638 368 1 8341 557 9 12381 661 8 7484 515 7 61422 675	88	2115 384 9		860			499	
236.531 3330 5397 4725 9536 6728 288 13742 8537 28274 710 2487 1404 3513 372 359 3764 1406 7200 268 14843 2820 3359 4374 2874 14843 2820 3359 243 2874 14843 2820 3359 243 243 243 2820 4350 343 3562 243 3562 243 3562 243 3562 243 3562 243 3562 243 3562 243 3562 243 3562 243 3562 243 3562 243 3562 374 3662 344 3661 3841 3674 3674 3675 3674 3675 3674 3675 3674 3675 3676 3676 3676 3676 3676 3676 3676 3676 3676 3676 3676 3677 3676 3676 3676 3677 3676 3676 3676 3676 3676 3676 3677 3676 3676	. 8	2232.731 0		576		938	184	53120 226 1
2487 130		2256 527 2		953			710	58433 248 7
2624,933 3744 254 5362 359 8 7705 287 4 16031 744 6 33595 273 2770 304 5 3969 909 7 5711 913 1 8245 657 5 17315 284 1 36619 848 2923 671 2 4209 104 2 6084 187 5 8823 853 5 18701 506 9 39916 635 3085 473 2 4462 650 5 6480 659 7 9442 523 3 20198 627 4 43510 132 3256 174 2 4731 409 5 6902 902 6 10104 499 9 21815 517 6 47427 044 3436 263 3 5318 271 8 7831 509 7 11570 712 0 25447 699 7 5636 478 3826 702 5 5638 368 1 8341 557 9 12381 661 8 27484 515 7 61422 675	7.0	2487 140 4		140		282	434	
2770 304 5 3969 909 7 5711 913 1 8245 657 5 17315 284 1 36519 848 2923 671 2 4209 104 2 6084 187 5 8823 853 5 18701 506 9 39916 635 3085 473 2 4462 650 5 6480 659 7 9442 523 3 20198 627 4 43510 132 3256 174 2 4731 409 5 6902 902 6 10104 499 9 21815 517 6 47427 044 3436 268 5 5016 294 1 7352 591 3 10812 814 9 23561 759 0 51696 478 3626 258 3 5318 271 8 7831 509 7 11570 712 0 25447 699 7 56350 161 3826 702 5 5638 368 1 8341 557 9 12381 661 8 27484 515 7 61422 675	200	2624.033.2		350			273	
2923 671 4209 104 6084 187 8823 853 18701 569 39916 635 3085 473 4462 650 5 6480 659 7 9442 523 20198 627 4 43510 132 3256 1742 4731 499 5 6902 902 6 10104 499 21815 517 6 47427 044 3436 266 258 5 166 294 1004 499 23561 759 5169 47427 044 3626 258 3 10812 814 2544 699 76550 169 478 778 7831 599 789	26	2770 304 5		913			848	
3085 473 2 4462 650 5 6480 659 7 9442 523 3 20198 627 4 43510 132 3256 174 2 4731 409 5 6902 902 6 10104 499 9 21815 517 6 47427 044 3436 263 8 5016 294 1 7352 591 3 10812 814 9 23561 759 0 51696 478 3626 258 3 5318 271 8 7831 509 7 11570 712 0 25447 699 7 56350 161 3826 702 5 5638 368 1 8341 557 9 12381 661 8 27484 515 7 61422 675	92	2923 671 2				506	635	85556 760 4
3256.1742 4731.409 6902.902 6902.902 499 21815.517 47427.044 3436.263 5016.2941 7352.5913 10812.814 23561.759 51696.478 3626.2583 5318.2718 7831.5097 11570.712 25447.6997 56350.161 3826.7025 5638.3681 8341.5579 12381.6618 27484.5157 61422.675	90					627	132	94113 436 4
3436.263 8 5016 294 1 7352 591 3 10812 814 9 23561 759 0 51696 478 3626 258 3 5318 271 8 7831 509 7 11570 712 0 25447 699 7 56350 161 826 702 5 5638 368 1 8341 557 9 12381 661 8 27484 515 7 61422 675	200			902		517	044	103225 780 1
3626 258 3 5318 2718 7831 599 7 11570 712 0 25447 699 7 56350 161 8826 702 5 5638 368 1 8341 557 9 12381 661 8 27484 515 7 61422 675	200					759	478	
3826 702 5 5638 368 1 8341 557 9 12381 661 8 27484 515 7 61422 675	66				712		191	
	8	3826 702 5	5638 368 I	8341 557 9	199	515	675	137796 123 3

EXPLANATION OF TABLE 24

ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME

Table 24 shows the annual investment necessary to accumulate one dollar in a given number of years at interest rates ranging from 2 to 10 per cent per annum. The annuity is assumed to be applied at the end of each year. The table shows in each case the sum of the annual installments plus the interest earnings.

This table is based on the following formula:

Let a_n' represent the annual installment which at compound interest in n years will amount to $\$_1$.

Let n represent the number of years required by the annuity to amount to \$1.

Let *i* represent the interest rate expressed in per cent, as 0.05 for 5 per cent.

Then

$$a_n' = \frac{i}{(1+i)^n - 1} \tag{28}$$

and it follows from equation (26) that

$$a_n' = \frac{1}{A}.$$
 (29)

In other words the annuity which will amount to \$1 in a given time can be found with the aid of a table showing the amount of an annuity of \$1. It is the reciprocal of the latter.

Example. — What annuity will amount at 5 per cent compound interest to \$7500 in 31 years?

From Table 24 the annuity which will amount to \$1 in 31 years at 5 per cent is found to be 0 014132. The annuity which will amount to \$7500 in 31 years is, therefore,

$$7500 \times 0.014132 = $105.99.$$

Or from Table 23 the amount of an annuity of \$1 for 31 years at 5 per cent is found to be \$70.76079, therefore, the annuity which will amount to \$7500 in 31 years will be

$$\frac{7500}{70.76079} = \$105.99.$$

TABLE $_{24}$. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME

1 2 2 4 70 A		1		,	to to to		
и и 4 го с	8						
ო 4 ო					490		
4 70 (
10 C							
٠	192 158	190 247	188 355	186 481	184 627	182 792	180 975
-							147 017
7							122 820
							104 722
6							069 060
10	091 327						079 505
II							070 389
12							062 825
13							056 456
14							051 024
15	057 825	055 766				048 114	046 342
91							042 270
17							.038 699
18							035 546
61							032 745
20							030 243
21	.038 785						966 220
22							025 971
23							024 137
24							022 471
22	.031 220	9/2 620.	027 428	025 674	024 012	022 439	020 952

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (Continued)

Years	2 per cent.	2½ per cent	3 per cent.	3½ per cent	4 per cent.	4½ per cent	5 per cent
26		1			\$ 022 567		\$ 019 564
27	028	026					
-82			023 293	021 603	020 013	018 521	017 123
29	025 778	023 891	022 115	020 445			
30		022 778	021 019	019 371	017 830	016 392	015 051
31		021 739	666 610		016 855		014 132
2 6	022 611	.020 768	019 047	.017 442	or5 949	014 563	013 280
33		019 859	018 156		ois 104		
348	020 819	019 007	017 322		014 315		
35	,020 002	018 206	016 539	014 998	o13 577	012 270	011 072
36	010 233	017 452	015 804	014 284	012 887	909 110	010 434
37	018 507	016 741	015 112	013 613		010 984	000 840
38.	017 821	016 070	or4 459	012 982	011 632	010 402	009 284
39	1/1 /10	.015 436	013 844	012 388		009 856	908 765
40	016 556	.014 836	013 262	011 827	010 523	009 343	
1	015 072	014 268	.012 712	011 298	010 017	008 862	007 822
42	.015 417	013 729	012 192	862 010	009 540	008 409	
43	,o14 890	013 217	011 698	010 325	060 600	007 982	
4	014 388	012 730	011 230	878 900	008 665	007 581	
45	013 910	012 268	010 785	009 453	008 262	007 202	006 262
46	.013 453	011 827	.010 363		007 882	006 845	005 928
47	013 018	011 407	196 600	699 800			
48	012 602	900 110	845 600.				
49	012 204	.010 623	.009 213	296 200			002 040
20	0.0	810	200 862	007 624	0,1,900	our for	444 700

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (Continued)

Year)
each
of
End
at
Applied
(Annuity

\$1 \$ 0.01 459 \$ 0.08 534 \$ 0.02 322 \$ 0.05 539 52 0.10 1744	\$334 \$007 912 \$007 913 \$005 913 \$005 913 \$005 919 \$005 910 \$	0006 0005 0005 0005 0004 0004 0004	\$ 005 332 005 077 004 835 004 835 004 388 004 181 003 985 003 799 003 622	\$ 004 529 004 294 004 294 003 864 .003 667 003 480 003 303 000 3136
11 109	334 626 626 626 634 634 634 635 635 635 635 635 635 635 635	000 000 000 000 000 000 000 000 000 00	600 600 600 600 600 600 600 600	4 4 4 6 6 6 6 8 6 8 6 8 6 8 8 8 8 8 8 8
. 010 774	913 925 926 927 927 927 927 927 927 927 927	005 719 005 719 005 231 005 005 004 789 004 584 004 202 004 202		
	93.5 93.1 93.1 93.1 93.1 93.5	005 719 005 231 005 005 004 789 004 584 004 202		
. 0.10 452	349 000 0084 005 831 005 336 005 133 005	005 409 005 231 005 005 004 789 004 584 004 202 004 024		
	349 006 084 005 831 005 831 005 356 005 133 005 133 005 005 005 005 005 005 005 005 005 0	005 231 005 005 004 789 004 584 004 202 004 202		
	084 005 831 005 588 005 356 005 133 005	005 005 004 789 004 584 004 388 004 202		
009 56i 008 102 006 83i 005 732 004 009 287 007 593 006 588 005 732 004 009 287 007 533 006 356 005 294 004 008 286 007 123 005 919 004 894 004 008 058 006 091 005 714 004 705 003 007 839 006 088 005 517 004 525 003 007 839 006 088 005 517 004 525 003 007 839 006 088 005 517 004 525 003 007 820 005 28 005 517 004 525 003 007 821 005 28 005 146 004 525 003 007 822 005 910 004 803 003 879 003 007 032 005 910 004 803 003 734 002 007 032 005 512 004 642 003 734 002 006 947 005 512 004 486 003 535 002 007 032 005 512 004 486	831 588 356 356 005 313 005 919 004	004 789 004 584 004 388 004 202		
. 009 287	588 005 356 005 133 005 919 004	004 584 004 388 004 202 004 024		
	356 005 133 005 919 004	004 388		
. 008 768	133 005 919 004	004 202		
	919	004 024		002 828
oo8 286 .oo6 901 .oo5 714 oo4 705 oo3 oo8 658 oo6 688 oo5 517 oo4 525 oo3 oo7 839 .oo6 482 oo5 318 oo4 555 oo3 .oo7 626 .oo6 285 oo5 146 oo4 188 oo3 .oo7 223 .oo6 094 oo4 971 oo4 030 oo3 .oo7 032 .oo5 733 oo4 642 oo3 879 oo2 .oo6 847 oo5 562 oo4 486 oo3 595 oo2 .oo6 668 .oo5 397 oo4 486 oo3 595 oo2 .oo6 494 oo5 238 oo4 193 oo3 333 oo2 .oo6 327 oo5 084 oo4 054 oo3 310 oo2 .oo6 327 oo5 084 oo4 054 oo3 312 oo2 .oo6 6165 oo4 936 oo4 054 oo3 312 oo2	71.4			002 686
0.007 8.58 0.05 517 0.04 525 0.03 0.007 8.59 0.006 482 0.05 318 0.04 353 0.03 0.007 6.56 0.006 685 0.05 146 0.04 188 0.03 0.007 2.23 0.005 910 0.04 803 0.03 879 0.03 0.007 0.32 0.005 910 0.04 803 0.03 879 0.03 0.006 8.47 0.005 562 0.04 486 0.03 595 0.02 0.006 668 0.005 397 0.04 486 0.03 595 0.02 0.006 644 0.005 337 0.04 193 0.03 333 0.02 0.006 668 0.005 384 0.04 193 0.03 310 0.02 0.006 676 0.006 327 0.00 084 0.04 054 0.03 092 0.02 0.006 676 0.006 676 0.007 032 0.007 032 0.002 0.002	+40 - ++1	003 854	003 143	002 552
.007 839 .006 482 005 328 004 353 003	517 004	003 692		002 424
	328 004	003 538		002 304
. 007 421.	146 004	003 390	002 730	002 189
. 007 223	971 004	003 249		002 081
.007 032 .005 733 .004 642 .003 734 .002 .005 847 .005 562 .004 642 .003 595 .002 .005 684 .005 562 .004 486 .003 595 .002 .005 684 .005 397 .004 337 .003 461 .005 684 .004 193 .003 313 .002 .005 684 .004 054 .003 921 .002 .002 .005 685 .004 936 .003 921 .002 .002 .002 .002 .002 .002 .003 921 .003 922 .003 922 .003 922 .003 922 .003 923 .003 9	803 .003			978 Ioo
.oof 847 oof 562 oo4 486 oo3 595 oo2 .oof 668 .oo5 397 oo4 337 oo2 461 oo2 .oof 494 oo5 238 oo4 193 oo3 333 .oo2 .oof 327 oo5 084 oo4 054 oo3 210 .oo2 .oof 165 oo4 936 oo3 991 oo3 992 oo2	642 003			oo1 880
oof 668 .oof 397 ood 337 oog 461 oo .oof 494 oo5 238 oo4 193 oo3 333 .oo .oof 327 oo5 084 oo4 054 oo3 210 .oo .oof 165 .oo4 936 oo3 991 oo3 og .oof 165 .oo4 936 oo3 992 oo2	486 003			287 100
.006 494	337 003		002 165	669 100
.006 327	193 003		.002 068	
.006 165 .004 936 003 911 003 092 002	054 003			
200 200 200 200 200 200 200 200 200 200	921 003		988 Ioo	001 46I
	792 .002			
.005 855 .004 654 003 668 002 869 .002	668 002	.002 229	001 721	001 322

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (Continued)

			ndder (ammer)	Annuity Applied at Ana of cases		-	
Years.	2 per cent	2½ per cent.	3 per cent.	3½ per cent	4 per cent.	4½ per cent.	5 per cent.
16	2 707 300	610	548	764	\$ 002 138 7	644	257
2 1	3 5	200	200	002 663	002 052 2		oor 195 8
201		96	222	002 567 2	4 696 100	Soi	137
0 5	202 4423 00	004 204 0	003 215 1	474	1 068 100	oo1 434 3	.001 082 2
2 8	167	950	4 111 600	287	001 814 I	7 001 370 7	001 029 6
200	7 001 500.	0 020 400.	003 111 /	500	-	5	,
81	.005 034 0	003 912 5	012	002 298 9	.001 741 3	001 310 0	9 626 000
8	I II0 700	903 802 5	002 915 8	216	001 671 5	252	000 932 I
8	707	003 606 I	002 822 8	136	604	196	900 880 9
28	675 8	.003 593 0	002 733 I	.002 060 2	oor 540 5	143	000 844 0
25	.004 563 2	003 493 1	002 646 5	9 986 100	001 479 г	.001 093 3	000 803 2
90	2 2 2 2 2 2	690000	8 292 200	8 510 100	001 420 2	001 045 2	764
8 9	.004 455 0	003 399 3	002 482 0	001 847 6	363	000 000 2	727
/0	.004 347 5	003 302 3	002 402 0		001 309 5	000 955 2	692
8 8	.004 244 2	.003 211 /	002 328 5	001 718 7	001 257 6	000 913 2	.000 658 9
3 8	, ctr too		1 1 1	6.7	8 70° 100	. 000 873 2	.000 627 I
8	004 046 0	003 038 I	002 255 0	.001 057 0	0 /07 100	z (/o ooo:	,
10	951	.002 955 2	002 185 1	599	0 091 100	.000 834 9	000 596 9
000	858	.002 874 9	002 116 9	542	001 114 1	.000 798 3	208
200	768	002 706 0	002 051 1	oo1 488 3	1 o/o 100	000 763 3	540
g 2	.003 681 2	.002 721 3	4 78g 100.	001 435 9	001 027 9	000 729 9	514
. 22		.002 647 9	8 326 100.	001 385 5	.000 987 4	0 869 000	000 490 0
4	000 512 1	9 923 600	2 998 100	336	000 948 5	000 667 5	000 466 5
0 0	42.0	002 507 5	808		116	000 638 3	000 444 I
780		002 440 3	752		000 875 4	000 610 5	000 422 7
S, S	.003 277 3	.002 375 2	6 869 100	201	841	000 583 8	000 402 4
100		002 311 9	.001 646 7	.001 159 3	0 808 000	000 558 4	000 383 I
i							

332		VALU	JA	110	JN	,	DF.	PF	Œ	CI	Α.	LIC	Ν	H	ΠN	ט	1.1	TLE.		X.A	LL	F-1)A	3E	,		
(Continued)		Io per cent.	\$1 000 000	476 190	302 115	215 47I	.163 797	129 607				062 745	053 963	040 703	040 779	035 740	o31 474	027 817	024 004	021 930	.019 547	or ₇ 460	015 624	or4 005	012 572	OII 300	010 168
GIVEN TIME		9 per cent		478 469			167 092	132 920				065 820	.056 947	049 651	043 567	038 433	034 059	030 300	027 046	024 212	021 730	or9 546		015 905			908 110
DOLLAR IN A	cn rear)	8 per cent.	\$I 000 000	480 769	308 034	221 921	170 456	136 315	112 072	094 015	080 080	620 690	940 090	052 695	046 522	041 297	036 830	032 977	629 620	026 702	.024 128	021 852	019 832	018 032		or4 978	or3 679
TO ONE	(Annuity Applied at End of each Year)	7 per cent.	\$1 000 000	483 092	311 052	.225 228	173 891	139 796			083 486	.072 378	063 357	055 902	049 651	.044 345	039 795	.035 858	032 425	029 413	026 753	024 393	022 289	020 406	018 714	681 710.	015 811
	(Annuity Appli	6§ per cent	\$1 000 000	484			.175 635	.141 568			085 238	074 105	065 055	057 568	051 283	.045 940	041 353	037 378	903 300	.030 855	028 156	025 756	023 613	021 691	196 610	018 398	186 910.
ANNUITY WHICH		6 per cent.	\$1 000 000	485	.314 110	228 591	177 396		.119 135	101 036	087 022	.075 868	. 066 793	059 277	.052 960	047 585	.042 963			.032 357		027 185		.023 046		649 610.	.018 227
TABLE 24. ANN		5} per cent	\$T 000 000	.486		230 295		.145 179	120 964	102 864	.088 839	.077 668	125 890'	920 190.	.054 684	.049 279	.044 626	.040 583			.031 150		.026 465			.021 036	.019 549
TA	,	Years.	-	1 (1)	67	4	ro	9	7	-∞	6	9	II	13	13	14.	12	91	17	81	19	20	1.0	22	23	24	22

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (Continued)

Years.	5½ per cent	6 per cent.	63 per cent.	7 per cent.	8 per cent	9 per cent.	10 per cent.
26		-			\$ 012 507 1	\$ oro 715 4	
27	910.	_	014 523	013 426	OII 448 I		
-82					oro 488 9	008 852 0	007 451 0
29	or4 679	or3 580	OI2 474	o11 449	009 618 5		
30	.013 805	012 649	011 577	010 586	008 827 4	007 336 4	2 620 900
31	.012 917	011 792		262 600		006 685 6	005 496 2
32	.012 095	011 002	266 600	009 073	007 450 8	2 960 900	004 971 7
33		010 273		008 408		2 195 200	004 499 4
34	010 630	865 600.	008 656	167 700		9 920 500	001 073 7
35	576 600.	008 974	008 062	007 234	005 803 3	004 635 8	003 689 7
36	998 600			006 715	005 344 7		003 343 I
37	.008 800.				004 924 4		003 029 9
38.	.008 272	007.358	006 535		004 538 9	003 538 2	002 746 9
39	087 700.	006 894	660 900	005 387	004 185 1	003 235 6	002 491 0
40	007 320	006 462	.005 694	005 009	003 860 2	002 959 6	002 259 4
41		650 900	818 300.	004 660	003 561 5	002 707 9	002 049 8
42		005 683	004 968	004.336	003 286 8	002 478 I	001 860 0
43		005 333	004 644	004 036	003 034 I	002 268 4	
4	192 500.	900 500.	.004 341	003 758	002 801 5	2 920 700.	001 532 2
46		004 700	004 000	003 500	002 587 3	7 106 100.	001 391 0
97					002 389 9	001 741 6	oo1 263 o
47	004 831	004 148	003 553	003 037	002 208 0	.001 595 2	oo1 146 8
-84				002 831	002 040 3	oor 461 4	001 041 5
46					oo1 885 6	oor 338 9	000 945 9
20	004 061	003 444	002 914	002 460	001 742 9	001 226 9	000 859 2

TABLE 24, ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (Continued)

_
Year)
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End
at
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Years.	5½ per cent	6 per cent.	6½ per cent	7 per cent	8 per cent.	9 per cent	10 per cent
;;	835	\$ 003 238 8				\$ 001 124 3	\$ 000 780 46
25	.003 621 9	003 046 2			001 489 6	001 030 4	00 604 000
	421	.002 865 5		1 566 100	377	944	644
4		002 696 0	002 242 7	1 198 100	001 273 7	000 865 7	000 585 23
20	.003 054 6	.002 537 0	,002 101 4	001 736 3	0 8/1 100	000 793 6	000 531 75
9,	002 887 0	002 387 6	696	оот 620 г	001 089 5	000 727 5	.000 483 17
7	,002 729 0	002 247 4	001 845 6	001 511 8	200	0 299 000	439
.00	002 580 1	002 115 7	001 730 0	9 014 100	000 932 3	9 119 000	$3\overline{9}$ 8
9	.002 439 6	001 992 O	621	001 316 9		000 560 8	.000 362 58
9	002 307 I	.001 875 7	.001 520 5	001 229 2	6 464 000	000 514 2	000 329 51
н	002 182 0	4 992 100	oo1 425 6	oo1 147 S	000 738 3	000 471 5	000 299 46
7	002 064 0	.001 663 7	001 336 8	001 071 3	000 683 I	000 432 4	000 272 17
	oor 952 6	0 267 o	001 253 7	2 000 100	000 632 I	000 396 5	000 247 36
4	.001 847 4	001 476 2	001 175 8	000 933 9	000 585 0	.000 363 7	.000 224 82
ور	.oor 748 o	2 001 390 7	001 102 8	000 872 0	000 541 3	000 333 5	000 204 34
9	.001 654 I	.001 310 2	001 034 4	000 814 3	000 201 0	000 305 9	000 185 73
7	.oor 565 4	00I 234 5	000 970 3	2 094 000	000 463 7	9 080 000	000 168 82
· ∞	oor 481 6	.001 163 3	000 010 3	000 710 2	000 429 I	.000 257 3	.000 153 45
ō.	.ooi 402 4	001 096 3	000 854 0	000 663 3	.000 397 2	.000 236 0	.000 139 48
0	oor 327 5	.001 033 1	000 801 2	000 619 5	9 292 000	000 216 5	000 126 78
H		.000 973 7	000 751 8	000 578 7	000 340 3	9 861 000	000 115 24
77	189	7 716 000	000 705 4	000 540 5	000 315 0	000 182 1	000 104 76
~~	.001 126 5	. 000 865 0	6 199 000	000 504 9	9 162 000	1 291 000	000 095 22
4	990	.000 815 4	000 621 1	000 471 6	6 692 000.	000 153 3	.000 086 56
10	0 010 100	000 768 7	.000 582 9	000 440 6	000 249 8	000 140 6	89 870 000.

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (Concluded)

EXPLANATION OF TABLE 25

THE PRESENT VALUE OF AN ANNUITY OF ONE DOLLAR

The present value of an annuity is the sum of the present values of the several annuity installments.

The present value of an annuity of \$1 receivable at the end of each year is presented for a few selected years in Table 25.

This table is based on the following formula:

Let P' represent the present value of an annuity of r receivable at the end of each year during r years.

Let n represent any number of years, the term of the annuity.

Let i represent the rate of interest expressed decimally; thus for 5 per cent, i = 0.05.

Then:

$$P' = \frac{\mathbf{I}}{i} - \frac{\mathbf{I}}{i(\mathbf{I} + i)^n} \quad \text{or} \quad \frac{\mathbf{I}}{i} \left(\mathbf{I} - \frac{\mathbf{I}}{(\mathbf{I} + i)^n} \right)$$
 (30)

This equation may be written:

$$P' = \frac{(\mathbf{I} + i)^n - \mathbf{I}}{i} \times \frac{\mathbf{I}}{(\mathbf{I} + i)^n}.$$
 (31)

Or by reference to equations (23) and (24)

$$P' = \frac{A''}{A'} {32}$$

In other words the present value of the annuity of \$1 is the amount of an annuity of \$1 in n years divided by the amount of \$1 at compound interest in n years.

The present value of an annuity of \$1 receivable at the end of each year is therefore ascertainable for any number of years to 100 from Tables 21 and 23 by dividing the amount of an annuity of \$1 found in Table 23 by the amount of \$1 at compound interest found in Table 21.

Example. — What is the present value of an annuity of \$53 receivable at the end of each year for 20 years at 5 per cent interest?

The amount of an annuity of \$1 for 20 years at 5 per cent interest is found in Table 23 to be \$33.06595. The amount of \$1 at 5 per cent compound interest for 20 years is found in Table 21 to be \$2.653298, consequently, the present value of an annuity of \$1 at 5 per cent for 20 years:

$$33.06595 \div 2.653298 = $12.46221.$$

And the present value of the annuity of \$53 for 20 years is $12.46221 \times 53 = 660.50 .

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Years.	2 per cent.	2½ per cent.	3 per cent	3½ per cent.	4 per cent	4½ per cent.	5 per cent
-	080	\$ 0 975 610	\$ 0 070 874	\$ o 966 184		\$ 0 956 938	\$ 0 952 381
	1.041	I 927	I 913	1 899	ı 886	н	1 859
. ~	883	856	2 828 611		2 775 091	2 748 964	723
4	3 807 729	3 761 974		3 673 079			3 545 950
19	4 713 460	4 645 829	4 579 707	4 515 052	4 451 822	4 389 977	4 329 477
	109		417	328	242	157	
7	6 471 991	6 349 39I	6 230 283	6 114 544	6 002 055	5 892 7or	5 786 373
. 80	325	170	7 019 692	873	732	595	463
6	162	970	786		7 435 332		7 107 822
10	8 982 585	8 752 064	8 530 203	8 316 605	8 110 896	7 912 718	7 721 735
ص	12 849 263	12 381 378	11 937 935	11 517 411	11 118 387	10 739 546	IO 379 658
20	16 351 433	15 589 162	14 877 475	14 212 403	13 590 326	13 007 936	12 462 210
2	19 523 457	18 424 376	17.413 148	16 481 515	15 622 080	14 828 209	14 093 945
•	22.396 456	20 930 293	19,600 441	18 392 045	17 292 033	16 288 889	15 372 451
35	24 998 619	23 145 157	21 487 220	20 000 661	18 664 613	17 461 012	16 374 194
40	27.355 479	25 102 775	23 114 772	21 355 072	19 792 774	18 401 584	17 159 086
10	29 490 160	833	24 518 713	22 495 450	20 720 040	19 156 347	17 774 070
_	31 423 606	28 362 312	25 729 764	23 455 618	21 482 185	19 762 008	18 255 925
_	34.760 887	30.908 657	27 675 564	24.944 734	22.623 490	20 638 022	18 929 290
_	37 498 620	32 897 857	29 123 421	26 000 397	23 394 515	21 202 112	19 342 677
88	39.744 514	34 451 817	30 200 763	26 748 776	23 915 392	21.565 345	19 596 460
_	41 586 929	35.665 768	31 002 407	27 279 311	24 267 278	21 799 241	rg 752 262
8	43 098 352	36 614 105	31 598 905	27 655 425	24 504 999	2I 949 853	19 847 910

THE DEPOEDUT VALUE OF AN ANNITTY OF ONE ONE OF A TABLE

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
н	\$ 0.947 867	\$ 0 943 396	\$ 0 938 967	934	\$ 0 925 926	\$ o 917 431	160 606 0\$
8	1.846 320	833	I 820 626	1 808 or8	1 783 265	111 654 1	
3	697	673	2 648 476	2 624 316		531	2 486 852
4	3 505 150	3 465 106			3 312 127	3 239 720	3 169 865
20	4.270 284	4 212 364	4 155 679	4 100 197	3 992 710	3 889 651	3 790 787
9	4 995 530		4 841 014		4 622 880	4 485 919	4 355 26I
7		582	484	389	206	032	4 868 4rg
∞	6 334 566	6 209 794	6 088 751	§ 971 299	5 746 639	5 534 819	
6	6 952 195	801	656	5^{15}	246	995	5 759 024
9	7 537 626	7 360 087	7.188 830	7 023 582	180 01/ 9	6 417 658	6 144 567
15	10.037 581	9 712 249	9 402 669	9 107 914	8 559 479	8 090 8	7.606 080
202	11.950 382	II 469 92I	11 018 507	10 594 o14	9 818 148	9 128 546	8 513 564
22	13.413 933	12 783 356	12 197 877	11 653 583	10 674 776	9 822 579	9 076 840
98	14.533 745	r3 764 836	13 058 676	12 409 041	11 257 783	10 273 654	9 426 914
35	15.390 552	14 498 246	13 686 957	12 947 672	11 654 568	10 566 821	9 644 159
9	16 046 125	15.046 297	14 145 527	13 331 709	11 924 613	10 757 360	150 644 6
46	16 547 726	IS 455 832	14 480 228	13 605 522	12 108 402	761 188 or	9 862 788
20	16 931 518	15 761 861	14 724 521	13 800 746	12 233 485	10 961 683	9 914 814
99	17 449 854	16 161 428	15 032 966	14 039 181	12 376 552	11 047 991	6 967 157
2	17 753 304	16 384 544	15 197 282	14 160 389	12 442 820	11 o84 449	9 987 338
8	17.930 953	16 509 131	15 284 900	14 222 005	12 473 514	11 o99 849	9.995 118
8	18.034 954	16.578 699	15 331 451	14 253 328	12 487 732	11 106 354	811 866 6
8	18.095 840	16 617 546	15.356 293	14 269 251	12 494 318	11 109 102	9 999 274

EXPLANATION OF TABLE 26

THE ANNUITY WHICH \$1 WILL PURCHASE

The annuity receivable at the end of each year which \$1 will purchase is noted in Table 26.

This table contains the reciprocals of the numbers appearing in Table 25. It is based on the following formula:

Let $a_n^{"}$ represent the annuity receivable at the end of each year which \$1 will buy for n years.

Let n represent any number of years, the term of the annuity. Let i represent the rate of interest expressed decimally;

thus for 5 per cent, i = 0.05.

Then:

$$a_n'' = \frac{i (1+i)^n}{(1+i)^n - 1}.$$
 (33)

And by reference to equations (23) and (24)

$$a_n^{\prime\prime} = \frac{A^{\prime}}{A^{\prime\prime}}. (34)$$

In other words the annuity receivable at the end of each year which r will buy for r years is the amount of r at compound interest divided by the amount of an annuity of r in r years.

The annuity which \$1 will purchase for any number of years is, therefore, ascertainable from Tables 21 and 23 by dividing the amount of \$1 at compound interest found in Table 21, by the amount of an annuity of \$1 found in Table 23.

Example. — What annuity for 20 years can be purchased for \$500 at 5 per cent interest?

The amount of \$1 at 5 per cent compound interest for 20 years in Table 21 is found to be \$2.653298. The amount of an annuity of \$1 for 20 years at 5 per cent interest, as found in Table 23, is \$33.06595; consequently the annuity which \$1 will purchase for 20 years at 5 per cent is

$$2.653298 \div 33.06595 = 0.0802426.$$

And the annuity which can be purchased for \$500 will be

$$0.0802426 \times 500 = $40.12.$$

TABLE 26. THE ANNUITY WHICH ONE DOLLAR WILL BUY

Years.	2 per cent.	2½ per cent.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	4½ per cent.	5 per cent.
н	\$1 020 000	\$1 025 000	\$1 030 000	\$1 035 000	\$I 040 000	\$I 045 000	\$1 050 000
7	515	518	522	526	530	533	537
3	.346 755	350 137	353 530	356 934	360 349	363 773	367 209
4	. 262 624		269 027			278 744	282 012
2	.212 158	.215 247	.218 355	221 481	.224 627	227 792	230 975
9	.178 526	181 550	184 598	, 899 /81	190 762	193 878	710 761
7	.154 512	157 495	160 506	.163 544	019 991	106 991	.172 820
∞	136 510	.139 467	142 456	145 477	148 528	151 610	154 722
6	122 515	125 457	128 434	131 446	134 493	137 574	.140 690
10	.111 327	114 259	117 231	120 241	123 291	126 379	129 505
9	077 825	994 080.	.083 767	086 825	.089 94r	093 114	096 342
25	751 190.	.064 147	.067 216	070 361	.073 582	928 920.	.080 243
25	.051 220	054 276	.057 428	060 674	.064 012	067 439	.070 952
8	.044 650	047 778	051 019	.054 371	.057 830	.061 392	065 051
25	040 002	043 206	046 539	.049 998	053 577	.057 270	220 190
<u>유</u>	.036 556	039 836	.043 262	046 827	050 523	.054 343	.058 278
5	o33 910	.037 268	040 785	044 453	048 262	.052 202	056 262
22	031 823	.035 258	038 865	.042 634	046 550	.050 602	.054 777
8	.028 768	.032 353	036 133	.040 089	044 202	.048 454	052 828
2	.026 668	030 397	.034 337	038 461	.042 745	.047 165	051 699
8	025 161	920 620	033 112	.037 385	041 814	.046 371	051 030
8	024 046	.028 038	032 256	036 658	041 208	.045 873	050 627
8	.023 203	.027 312	031 647	.036 159	040 808	.045 558	050 383

Years	5½ per cent	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent	10 per cent.
-	\$ 000 July	%I obo ooo		\$1 070 000	\$1 080 000		\$1 100 000
1 0	541	545	549	553	260 769	568 469	925
~	370 654	374 110		381 052	388 034		402 115
4	285 294		291 903	.295 228	301 921		
۵	234 176	.237 396	240 635	243 891	250 456	257 092	263 797
9	200 179		206 568	962 602	216 315	222 920	229 607
7	175 964		182 331	185 553	.192 072	169 861	205 406
.∞	157 864	161 036	164 237	167 468	174 015	180 674	.187 444
6	.143 839	147 022	150 238	153 486	160 080	106 799	173 641
10	132 668	135 868	139 105	142 378	149 029	155 820	162 745
10	929 660	.102 963	106 353	109 795	116 830	124 059	131 474
20	083 679	.087 185	952 060	094 393	101 852	109 546	117 460
25	.074 549	078 227	186 180	085 811	093 679	101 806	110 168
30	.068 805	072 649	076 577	080 586	088 827	097 336	105 879
35	064 975	974	073 062	077 234	085 803	094 636	103 690
40	062 320	066 462	070 694	002 900	083 860	092 960	102 259
46	060 431	.064 700	090 690	073 500	082 587	091 902	101 391
20	190 650	063 444	067 914	072 460	o81 743	091 227	100 859
9	.057 307	928 190	066 520	071 229	962 080	090 514	100 330
20	056 328	.061 033	o65 801	070 620	080 368	090 216	100 127
80	692 550	060 573	065 424	070 314	080 170	160 060	. 100 049
8	.055 448	060 318	065 225	070 159	620 080	030 033	010 001
901	.055 261	221 090	065 120	180 070	080 036	910 060	700 001.

EXPLANATION OF TABLE 27

AMORTIZATION AND DEPRECIATION

Definitions of amortization and depreciation have already been given

Table 27 is an amortization table or a set of tables computed for the interest rates 4 per cent to 7 per cent per annum and for all terms of indebtedness, terms of bonds or probable life terms of perishable articles, from 2 to 75 years, which are likely to come under consideration.

By adding to this table headings at the bottom of the columns it has also been made convenient for use as a depreciation table. Care must be taken in its use to preserve the distinction indicated by the top and bottom headings and not to assume that the years of expectancy in the last column apply to the age or number of years noted in the first column.

All values noted in this table are referred to \$100 as the basic amount in order that the figures in the table may be used as percentages when so desired.

The table has been computed by the use of the following formulæ which may be used when values are to be ascertained for interest rates or terms not covered by the table.

- Let a_n be the amortization installment which must be invested annually in order to amount at compound interest to \$100 in n years.
- Let i be the interest rate expressed decimally (thus 0.05 for 5 per cent).
- Let A_m be the accrued amortization in m years when the annual amortization installment is a_n and the interest rate is i.

Let m be any number of years.

- Let n represent the amortization term expressed in years.
- Let a_m be the current amortization in the mth year, i.e., the amortization increment a_n plus interest on the amortization fund already accumulated.
- Let e be the remaining years of usefulness of any article whose probable life new is n years.

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The basic formula for \$100 derived from (28) from which all other formulæ are derived will be

$$a_n = \frac{100 i}{(1+i)^n - 1}$$
 (35)

Note. — This formula is for the special case in which the amount of the annuity is \$100 which fact should not be overlooked in applying it.

The accrued amortization in m years will be:

$$A_m = \frac{100 (1+i)^m - 1}{(1+i)^n - 1}.$$
 (36)

The remaining investment at the end of the mth year will be:

$$100 - A_m = 100 \left[1 - \frac{(1+i)^m - 1}{(1+i)^n - 1} \right]$$
 (37)

The current amortization a_m in the *m*th year will be:

$$a_m = 100 \left[\frac{(1-i)^m - (1+i)^{m-1}}{(1+i)^n - 1} \right]$$
 (38)

For depreciation these formulæ may be written as follows:

The basic depreciation increment for a value of \$100 will be:

$$a_n = \frac{100 i}{(1+i)^n - 1} \tag{35}$$

The accrued depreciation will be:

$$A_m = \frac{100 (1+i)^{n-e} - 1}{(1+i)^n - 1}.$$
 (39)

The remaining or present value at the time when the expectancy is e years will be

$$100 - A_m = 100 \left[1 - \frac{(1+i)^{n-e} - 1}{(1+i)^n - 1} \right]$$
 (40)

The current annual depreciation when the expectancy is e years will be

$$a_m = \operatorname{IOO}\left[\frac{(\mathbf{I}+i)^{n-\theta} - (\mathbf{I}+i)^{n-\theta-1}}{(\mathbf{I}+i)^n - \mathbf{I}}\right]. \tag{41}$$

The remaining value of any perishable article depends upon the cost, all circumstances considered, of replacing it when it

ceases to be useful, and the time when the article will go out of use. But this value is dependent too upon the probable life of a new article of the same kind. The remaining value of several articles with the same expectancy of which one is in the 10-year life class, another in the 20-year class and another in the 40-year class will not be the same, because the proportional service yet to be expected when compared with that of new articles will, in these cases, vary inversely as 10 to 20 to 40, and the remaining values will depart widely from each other. If the expectancy, for example, of each of three such articles is 5 years, and 6 per cent interest be made the basis of the calculation, the remaining values will be 57.23 per cent, 36.73 per cent and 28.00 per cent of the cost of replacement. The article with the probable life new of 10 years has the highest value for the reason that its remaining service years are a larger proportion of its probable life new than in the case of the other two articles with longer probable life new.

Illustration of the Use of Table 27

What amount at the end of the 26th year will be in a sinking fund for the retirement of a bond issue of \$100,000 running 40 years, if the money accumulating in the sinking fund earns 6 per cent per annum?

On page 361 in the 40-year life section of Table 27 at year 27 (beginning of year 27 is the end of year 26) in the left-hand column, the accrued amortization in the 6 per cent column for each \$100 will be found to be \$41.1637. Consequently, the amount in the sinking fund to retire \$100,000 will be \$41,163.70. The amount which will be added to the sinking fund in the 27th year will be \$2030.60.

Let it be assumed that by any means, such as inspection by experts, the probable remaining term of usefulness of an electric generator 13 years old has been found to be 11 years and that the type of generator to which it belongs has a probable life new of 20 years. What at 6 per cent interest is the accrued depreciation if the cost of the generator was \$3000 and what is its re-

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maining value if the cost of replacement at the end of its period of usefulness may be estimated at \$2500?

On page 353 in the 20-year life section of Table 27 at 11 years in the right-hand column there are found in the column with the bottom heading "Present Value" the figures 68.7614 which represent percentage. The remaining value estimated from cost would be $0.687614 \times 3000 = \$2062.84$; consequently the accrued depreciation, estimated from cost, would be 3000 - 2062.84 = \$937.16. The remaining value estimated from the cost to replace will be $0.687614 \times 2500 = \$1719.04$.

The actual accrued depreciation, therefore, will be

$$3000 - 1719.04 = $1280.96.$$

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES

The Current Annual and Total Amortization and the Remaining Investment also the Current Annual and Accrued Depreciation and the Present Value for Each \$100 of Bonds or of Investment.

Computed by the compound interest sinking fund method

	I		o and o dance		iming rund	1110 VIIO G	
Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year.	
2 YEA	r Life					2 YEA	r Life
		4 per cent			5 per cent.		
1 2	\$100 0000 50 9804		\$49 0196 100 0000	\$100 0000 51 2195	\$48 7805 51.2195	\$48 7805 100 0000	2 I
		6 per cent.			7 per cent.		
I 2	\$100 0000 51 4563	\$48 5437 51 4563	\$48 5437 100 0000	\$100 0000 51.6908	\$48 3092 51 6908	\$48 3092 100 0000	2 I
3 YEA	R LIFE					3 YeA	r Life
		4 per cent			5 per cent		
1 2 3	\$100 0000 67 9651 34 6489	\$32 0349 33 3162 34 6489	\$32 0349 65 3511 100 0000	\$100 0000 68 2791 34 9722	\$31 7209 33 3069 34 9722	\$31 7209 65 0278 100 0000	3 2 1
		6 per cent.					
1 2 3	\$100 0000 68.5890 35 2934	\$31 4110 33 2956 35 2934	\$31 4110 64 7066 100 0000	\$100 0000 68 8948 35 6123	\$31 1052 33 2825 35 6123	\$31 1052 64 3877 100 0000	3 2 1
4 YEA	r Life				ı	4 Үел	R LIFE
		4 per cent			5 per cent		
1 2 3 4	\$100 0000 76 4510 51 9600 26 4894	24 4910 25 4706	\$23 5490 8 0400 73 5106 100 0000	\$100 0000 76 7988 52. 376 26 8583	\$23 2012 24.3612 25 5793 26 8583	\$23 2012 47 5624 73.1417 100 0000	4 3 2 1
		6 per cent					
1 2 3 4	\$100.0000 77.1409 52 9102 27 2257	24 2307 25 6845	\$22 8591 47 0898 72 7743 100 0000	\$100.0000 77 4772 53.3778 27 5914	\$22 5228 24 0994 25 7864 27.5914	\$22 5228 46 6222 72.4086 100 0000	4 3 2 1
	Present value.	Current depreciation during year.	,	Present value.	Current depreciation during year.		Expect- ancy Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year	
5 YEAR	R LIFE					5 YEA	r Life
		4 per cent			5 per cent		
1 2 3 4 5	\$100 0000 81 5373 62 3361 42 3668 21 5988	\$18 4627 19 2012 19 9693 20 7680 21 5988	\$18 4627 37 6639 57 6332 78 4012	\$100 0000 81 9025 62 9002 42 9477 21 9976	\$18 0975 19 0023 19 9525 20 9501 21 9976	\$18 0975 37 0998 57 0523 78 0024	5 4 3 2 I
		6 per cent			7 per cent		
1 2 3 4 5	\$100 0000 82 2604 63 4563 43 5241 22 3959	\$17 7396 18 8041 19 9322 21 1282 22 3959	\$17 7396 36 5437 56 4759 77 6041	\$100 0000 82 6109 64 0046 44 0959 22 7935	\$17 3891 18 6063 19 9087 21 3024 22 7935	\$17 3891 35 9954 55 9041 77 2065 100 0000	5 4 3 2
6 Үел	r Life					6 Үел	r Life
		4 per cent			5 per cent		
1 2 3 4 5	\$100 0000 84 9238 69 2446 52 9382 35 9795 18 3425	\$15 0762 15 6792 16 3064 16 9587 17 6370 18 3425	\$15 0762 30 7554 47 0618 64 0205 81 6575	\$100 0000 85 2983 69 8615 53 6528 36 6337 18 7636	\$14 7017 15 4368 16 2087 17 0191 17 8701 18 7636	\$14 7017 30 1385 46 3472 63 3663 81 2364 100 0000	6 5 4 3 2
		6 per cent	· · · · · · · · · · · · · · · · · · ·				
1 2 3 4 5 6	\$100 0000 85 6637 70 4673 54 3591 37 2844 19 1852	\$14 3363 15.1964 16 1082 17 0747 18 0992 19 1852	\$14 3363 29.5327 45 6409 62.7156 80.8148	\$100 0000 86 0204 71 0623 55 0571 37 9315 19 6071	\$13 9796 14 9581 16 0052 17 1256 18 3244 19 6071	\$13 9796 28 9377 44 9429 62.0685 80 3929 100 0000	6 5 4 3 2 1
all the second s	Present value.	Current depreciation during year		Present value.	Current depreciation during year		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.		
7 YEA	r Life					7 YEA	r Life	
	4 per cent.				5 per cent.			
1 2 3 4 5 6 7	\$100 0000 87 3390 74 1716 60 4775 46 2356 31 4241 16 0201	\$12 6610 13 1674 13 6941 14 2419 14 8115 15 4040 16 0201	\$12 6610 25 8284 39 5225 53 7644 68 5759 83 9799 100 0000	\$100 0000 87 7180 74 8219 61 2810 47 0631 32 1343 16 4590	\$12 2820 12.8961 13 5409 14 2179 14 9288 15 6753 16 4590	\$12 2820 25 1781 38 7190 52 9369 67 8657 83 5410 100 0000	7 6 5 4 3 2	
		6 per cent			7 per cent.	l .		
1 2 3 4 5 6 7	\$100 0000 88 0865 75 4582 62 0722 47 8830 32 8425 16 8995	\$11 9135 12 6283 13 3860 14 1892 15 0405 15 9430 16 8995	\$11 9135 24 5418 37 9278 52 1170 67 1575 83 1005 100 0000	\$100 0000 88 4447 76 0805 62 8508 48 6950 33 5483 17 3414	\$11 5553 12 3642 13 2297 14 1558 15 1467 16 2069 17 3414	\$11 5553 23 9195 37 1492 51 3050 66 4517 82 6586 100 0000	7 6 5 4 3 2	
8 Yea	r Life					8 Yea	r Life	
		4 per cent.			5 per cent.			
1 2 3 4	\$100 0000 89 1472 77 8603 66 1219	\$10 8528 11 2869 11 7384 12 2079	\$10 8528 22 1397 33 8781 46 0860	\$100 0000 89 5278 78 5320 66.9864	\$10 4722 10 9958 11 5456 12 1229	\$10 4722 21 4680 33 0136 45 1365	8 7 6 5	
5 6	53 9140	12.6962	58 7822 71 9863	54 8635 42 1345	12 7290 13 36,4	57 8655 71 2309	4 3	
7 8	28 0137 14 2815	13 7322 14 2815	85 7185 100 0000	28 7691 14 7354	14.0337 14.7354	85 2646	2 I	
	Present value	Current depreciation during year.		Present value.	Current depreciation during year		Expect- ancy, Years,	

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

			(00111				
Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	
8 YEA	R LIFE					8 YEA	R LIFE
		6 per cent			7 per cent		
1 2 3 4 5	\$100 0000 89 8964 79 1866 67 8342 55 8007	\$10 1036 10 7098 11 3524 12 0335 12 7556	\$10 1036 20 8134 32 1658 44 1993 56 9549	\$100 0000 90 2532 79 8242 68 6651 56.7249	\$ 9 7468 10 4290 11 1591 11 9402 12 7760	\$ 9 7468 20 1758 31 3349 43 2751 56 0511	8 7 6 5
6 7 8	43 0451 29 5242 15 1921	13 5209 14 3321 15 1921	70 4758 84 8079 100 0000	43 9489 30 2785 15 6512	13 6704 14 6273 15 6512	69 7215 84 3488 100 0000	3 2 1
9 Yea	R LIFE					9 Үел	R LIFE
		4 per cent			5 per cent		
1 2 3 4 5 6 7 8	\$100 0000 90 5507 80 7234 70 5030 59 8738 48 8195 37 3230 25 3666 12 9320	\$ 9 4493 9 8273 10 2204 10 6292 11 0543 11 4965 11 9564 12 4346 12 9320	\$ 9 4493 19 2766 29 4970 40 1262 51 1805 62 6770 74 6334 87 0680 100 0000	\$100 0000 90 9310 81 4085 71 4099 60 9114 49 8880 38 3134 26 1601 13 3991	\$ 9 0690 9 5225 9 9986 10 4985 11 0234 11 5746 12 1533 12 7610 13 3991	\$ 9 0690 18 5915 28 5901 39 0886 50 1120 61 6866 73 8399 86 6009 100 0000	9 8 7 6 5 4 3 2 1
		6 per cent	1		7 per cent.		
1 2 3 4 5 6 78 9	\$100 0000 91 2978 82 0734 72 2956 61 9311 50 9447 39 2992 26 9549 13 8700	\$ 8 7022 9 2244 9 7778 10 3645 10 9864 11 6455 12 3443 13 0849 13 8700	\$ 8 7022 17 9266 27.7044 38 0689 49 0553 60.7008 73 0451 86 1300 100 0000	\$100 0000 91 6514 82 7184 73 1600 62 9325 51 9891 40 2797 27 7506 14 3445	\$ 8 3486 8 9330 9 5584 10 2275 10 9434 11 7094 12.5291 13.4061 14 3445	\$ 8 3486 17 2816 26 8400 37.0675 48.0109 59 7203 72 2494 85.6555 100 0000	9 8 7 6 5 4 3 2
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year.	amo tıor	otal rtiza- n end year	inve beg	naining estment inning year	Amortiza- tion during year	Total amortiza- tion end of year.	
10 УЕ	ar Life							10 УЕА	r Life
		4 per cent					5 per cent.		
1 2 3 4 5 6 7 8 9 10	\$100 0000 91 6709 83 0086 73 9998 64 6308 54 8869 44 7533 34 2144 23 2538 11 8549	\$ 8 3291 8.6623 9 0088 9 3690 9 7439 10 1336 10 5389 10 9606 11 3989 11 8549	16 26 35 45 55 65 76 88	3291 9914 0002 3692 1131 2467 7856 7462 1451 0000	92 83 72 65 56 45 35	0000 0495 7015 9361 7324 0686 9216 2672 0801	\$ 7 9505 8 3480 8 7654 9 2037 9 6638 10 1470 10 6544 11 1871 11 7464 12 3337	\$ 7 9505 16 2985 25 0639 34 2676 43 9314 54 0784 64 7328 75 9199 87 6663 100 0000	10 9 8 7 6 5 4 3 2 1
******************		6 per cent			7 per cent				
1 2 3 4 5 6 7 8 9	\$100 0000 92 4132 84 3712 75 8467 66 8107 57 2325 47 0797 36 3177 24 9099 12 8177	\$ 7 5868 8 0420 8 5245 9 0360 9 5782 10 1528 10.7620 11 4078 12 0922 12 8177	15 24 33 42 52 63 75 87	5868 6288 1533 1893 7675 9203 6823 0901 1823	92 85 76 62 58 48 32	7623 7623 70179 7314 8648 3776 2263 3644 7421 3063	\$ 7 2377 7 7444 8 2865 8 8666 9 4872 10 1513 10 8619 11 6223 12 4358 13 3063	\$ 7 2377 14 9821 23 2686 32 1352 41 6224 51 7737 62 6356 74 2579 86 6937 100.0000	10 9 8 7 6 5 4 3 2
15 YE	ar Life							15 YEA	r Life
		4 per cent					5 per cent.		
1 2 3 4 5 6	\$100 0000 95 0059 89 8120 84 4104 78 7927 72 9503 66 8743	\$ 4 9941 5 1939 5 4016 5 6177 5 8424 6 0760 6 3192	10 15 21 27 33	9941 1880 .5896 2073 0497 1257 4449	95 96 85 86	9 0000 3658 9 4998 3906 9 0260 1 3930 4784	\$ 4 6342 4 8660 5 1092 5-3646 5 6330 5 9146 6 2103	\$ 4 6342 9 5002 14.6094 19 9740 25 6070 31 5216 37 7319	15 14 13 12 11 10 9
	Present value	Current depreciation during year				resent alue.	Current depreciation during year		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

	(Continued)						
Year.	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
15 YE	ar Life					15 YEAR	R LIFE
		4 per cent.			5 per cent		
8 9 10	\$60 5551 53 9832 47 1484	\$6 5719 6 8348 7 1081	\$46 0168 52 8516 59 9597	\$62 2681 55 7472 48 9004	\$6 5209 6 8468 7 1892	\$44 2528 51 0996 58 2888	8 7 6
11 12 13 14 15	40 0403 32 6478 24 9596 16 9639 8 6483	7 3925 7 6882 7 9957 8 3156 8 6483	67 3522 75 0404 83 0361 91 3517 100 0000	41 7112 34 1625 26 2364 17 9140 9 1755	7 5487 7 9261 8 3224 8 7385 9 1755	65 8375 73 7636 82 0860 90 8245 100 0000	5 4 3 2 1
		6 per cent			7 per cent		
1 2 3 4 5	\$100 0000 95 7037 91 1497 86 3224 81 2054	\$4 2963 4 5540 4 8273 5 1170 5 4239	\$ 4 2963 8 8503 13 6776 18 7946 24 2185	\$100 0000 96 0205 91 7625 87 2064 82 3314	\$ 3 9795 4 2580 4 5561 4 8750 5 2162	\$ 3 9795 8 2375 12 7936 17 6686 22 8848	15 14 13 12
6 7 8 9 10	75 7815 70 0322 63 9378 57 4777 50 6301	5 7493 6 0944 6 4601 6 8476 7 2584	29 9678 36 0622 42 5223 49 3699 56 6283	77 1152 71 5338 65 5616 59 1715 52 3340	5 5814 5 9722 6 3901 6 8375 7 3160	28 4662 34 4384 40 8285 47 6660 54 9820	10 9 8 7 6
11 12 13 14 15	43 3717 35 6776 27 5221 18 8771 9 7135	7 6941 8 1555 8 6450 9 1636 9 7135	64 3224 72 4779 81 1229 90 2865 100 0000	45 0180 37 1898 28 8136 19 8511 10 2612	7 8282 8 3762 8 9625 9 5899 10 2612	62 8102 71 1864 80 1489 89 7388 100 0000	5 4 3 2 1
20 YE	ar Life					20 YEA	r Life
		4 per cent.			5 per cent		
1 2 3 4 5	\$100 0000 96 6418 93 1493 89 5171 85 7396 81 8110	\$3 3582 3 4925 3 6322 3 7775 3 9286 4 0857	\$ 3 3582 6 8507 10 4829 14 2604 18 1890 22 2747	\$100 0000 96 9757 93 8002 90 4660 86 9650 83 2890	\$3 0243 3 1755 3 3342 3 5010 3 6760 3 8598	\$ 3 0243 6 1998 9 5340 13 0350 16 7110 20 5708	20 19 18 17 16 15
	Present value.	Current depreciation during year.		Present value	Current depreciation during year.		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
20 YE	AR LIFE					20 YEA	r Life
		4 per cent.			5 per cent.		
7 8 9 10	\$77 7253 73 4761 69 0570 64 4611 59 6814 54 7105	\$4 2492 4 4191 4 5959 4 7797 4 9709 5 1698	\$26 5239 30 9430 35 5389 40 3186 45 2895 50 4593	\$79 4292 75 3764 71 1210 66 6528 61 9612 57 9359	\$4 0528 4 2554 4 4682 4 6916 4 9262 5 1725	\$24.6236 28 8790 33 3472 38 0388 42 9650 48 1375	14 13 12 11 10 9
13 14 15	49 5407 44 1641 38.5725	5 3766 5 5916 5 8152	55 8359 61 4275 67 2427	51 8625 46 4314 40 7287	5 4311 5 7027 5 9878	53 5686 59 2713 65 2591	8 7 6
16 17 18 19 20	32 7573 26 7094 20 4196 13 8782 7 0751	6 0479 6 2898 6 5414 6 8031 7 0751	73 2906 79 5804 86 1218 92 9249 100 0000	34 7409 28 4537 21 8521 14 9204 7 6422	6 2872 6 6016 6 9317 7 2782 7 6422	71 5463 78.1479 85.0796 92.3578 100 0000	5 4 3 2 1
		6 per cent			7 per cent.		
1 2 3 4 5	\$100 0000 97 2815 94 4000 91 3455 88 1078	\$2 7185 2 8815 3 0545 3 2377 3 4320	\$ 2 7185 5 6000 8 6545 11 8922 15 3242	\$100 0000 97 5607 94 9507 92 1579 89 1697	\$2 4393 2 6100 2 7928 2 9882 3 1974	\$ 2 4393 5 0493 7 8421 10.8303 14 0277	20 19 18 17 16
6 7 8 9 10	84 6758 81 0379 77 1817 73 0942 68 7614	3 6379 3 8562 4 0875 4 3328 4 5928	18 9621 22 8183 26 9058 31 2386 35 8314	85 9723 82 5510 78 8903 74 9734 70 7822	3 4213 3 6607 3 9170 4 1911 4 4846	17 4490 21.1097 25.0267 29.2178 33 7024	15 14 13 12 11
11 12 13 14 15	64 1686 59 3002 54 1398 48 6697 42 8715	4 8683 5 1604 5 4701 5.7982 6 1462	40 6998 45 8602 51 3303 57 1285 63 2747	66 2976 61 4992 56 3648 50 8711 44 9928	4 7984 5 1344 5 4937 5 8783 6 2898	38 5008 43 6352 49 1289 55 0072 61 2970	9 8 7 6
16 17 18 19 20	36 7253 30 2104 23 3046 15 9844 8.2250	6 5149 6 9058 7 3202 7 7594 8 2250	69 7896 76 6954 84 0156 91 7750 100 0000	38 7030 31.9729 24 7717 17 0664 8 8218	6 7301 7 2012 7.7053 8 2446 8 8218	68 0271 75 2283 82 9336 91.1782 100 0000	5 4 3 2 1
	Present value	Current deprectation during year.		Present value	Current depreciation during year		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

	(Constitution)						
Year	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
25 YE	ar Life					25 YEA	r Life
		4 per cent			5 per cent		
1 2 3 4 5	\$100 0000 97 5988 95 1016 92 5044	\$2 4012 2 4972 2 5972 2 7010	\$ 2 4012 4 8984 7 4956 10 1966	\$100 0000 97 9048 95 7047 93 3947	2 2001 2 3100 2 4254	\$ 2 0952 4 2952 6 6053 9 0307	25 24 23 22 21
6 7 8 9	89 8034 86 9943 84 0729 81 0346 77 8748 74 5886	2 8091 2 9214 3 0383 3 1598 3 2862 3 4176	13 0057 15 9271 18 9654 22 1252 25 4114 28 8290	90 9693 88 4225 85 7483 82 9105 79 9923 76 8966	2 5468 2 6742 2 8078 2 9482 3 0957 3 2504	11 5775 14 2517 17 0595 20 0077 23 1034 26 3538	20 19 18 17
11 12 13 14 15	71 1710 67 6166 63 9201 60 0757 56 0775	3 5544 3 6965 3 8444 3 9982 4 1581	32 3834 36.0799 39 9243 43 9225 48 0806	73 6462 70 2333 66 6497 62 8870 58 9361	3 4129 3 5836 3 7627 3 9509 4.1485	29 7667 33 3503 37 1130 41 0639 45 2124	15 14 13 12
16 17 18 19 20	51 9194 47 5950 43 0976 38 4203 33 5560	4 3244 4 4974 4 6773 4 8643 5 0590	52 4050 56 9024 61 5797 66 4440 71 5030	54 7876 50 4317 45 8581 41 0557 36 0133	4 3559 4 5736 4 8024 5 0424 5 2946	49 5683 54 1419 58.9443 63 9867 69 2813	9 8 7 6
21 22 23 24 25	28 4970 23 2357 17 7639 12 0733 6 1550	5 2613 5 4718 5 6906 5 9183 6 1550	76 7643 82 2361 87 9267 93 8450 100 0000	30 7187 25 1594 19 3221 13 1930 6 7574	5 5593 5 8373 6 1291 6 4356 6 7574	74 8406 80 6779 86 8070 93 2426 100 0000	5 4 3 2 1
		6 per cent.			7 per cent.		
1 2 3 4 5 6 7 8 9	\$100 0000 98 1773 96 2453 94.1973 92.0265 89 7254 87 2863 84.7008 81 9602 79.0551	\$ I 8227 I.9320 2 0480 2.1708 2 3011 2 4391 2 5855 2 7406 2 9051 3 0794	\$ I 8227 3 7547 5 8027 7 9735 10 2746 12 7137 15 2992 18 0398 20 9449 24 0243	\$100 0000 98 4189 96 7272 94 9171 92 9802 90 9078 88 6903 86 3175 83 7787 81 0622	\$1.5811 1 6917 1.8101 1.9369 2.0724 2 2175 2.3728 2 5388 2 7165 2.9067	\$ 1.5811 3 2728 5 0829 7 0198 9 0922 11 3097 13.6825 16 2213 18.9378 21.8445	25 24 23 22 21 20 19 18 17 16
	Present value.	Current depreciation during year		Present value.	Current depreciation during year.	,	Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	
25 YE	AR LIFE					25 YEAR	LIFE
		6 per cent.			7 per cent		
11 12 13 14 15	\$75 9757 72 7116 69 2516 65 5841 61 6964	\$3 2641 3 4600 3 6675 3 8877 4 1208	\$27 2884 30 7484 34 4159 38 3036 42 4244	\$78 1555 75 9453 71 7174 68 1566 64 3465	\$3 1102 3 3279 3 5608 3 8101 4 0768	\$24 9547 28 2826 31 8434 35 6535 39 73°3	15 14 13 12 11
16 17 18 19 20	57 5756 53 2074 48 5772 43 6691 38 4666	4 3682 4 6302 4 9081 5 2025 5 5147	46.7926 51 4228 56 3309 61 5334 67 0481	60 2697 55 9075 51 2400 46 2458 40 9019	4 3622 4 6675 4 9942 5 3439 5.7179	44 0925 48 7600 53 7542 59 0981 64 8160	9 8 7 6
21 22 23 24 25	32 9519 27 1064 20 9101 14 3420 7 3799	5 8455 6 1963 6 5681 6 9621 7 3799	72 8936 79 0899 85 6580 92 6201 100 0000	35 1840 29 0659 22 5194 15 5147 8 0197	6 1181 6 5465 7 0047 7 4950 8 0197	70 9341 77 4806 84 4853 91 9803 100 0000	5 4 3 2 1
30 YE	AR LIFE					30 УЕА	R LIFE
		4 per cent.			5 per cent		
1 2 3 4 5 6 7 8	\$100 0000 98.2170 96 3627 94.4342 92 4285 90 3426 88.1733	\$1 7830 1 8543 1 9285 2 0057 2 0859 2 1693 2 2560	\$ 1 7830 3 6373 5 5658 7 5715 9 6574 11 8267 14 0827	\$100 0000 98 4949 96 9145 95.2550 93 5126 91 6831 89 7621	\$1 5051 1 5804 1.6595 1 7424 1 8295 1 9210 2 0170	\$ 1 5051 3 0855 4 7450 6 4874 8.3169 10 2379 12 2549	30 29 28 27 26 25 24
9 10	85 9173 83 5710 81 1308	2 3463 2 4402 2 5378	16 4290 18 8692 21 4070	87.7451 85 6272 83 4034	2 1179 2 2238 2 3349	14 3728 16 5966 18 9315	23 22 21
11 12 13 14 15	78 5930 75 9537 73 2088 70 3543 67.3854	2 6393 2 7449 2 8545 2 9689 3 0877	24 0463 26 7912 29 6457 32.6146 35 7023	81 0685 78 6168 76.0425 73 3394 70 5013	2 4517 2 5743 2 7031 2 8381 2 9801	21 3832 23 9575 26 6606 29 4987 32 4788	20 19 18 17 16
16 17	64 2977 61 0866	3 3395	38 9134 42 2529	67 5212 64 3921	3 1291 3 2855	35 6079 38.8934	15
	Present value.	Current depreciation during year	Accrued depreciation	Present value.	Current depreciation during year	Accrued depreciation	Expect- ancy Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
30 YE	ar Life					30 Үеаг	LIFE
		4 per cent			5 per cent		
18 19 20	\$57 7471 54 2740 50 6619	\$3 473 3 6121 3.7565	\$45 7260 49 3381 53 0946	\$61 1066 57.6567 54 0344	\$3 4499 3 6223 3 8034	\$42 3433 45 9656 49 7690	13 12 11
21 22 23 24 25	46 9054 42 9986 38 9355 34 7099 30 3153	3 9068 4 0631 4 2256 4 3946 4 5704	57 0014 61 0645 65 2901 69 6847 74 2551	50 2310 46 2374 42 0442 37 6412 33 0181	3 9936 4 1932 4 4030 4 6231 4 8542	53 7626 57 9558 62 3588 66 9819 71 8361	9 8 7 6
26 27 28 29 30	25 7449 20 9917 16 0484 10 9073 5 5606	4 7532 4 9433 5 1411 5 3467 5 5606	79 0083 83 9516 89 0927 94 4394 100 0000	28 1639 23 0670 17 7152 12 0958 6 1954	5 0969 5 3518 5 6194 5 9004 6 1954	76 9330 82 2848 87 9042 93 8046 100 0000	5 4 3 2 1
		6 per cent			7 per cent		
1 2 3 4 5	\$100 0000 98 7351 97 3943 95 9731 94 4666	\$1 2649 1 3408 1 4212 1 5065 1 5969	\$ 1 2649 2 6057 4 0269 5 5334 7 1303	\$100.0000 98 9414 97 8086 96 5966 95 2997	\$1 0586 1 1328 1 2120 1 2969 1 3877	\$ 1 0586 2 1914 3 4034 4 7003 6 0880	30 29 28 27 26
6 7 8 10	92 8697 91 1770 89 3827 87 4808 85 4647	1 6927 1 7943 1 9019 2 0161 2 1370	8 8230 10 6173 12 5192 14 5353 16 6723	93 9120 92 4272 90 8385 89 1386 87 3196	1 4848 1 5887 1 6999 1 8190 1 9462	7 5728 9 1615 10 8614 12 6804 14 6266	25 24 23 22 21
11 12 13 14 15	83 3277 81 0625 78 6614 76 1162 73 4182	2 2652 2 4011 2 5452 2 6980 2.8597	18 9375 21 3386 23 8838 26.5818 29 4415	85 3734 83 2909 81 0626 78 6783 76 1271	2 0825 2 2283 2 3843 2 5512 2 7297	16 7091 18 9374 21 3217 23 8729 26.6026	20 19 18 17 16
16 17 18 19 20	70 5585 67 5271 64 3138 60.9077 57.2973	3 0314 3 2133 3 4061 3 6104 3.8271	32 4729 35.6862 39 0923 42 7027 46 5298	73.3974 70.4766 67.3513 64.0073 60.4291	2 9208 3.1253 3 3440 3 5782 3 8286	29 5234 32 6487 35 9927 39 5709 43 3995	15 14 13 12 11
	Present value	Current deprectation during year.	Accrued depreciation	Present value.	Current depreciation during year	Accrued depreciation	Expect- ancy Years

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	
30 YE	ar Life					30 УЕЛЬ	R LIFE
		6 per cent			7 per cent.		
21 22 23 24 25	\$53 4702 49 4136 45 1135 40 5554 35 7238	4 3001 4 5581 4 8316 5 1214	\$50 5864 54 8865 59 4446 64 2762 69 3976	\$56 6005 52 5039 48 1206 43 4304 38 4118	\$4 0966 4 3883 4 6902 5 0186 5 3698	\$47 4961 51 8794 56 5696 61 5882 66 9580	9 8 7 6
26 27 28 29 30	30 6024 25 1736 19 4192 13 3194 6 8537	5 4288 5 7544 6 0998 6 4657 6 8537	74 8264 80 5808 86 6806 93 146 100 0000	33 0420 27 2963 21 1484 14 5702 7 5315	5 7457 6 1479 6 5782 7 0387 7 5315	72 7037 78 8516 85 4298 92 4685 100 0000	5 4 3 2 1
35 YE	ar Life					35 YEAR	R LIFE
		4 per cent			5 per cent		
1 2 3 4 5	\$100 0000 98 6423 97 2302 95 7617 94 2344	\$1 3577 1 4121 1 4685 1 5273 1 5884	\$1 3577 2 7698 4 2383 5 7656 7 3540	\$100 0000 98 8928 97 7303 96 5096 95 2880	\$1 1072 1 1625 1 2207 1 2816 1 3458	\$1 1072 2 2697 3 4904 4 7720 6 1178	35 34 33 32 31
6 7 8 9 10	92 6460 90 9941 89 2762 87 4895 85 6314	1 6519 1 7179 1 7867 1 8581 1 9325	9 0059 10 7238 12 5105 14 3686 16 3011	93 8822 92 4691 90 9854 89 4275 87 7917	1 4131 1 4837 1 5579 1 6358 1 7176	7 5309 9 0146 10 5725 12 2083 13 9259	30 29 28 27 26
11 12 13 14 15	83 6989 81 6891 79 5989 77 4252 75 1645	2 0098 2 0902 2 1737 2 2607 2 3512	18 3109 20 4011 22 5748 24.8355 27 1867	86 0741 84 2706 82 3770 80 3887 78 3010	1 8035 1 8936 1 9883 2 0877 2 1922	15 7294 17 6230 19 6113 21 6990 23 8912	25 24 23 22 21
16 17 18 19 20	72 8133 70 3681 67 8251 65 1804 62 4299	2 4452 2 5430 2 6447 2 7505 2 8605	29 6319 32 1749 34 8196 37 5701 40.4306	76 1088 73 8071 71 3903 68 8526 66 1881	2 3017 2 4168 2 5377 2 6645 2 7978	26 1929 28 6097 31 1474 33.8119 36 6097	20 19 18 17 16
21 22 23	59 5694 56 5944 53 5005	2 9750 3 0939 3 2178	43 4056 46 4995 49 7173	63 3903 60.4527 57 3682	2 9376 3 0845 3 2388	39·5473 42 6318 45 8706	15 14 13
	Present value.	Current depreciation during year	Accrued depreciation	Present value	Current depreciation during year.	Accrued depreciation	Expect- ancy. Years.

TABLE 27 AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	
35 YE	AR LIFE					35 YEA	R LIFE
		4 per cent			5 per cent		
²⁴ 25	\$50 2827 46 9363	\$3 3464 3 4803	\$53 0637 56 5440	\$54 1294 50 7287	\$3 4007 3 5708	\$49 2713 52 8421	12 11
26 27 28 29 30	43 4560 39 8365 36 0723 32 1574 28 0860	3 6195 3 7642 3 9149 4 0714 4 2343	60 1635 63 9277 67 8426 71 9140 76 1483	47 1579 43 4087 39 4720 35 3383 30 9980	3 7492 3 9367 4 1337 4 3403 4 5572	56 5913 60 5280 64 6617 69.0020 73.5592	9 8 7 6
31 32 33 34 35	23.8517 19 4480 14 8682 10 1052 5 1517	4 4037 4 5798 4 7630 4 9535 5 1517	80 5520 85 1318 89 8948 94 8483 100 0000	26 4418 21 6557 16 6313 11 3557 5 8163	4.7851 5 0244 5 2756 5 5394 5 8163	78 3443 83 3687 88 6443 94 1837 100 0000	5 4 3 2 1
	6 per cent 7 per cent.						
1 2 3 4 5	\$100 0000 99 1026 98 1514 97 1431 96 0743	\$0 8974 0 9512 1 0083 1 0688 1 1329	\$ 0 8974 1 8486 2 8569 3 9257 5 0586	\$100 0000 99 2766 98 5026 97 6744 96 7882	\$0 7234 0 7740 0 8282 0 8862 0 9483	\$0 7234 1 4974 2.3256 3 2118 4 1601	35 34 33 32 31
6 7 8 9 10	94 9414 93 7404 92 4675 91 1182 89 6879	1 2010 1 2729 1 3493 1 4303 1 5162	6 2596 7 5325 8 8818 10 3121 11 8283	95 8399 94 8253 93 7397 92 5781 91 3352	1 0146 1 0856 1 1616 1 2429 1 3300	5 1747 6.2603 7 4219 8 6648 9 9948	29 28 27 26
11 12 13 14 15	88 1717 86 5646 84 8612 83.0554 81 1414	1 6071 1 7034 1.8058 1 9140 2 0289	13 4354 15 1388 16 9446 18 8586 20 8875	90 0052 88.5822 87 0596 85 4303 83 6871	1 4230 1.5226 1 6293 1 7432 1 8653	11 4178 12 9404 14 5697 16 3129 18 1782	25 24 23 22 21
16 17 18 19 20	79 1125 76 9619 74.6822 72 2657 69 7043	2 1506 2 2797 2 4165 2 5614 2.7152	23 0381 25 3178 27 7343 30 2957 33 0109	81 8218 79 8259 77 6903 75 4052 72.9602	1.9959 2 1356 2 2851 2.4450 2 6162	20.1741 22 3097 24 5948 27 0398 29 6560	20 19 18 17 16
2I 22	66 9891	2 8780 3 0507	35 8889 38.9396	70 3440 67 5447	2 7993 2 9953	32 4553 35 4506	15 14
	Present value.	Current depreciation during year		Present value.	Current depreciation during year		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

	(Continuea)							
Year.	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.		
35 YEA	R LIFE				e Total	35 YEAR	Life	
	(6 per cent			7 per cent.			
23 24 25 26	\$61 0604 57 8266 54 3988 50 7654	3 4278 3 6334 3 8515	\$42 1734 45 6012 49 2346 53 0861	\$64 5494 61 3445 57 9152 54 2459	\$3 2049 3 4293 3 6693 3 9262	\$38 6555 42 0848 45 7541 49 6803	13 12 11 10	
27 28 29 30	46 9139 42 8313 38 5038 33 9167	4 0826 4 3275 4 5871 4 8624	57 1687 61 4962 66 0833 70 9457	50 3197 46 1197 41 6236 36 8139	4 2010 4 4951 4 8097 5 1464	53 8813 58 3764 63 1861 68 3325	9 8 7 6	
31 32 33 34 35	29 0543 23 9002 18 4368 12 6457 6 5070	5 1541 5 4634 5 7911 6 1387 6 5070	76 0998 81 5632 87 3543 93 4930 100 0000	31 6675 26 1608 20 2686 13 9640 7 2181	5 5067 5 8922 6 3046 6 7459 7 2181	73 8392 79 7314 86 0360 92 7819 100 0000	5 4 3 2 1	
40 YE	ar Life	•				40 YEAI	LIFE	
		4 per cent.		A.	5 per cent			
1 2 3 4 5 6	\$100 0000 98 9476 97 8532 96 7150 95 5312 94 3001 93 0198	\$1 0524 1 0944 1 1382 1 1838 1 2311 1 2803 1 3316	\$ 1 0524 2 1468 3 2850 4 4688 5 6999 6 9802 8 3118	\$100 0000 99 1722 98 3030 97 3903 96 4320 95 4258 94 3693	\$0 8278 0 8692 0 9127 0 9583 1 0062 1 0565 1 1093	\$ 0 8278 1 6970 2 6097 3.5680 4 5742 5 6307 6 7400	40 39 38 37 36 35 34	
7 8 9 10	91 6882 90 3034 88 8632	1 3848 1 4402 1 4978	9 6966 11 1368 12 6346	93 2600 92 0952 90.8722	1 1648 1 2230 1.2842	7 9048 9 1278 10 4120	33 32 31	
11 12 13 14 15	87 3654 85 8077 84 1876 82 5028 80 7505	I 5577 I 6201 I 6848 I 7523 I 8223	14 1923 15 8124 17 4972 19 2495 21 0718	89 5880 88 2396 86 8238 85 3372 83 7762	1 3484 1 4158 1 4866 1 5610 1 6390	11 7604 13 1762 14 6628 16 2238 17 8628	30 29 28 27 26	
16 17 18 19 20	78.9282 77 0330 75.0620 73.0121 70 8802	1.8952 1 9710 2 0499 2 1319 2 2171	22 9670 24 9380 26 9879 29.1198 31 3369	82 1372 80 4162 78 6092 76 7118 74.7195	1 7210 1 8070 1 8974 1 9923 2 0919	19 5838 21 .3908 23 2882 25 2805 27 3724	25 24 23 22 21	
	Present value.	Current deprectation during year		Present value.	Current depreciation during year		Expect- ancy. Years.	

TABLE 27 AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
ю УЕ	ar Life					40 ҮЕАН	R LIFE
		4 per cent			5 per cent		
21 22 23 24 25	\$68 6631 66 3572 63 9592 61 4652 58 8715	\$2 3059 2 3980 2 4940 2 5937 2 6975	\$33 6428 36 0408 38 5348 41 1285 43 .8260	\$72 6276 70 4311 68 1248 65 7032 63 1605	\$2 1965 2 3063 2 4216 2 5427 2 6698	\$29 5689 31 8752 34 2968 36 8395 39.5093	20 19 18 17 16
26 27 28 29 30	56 1740 53 3686 50 4510 47 4167 44 2610	2 8054 2.9176 3 9343 3 1557 3 2819	46 6314 49 5490 52 5833 55 7390 59 0209	60 4907 57 6874 54 7440 51 6534 48 4083	2 8033 2 9434 3 0906 3 2451 3 4074	42 3126 45 2560 48 3466 51 5917 54 9991	15 14 13 12 11
31 32 33 34 35	40 9791 37 5659 34 0161 30 3245 26 4852	3 4132 3 5498 3 6916 3 8393 3 9930	62 4341 65 9839 69 6755 73 5148 77 5078	45 0009 41 4231 37 6664 33 7219 29 5802	3 5778 3 7567 3 9445 4 1417 4 3488	58 5769 62 3336 66 2781 70 4198 74 7686	9 8 7 6
36 37 38 39 40	22 4922 18 3396 14 0207 9 5292 4 8580	4 1526 4 3189 4 4915 4 6712 4 8580	81 6604 85 9793 90 4708 95 1420 100 0000	25 2314 20 6652 15 8706 10 8363 5 5503	4 5662 4 7946 5 0343 5 2860 5 5503	79 3348 84 1294 89 1637 94 4497 100 0000	5 4 3 2 1
		6 per cent.			7 per cent		
1 2 3 4 5	\$100 0000 99 3538 98 6689 97 9429 97 1733	\$0 6462 0 6849 0 7260 0 7696 0 8157	\$ 0 6462 I 33II 2 057I 2.8267 3 6424	\$100 0000 99 4991 98 9631 98 3896 97.7760	\$0 5009 0 5360 0 5735 0 6136 0 6566	\$ 0.5009 1 0369 1 6104 2 2240 2 8806	40 39 38 37 36
6 7 8 . 9	96 3576 95 4929 94 5763 93 6047 92.5748	0 8647 0.9166 0 9716 1 0299 1 0916	4 5071 5 4237 6 3953 7 4252 8 5168	97 1194 96 4168 95 6651 94 8607 94 0001	o 7026 o 7517 o.8044 o 8606 o 9210	3 5832 4 3349 5 1393 5 9999 6 9209	35 34 33 32 31
11 12 13 14 15	91 4832 90 3260 89 0994 87 7992 86.4210	I 1572 I.2266 I 3002 I 3782 I.4609	9 6740 10 9006 12.2008 13 5790 15 0399	93.0791 92 0938 91 0394 89 9113 88 7042	0.9853 1 0544 1 1281 1.2071 1.2917	7 9062 8 9606 10 0887 11 2958 12 5875	30 29 28 27 26
	Present value.	Current depreciation during year		Present value.	Current depreciation during year		Expect ancy. Years

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

			<u> </u>				
Year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	
40 YE	ar Life					40 YEAR	LIFE
		6 per cent			7 per cent		
16 17 18 19 20	\$84 9601 83 4116 81 7701 80 0302 78 1859	1 6415 1 7399 1 8443	\$16 5884 18 2299 19 9698 21 8141	\$87 4125 86 0304 84 5517 82 9694	\$1 3820 1 4788 1 5823 1 6930	\$13 9695 15 4483 17 0306 18 7236	25 24 23 22
21 22 23 24 25	76 2309 74 1585 71 9619 69 6335 67 1653	1 9550 2 0724 2 1966 2 3284 2 4682 2 6162	23 7691 25 8415 28 0381 30 3665 32 8347 35 4509	81 2764 79 4648 77 5264 75 4523 73 2331 70 8585	1 8116 1 9384 2 0741 2 2192 2 3746 2 5408	20 5352 22 4736 24 5477 26 7669 29.1415 31 6823	21 20 19 18 17 16
26 27 28 29 30	64 5491 61 7759 58 8363 55 7203 52 4174	2 7732 2 9396 3 1160 3 3029 3 5011	38 2241 41 1637 44 2797 47 5826 51 0837	68 3177 65 5990 62 6900 59 5774 56 2469	2 7187 2 9090 3 1126 3 3305 3 5636	34 4010 37 3100 40 4226 43 7531 47.3167	15 14 13 12
31 32 33 34 35	48 9163 45 2051 41 2712 37 1013 32 6813	3 7112 3 9339 4 1699 4 4200 4 6853	54 7949 58 7288 62 8987 67 3187 72 0040	52 6833 48 8702 44 7902 40 4246 35 7534	3 8131 4 0800 4 3656 4 6712 4 9982	51.1298 55 2098 59 5754 64 2466 69 2448	10 9 8 7 6
36 37 38 39 40	27 9960 23 0296 17.7652 12.1850 6.2699	4 9664 5 2644 5 5802 5 9151 6 2699	76.9704 82 2348 87 8150 93 7301 100 0000	30 7552 25 4072 19 6847 13 5618 7 0102	5 3480 5 7 ²² 5 6 1229 6 5516 7 0102	74 5928 80 3153 86 4382 92 9898 100 0000	5 4 3 2 1
45 YE	ar Life					45 YEA	r Life
		4 per cent			5 per cent		
3 4 5 6 7	\$100 0000 99 1738 98 3145 97 4208 96 4914 95 5248 94 5195 93 4740	\$0 8262 0 8593 0 8937 0 9294 0 9666 1 0053 1 0455 1 0872	\$0 8262 1 6855 2 5792 3 5086 4 4752 5 4805 6 5260 7 6132	\$100 0000 99 3738 98 7163 98 0260 97 3011 96 5400 95 7408 94 9016	\$0 6262 0 6575 0 6903 0 7249 0 7611 0 7992 0 8392 0 8810	\$0 6262 1 2837 1 9740 2 6989 3 4600 4 2592 5 0984 5 9794	45 44 43 42 41 40 39 38
	Present value	Current depreciation during year	L	Present value.	Current depreciation during year		Expect- ancy Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
45 YE	AR LIFE				<u> </u>	45 YEA	r Life
		4 per cent			5 per cent.		
10	\$92 3868	\$1 1308	\$ 8 7440	\$94 0206	\$0 9251	\$ 6 9045	37
	91 2560	1 1760	9 9200	93 0 955	0 9714	7 8759	36
11	90 0800	I 2230	11 1430	92 1241	1 0200	8 8959	35
12	88 8570	I 2720	12 4150	91 1041	1 0710	9 9669	34
13	87 5850	I 3229	13 7379	90 0311	1 1245	11 0914	33
14	86 2621	I 3757	15 1136	88 9086	1 1807	12 2721	32
15	84 8864	I 4308	16 5444	87 7279	1 2398	13 5119	31
16	83 4556	1 4880	18 0324	86.4881	1 3018	14 8137	30
17	81 9676	1 5476	19 5800	85 1863	1 3668	16 1805	29
18	80 4200	1 6094	21 1894	83 8195	1 4352	17 6157	28
19	78 8106	1 6738	22 8632	82 3843	1 5070	19 1227	27
20	77 1368	1 7408	24 6040	80 8773	1 5823	20 7050	26
21	75 3960	1 8104	26 4144	79 2950	1 6614	22 3664	25
22	73 5856	1 8828	28 2972	77 6336	1 7445	24 1109	24
23	71 7028	1 9581	30 2553	75 8891	1 8317	25 9426	23
24	69 7447	2 0365	32 2918	74 0574	1 9223	27 8659	22
25	67 7082	2 1180	34 4098	72 1341	2 0195	29 8854	21
26	65 5902	2 2026	36 6124	70 1146	2 1205	32 0059	20
27	63 3876	2 2907	38 9031	67 9941	2 2264	34 2323	19
28	61 0969	2 3824	41 2855	65 7677	2 3378	36 5701	18
29	58 7145	2 4777	43 7632	63 4299	2 4548	39 0249	17
30	56 2368	2 5768	46 3400	60 9751	2 5773	41 6022	16
31 32 33 34 35	53 6600 50 9802 48 1932 45 2946 42 2802	2 6798 2 7870 2 8986 3 0144 3 1350	49 0198 51 8068 54.7054 57 7198 60 8548	58 3978 55 6916 52 8499 49 8662 46 7334	2 7062 2 8417 2 9837 3 1328 3 2896	44 3084 47 1501 50 1338 53.2666 56 5562	15 14 13 12
36	39 1452	3 2605	64 1153	43 4438	3 4539	60 0101	10
37	35 8847	3 3909	67 5062	39 9899	3 6267	63.6368	9
38	32 4938	3 5264	71 0326	36.3632	3 8081	67 4449	8
39	28.9674	3 6676	74 7002	32 5551	3 9983	71 4432	7
40	25.2998	3.8142	78 5144	28 5568	4 1984	75.6416	6
41 42 43 44 45	21 4856 17.5187 13.3932 9.1027 4 6406	3 9669 4 1255 4 2905 4 4621 4 6406	82 4813 86 6068 90 8973 95 3594 100.0000	24 3584 19 9501 15 3215 10 4615 5.3583	4 4083 4 6286 4 8600 5.1032 5 3583	80 0499 84 6785 89 5385 94 6417	5 4 3 2 1
	Present value.	Current depreciation during year		Present value.	Current depreciation during year		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year.	
45 YE.	ar Life					45 YEAR	LIFE
		6 per cent.			7 per cent.		
1	\$100 0000	\$0 4700	\$ 0 4700	\$100 0000	\$0 3500	\$ 0 3500	45
2	99 5300	0 4983	0 9683	99.6500	0 3744	0 7244	44
3	99 0317	0 5281	1 4964	99.2756	0 4007	1 1251	43
4	98 5036	0 5599	2 0563	98.8749	0 4287	1 5538	42
5	97 9437	0 5934	2 6497	98.4462	0 4587	2 0125	41
6 7 8 10	97 35°3 96 7213 96 °545 95 3477 94 5985	o 6290 o 6668 o 7068 o 7492 o 7941	3 2787 3 · 9455 4 6523 5 4015 6 · 1956	97 9875 97 4967 96 9715 96 4095 95.8082	0 4908 0 5252 0 5620 0 6013 0 6434	2 5033 3 0285 3 5905 4 1918 4 8352	40 39 38 37 36
11	93 8044	0 8418	7 9374	95 1648	o 6884	5 5236	35
12	92 9626	0 8923	7 9297	94 4764	o 7366	6 2602	34
13	92 0703	0 9458	8 8755	93 7398	o 7882	7 0484	33
14	91 1245	1 0026	9 8781	92.9516	o 8433	7 8917	32
15	90 1219	1 0628	10 9409	92 1083	o 9024	8 7941	31
16	89 0591	1 1265	12 0674	91 2059	0 9655	9 7596	30
17	87 9326	1 1941	13 2615	90.2404	1 0331	10 7927	29
18	86 7385	1 2657	14.5272	89 2073	1 1055	11 8982	28
19	85 4728	1 3417	15 8689	88 1018	1 1828	13 0810	27
20	84.1311	1.4222	17 2911	86 9190	1 2657	14 3467	26
21	82 7089	1 5075	18 7986	85 6533	I 3542	15 7009	25
22	81 2014	1 5979	20.3965	84 2991	I 4490	17 1499	24
23	79.6035	1.6939	22.0904	82 8501	I 5504	18 7003	23
24	77 9096	1.7954	23.8858	81 2997	I 6590	20 3593	22
25	76 1142	1.9032	25.7890	79.6407	I 775I	22 1344	21
26	74 2110	2.0174	27 8064	77 8656	1 8994	24 0338	20
27	72 1936	2 1385	29 9449	75 9662	2 0323	26 0661	19
28	70 0551	2 2667	32 2116	73 9339	2 1746	28 2407	18
29	67 7884	2.4028	34 6144	71.7593	2 3268	30 5675	17
30	65 3856	2 5469	37.1613	69 4325	2 4897	33 0572	16
31	62 8387	2 6997	39 8610	66 9428	2 6640	35.7212	15
32	60.1390	2.8617	42.7227	64 2788	2 8504	38.5716	14
33	57.2773	3 0334	45.7561	61 4284	3 0499	41 6215	13
34	54 2439	3.2154	48 9715	58 3785	3 2635	44.8850	12
35	51 0285	3 4083	52 3798	55 1150	3 4920	48.3770	11
	Present value.	Current deprecia- tion during year.		Present value.	Current deprecia- tion during year.	5	Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

			,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,				
Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
45 YE	ar Life					45 YEA	R LIFE
		6 per cent			7 per cent		
36 37 38 39 40	\$47 6202 44 0073 40 1777 36 1183 31 8153	\$3 6129 3 8296 4 0594 4 3030 4 5611	\$55 9927 59 8223 63 8817 68 1847 72 7458	\$51 6230 47 8867 43 8888 39 6111 35 9339	\$3 7363 3 9979 4 2777 4 5772 4 8976	\$52 1133 56 1112 60 3889 64.9661 69 8637	10 9 8 7 6
41 42 43 44 45	27 2542 22 4194 17 2945 11 8621 6 1038	4 8348 5 1249 5 4324 5 7583 6 1038	77 5806 82 7055 88 1379 93 8962 100 0000	30 1363 24 8959 19 2886 13 2889 6 8692	5 2404 5 6073 5 9997 6 4197 6 8692	75 1041 80 7114 86 7111 93 1308 100 0000	5 4 3 2 1
50 YE	ar Life					50 YEA	R LIFE
		4 per cent			5 per cent		
1 2 3 4 5	\$100 0000 99 3450 98 6638 97 9553 97 2185	\$0 6550 0 6812 0 7085 0 7368 0 7663	\$ 0 6550 1 3362 2 0447 2 7815 3 5478	\$100 0000 99 5223 99 0208 98 4941 97 9412	\$0 4777 0 5015 0 5267 0 5529 0 5806	\$ 0 4777 0 9792 1 5059 2 0588 2 6394	50 49 48 47 46
6 7 8 10	96 4522 95 6553 94 8265 93 9645 93 0681	o 7969 o 8288 o 8620 o 8964 o 9323	4 3447 5 1735 6 0355 6 9319 7 8642	97 3606 96 7509 96 1108 95 4386 94 7329	o 6097 o 6401 o 6722 o 7057 o 7410	3 2491 3 8892 4 5614 5 2671 6 0081	45 44 43 42 41
11 12 13 14 15	92 1358 91 1662 90 1578 89 1091 88 0185	0 9696 1 0084 1 0487 1 0906 1 1343	8 8338 9 8422 10 8909 11 9815 13 1158	93 9919 93 2138 92 3968 91 5390 90 6382	o 7781 o 8170 o 8578 o 9008 o 9457	6 7862 7 6032 8 4610 9 3618 10 3075	40 39 38 37 36
16 17 18 19 20	86 8842 85 7045 84 4777 83 2018 81.8748	1.1797 1.2268 1 2759 1.3270 1 3800	14 2955 15 5223 16 7982 18 1252 19 5052	89 6925 88 6994 87 6567 86 5619 85 4123	0 9931 1 0427 1 0948 1 1496 1 2071	11 3006 12 3433 13 4381 14 5877 15 7948	35 34 33 32 31
21 22 23	80 4948 79 0595 77.5669	1.4353 1 4926 1 5523	20 9405 22 4331 23 9854	84.2052 82 9378 81 6017	I 2674 I 3307 , I 3974	17 0622 18 3929 19 7903	30 29 28
	Present value.	Current depreciation during year		Present value	Current depreciation during year		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
50 YE.	ar Life					50 YEAR	LIFE
		4 per cent			5 per cent.		
24 25	\$76 0146 74 4002	\$1 6144 1 6791	\$25 5998 27 2789	\$80 2097 78 7425	\$1 4672 1 5405	\$2I 2575 22 7980	27 26
26 27 28 29 30	72 7211 70 9750 69 1589 67 2702 65 3060	1 7461 1 8161 1 8887 1 9642 2 0427	29 0250 30 8411 32 7298 34 6940 36 7367	77 2020 75 5844 73 8860 72 1026 70 2300	1 6176 1 6984 1 7834 1 8726 1 9661	24 4156 26 1140 27 8974 29 7700 31 7361	25 24 23 22 21
31 32 33 34 35	63 2633 61 1388 58 9293 56 6314 54 2417	2 1245 2 2095 2 2979 2 3897 2 4854	38 8612 41 0707 43 3686 45 7583 48 2437	68 2639 66 1994 64 0317 61 7556 59 3657	2 0645 2 1677 2 2761 2 3899 2 5095	33 8006 35 9683 38 2444 40 6343 43 1438	20 19 18 17 16
36 37 38 39 40	51 7563 49 1716 46 4834 43 6877 40 7802	2 5847 2 6882 2 7957 2 9075 3 0238	50 8284 53 5166 56 3123 59 2198 62 2436	56 8562 54 2215 51 4549 48 5499 45 4998	2 6347 2 7666 2 9050 3 0501 3 2027	45 7785 48 5451 51 4501 54 5002 57 7029	15 14 13 12
41 42 43 44 45	37 7564 34 6116 31 3410 27 9397 24 4022	3 1448 3 2706 3 4013 3 5375 3 6789	65 3884 68 6590 72 0603 75 5978 79 2767	42 2971 38 9342 35 4033 31 6958 27.8029	3 3629 3 5309 3 7075 3 8929 4 0875	61.0658 64.5947 68.3042 72.1971 76.2846	9 8 7 6
46 47 48 49 50	20 7233 16 8972 12 9181 8 7798 4 4760	3 8261 3 9791 4 1383 4 3038 4 4760	83 1028 87 0819 91 2202 95 5240 100 0000*	23.7154 19 4235 14 9170 10 1851 5 2168	4 2919 4 5065 4 7319 4 9683 5 2168	80 5765 85 0830 89 8149 94 7832 100 0000	5 4 3 2
		6 per cent.			7 per cent		
1 2 3 4 5	\$100 0000 99 6556 99 2905 98 9035 98 4933 98 0584	\$0 3444 0 3651 0 3870 0 4102 0 4349 0 4609	\$0 3444 0 7095 1 0965 1 5067 1 9416 2 4025	\$100 0000 99 7540 99 4908 99 2092 98 9078 98 5852	0 2632 0 2816 0 3014 0 3224	1 4146	50 49 48 47 46 45
	Present value	Current depreciation during year		Present value.	Current depreciatio during yea		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year	
50 YE	ar Life					50 Үел	r Life
		6 per cent		•	7 per cent		
7 8 9 10	\$97 5975 97 1089 96 5910 96 0421 95 4602 94 8433	\$0 4886 0 5179 0 5489 0 5819 0 6169 0 6538	\$ 2 8911 3 4090 3 9579 4 5398 5 1567 5 8105	\$98 2404 97 8712 97 4762 97 0536 96 6014 96 1175	\$0 3692 0 3950 0 4226 0 4522 0 4839 0 5178	\$ 2 1288 2 5238 2 9464 3 3986 3 8825 4 4003	44 43 42 41 40 39
13 14 15	94 1895 93 4964 92 7618	o 6931 o 7346 o 7787	6 5036 7 2382 8 0169	95 5997 95 0457 94 4529	o 5540 o 5928 o 6343	4 9543 5 5491 6 1814	38 37 36
16 17 18 19 20	91 9831 91 1576 90 2827 89 3552 88 3721	0 8255 0 8749 0 9275 0 9831 1 0421	8 8424 9 7173 10 6448 11 6279 12 6700	93 8186 93 1400 92 4138 91 6367 90 8053	o 6786 o 7262 o 7771 o 8314 o 8896	6 8600 7 5862 8 3633 9 1947 10 0843	35 34 33 32 31
21 22 23 24 25	87 3300 86 2254 85 0544 83 8133 82 4977	1 1046 1 1710 1 2411 1 3156 1 3946	13 7746 14 9456 16.1867 17 5023 18 8969	89 9157 88 9638 87 9453 86 8555 85 6894	0 9519 1 0185 1 0898 1.1661 1.2477	11 0362 12 0547 13 1445 14 3106 15 5583	29 28 27 26
26 27 28 29 30	81 1031 79 6249 78 0579 76 3969 74 6363	1 4782 1 5670 1 6610 1 7606 1 8662	20 3751 21 9421 23 6031 25 3637 27 2299	84.4417 83 1066 81 6781 80.1496 78 5141	1.3351 1 4285 1 5285 1.6355 1.7500	16 8934 18 3219 19 8504 21 4859 23 2359	25 24 23 22 21
31 32 33 34 35	72 7701 70 7918 68 6949 66 4722 64 1161	1.9783 2 0969 2 2227 2 3561 2 4975	29 2082 31 3051 33.5278 35 8839 38 3814	76 7641 74.8916 72 8880 70.7442 68 4503	1 8725 2.0036 2 1438 2.2939 2.4545	25 1084 27 1120 29 2558 31 5497 34 0042	20 19 18 17 16
36 37 38 39 40	61 6186 58 9713 56 1652 53 1907 50.0377	2 6473 2 8061 2 9745 3 1530 3 3422	41.0287 43.8348 46 8093 49 9623 53 3045	65 9958 63 3695 60 5594 57 5526 54 3353	2.6263 2 8101 3 0068 3.2173 3 4425	36 6305 38 4406 42 4474 45 6647 49 1072	15 14 13 12 11
41	46 6955	3 5427	56 8472	50.8928	3 6835	52 7907	10
	Present value	Current deprecia- tion during year.		Present value	Current deprecia- tion during year.		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

			(Cont.	inueu)	_		
Year	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year	
50 YE	ar Life					50 YEAR	R LIFE
		6 per cent			7 per cent		
42 43 44 45 46 47 48	\$43 1528 39 3975 35 4170 31 1976 26 7250 21 9840 16 9587	\$3 7553 3 9805 4 2194 4 4726 4 7410 5 0253 5 3270	\$60 6025 64 5830 68 8024 73 2750 78.0160 83 0413 88 3683	\$47 2093 43 2679 39 0507 34 5383 29 7099 24 5437 19 0157	\$3 9414 4 2172 4 5124 4 8284 5 1662 5 5280 5 9149	\$56 7321 60 9493 65 4617 70 2901 75 4563 80 9843 86 8992	9 8 7 6 5 4
49 50	11 6317	5 6465 5 9852	94 0148	13 1008	6 3289	93 2281	3 2 I
60 YE	ar Life				L	60 YEAR	R LIFE
	4 per cent			5 per cent			
1 2 3 4 5	\$100 0000 99 5798 99 1428 98 6884 98 2157	\$0 4202 0 4370 0 4544 0 4727 0 4916	\$ 0 4202 0 8572 1 3116 1 7843 2 2759	\$100 0000 99 7172 99 4202 99 1084 98 7810	\$0 2828 0 2970 0 3118 0 3274 0 3437	\$ 0 2828 0 5798 0 8916 1 2190 1.5627	60 59 58 57 56
6 7 8 9 10	97 7241 97 2129 96 6812 96 1283 95 5533	0 5112 0 5317 0 5529 0 5750 0 5981	2 7871 3 3188 3 8717 4 4467 5 0448	98 4373 98 0763 97 6973 97 2993 96 8815	0 3610 0 3790 0 3980 0 4178 0 4388	1 9237 2 3027 2 7007 3 1185 3 5573	55 54 53 52 51
11 12 13 14 15	94 9552 94 3332 93 6864 93 0136 92 3140	0 6220 0 6468 0 6728 0 6996 0 7276	5 6668 6 3136 6 9864 7 6860 8 4136	96 4427 95.9821 95 4984 94 9905 94 4572	o 4606 o 4837 o 5079 o 5333 o 5600	4 0179 4 5016 5 0095 5 5428 6 1028	50 49 48 47 46
16 17 18 19 20	91 5864 90 8297 90 0427 89 2242 88 3730	o 7567 o 7870 o 8185 o.8512 o 8853	9 1703 9 9573 10 7758 11 6270 12 5123	93 8972 93 3092 92 6918 92 0437 91 3630	o 5880 o 6174 o 6481 o.6807 o 7146	6 6908 7 3082 7 9563 8 6370 9 3516	45 44 43 42 41
21 22 23	87 4877 86 5670 85.6095	0.9207 0 9575 0 9958	13 4330 14 3905 15 3863	90 6484 89 8980 89 1100	o 7504 o 7880 o 8273	10 1020 10 8900 11.7173	40 39 38
	Present value.	Current depreciation during year		Present value	Current depreciation during year		Expect- ancy Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
60 ҮЕ	ar Life					60 Уел	r Life
		4 per cent		5 per cent			
24	\$84 6137	\$1 0356	\$16 4219	\$88 2827	\$0 8686	\$12 5859	37
25	83 5781	1 0771	17 4990	87 4141	0 9122	13 4981	36
26	82 5010	1 1201	18 6191	86 5019	0 9577	14 4558	35
27	81 3809	1 1650	19 7841	85 5442	1 0056	15 4614	34
28	80 2159	1 2115	20 9956	84 5386	1 0559	16 5173	33
29	79 0044	1 2600	22 2556	83 4827	1 1087	17 6260	32
30	77 7444	1 3104	23 5660	82 3740	1 1641	18 7901	31
31	76 4340	I 3629	24 9289	81 2099	1 2223	20 0124	30
32	75 0711	I 4173	26 3462	79 9876	1 2835	21 2959	29
33	73 6538	I 4741	27 8203	78 7041	1 3476	22 6435	28
34	72 1797	I 5329	29 3532	77 3565	1 4149	24 0584	27
35	70 6468	I 5944	30 9476	75 9416	1 4858	25 5442	26
36	69 0524	1 6580	32 6056	74 4558	1 5600	27 1042	25
37	67 3944	1 7245	34 3301	72 8958	1 6380	28 7422	24
38	65 6699	1 7934	36 1235	71 2578	1 7200	30 4622	23
39	63 8765	1 8651	37 9886	69 5378	1 8059	32 2681	22
40	62 0114	1 9397	39 9283	67 7319	1 8962	34 1643	21
41	60 0717	2 0173	41 9456	65 8357	1 9911	36 1554	20
42	58 0544	2 0980	44 0436	63 8446	2 0906	38 2460	19
43	55 9564	2 1820	46 2256	61 7540	2 1951	40 4411	18
44	53 7744	2 2692	48 4948	59 5589	2 3049	42 7460	17
45	51 5052	2 3599	50 8547	57 2540	2 4201	45 1661	16
46	49 1453	2 4544	53 3091	54 8339	2 5411	47 7072	15
47	46 6909	2 5526	55 8617	52 2928	2 6682	50 3754	14
48	44 1383	2 6546	58 5163	49 6246	2 8016	53 1770	13
49	41 4837	2 7609	61.2772	46.8230	2 9416	56 1186	12
50	38 7228	2 8712	64 1484	43 8814	3 0888	59 2074	11
51 52 53 54 55	35 8516 32 8655 29 7599 26 5301 23 1711	2 9861 3 1056 3 2298 3 3590 3 4933	67 1345 70 2401 73 4699 76 8289 80 3222	40 7926 37.5494 34 1441 30 5685 26 8141	3 2432 3 4053 3 5756 3 7544 3 9421	62 4506 65.8559 69 4315 73 1859 77 1280	9 8 7 6
56	19 6778	3 6331	83 9552	22.8720	4 1392	81.2672	5
57	16 .0447	3 7784	87 7337	18 7328	4.3462	85 6134	4
58	12 2663	3 9295	91 6632	14 3866	4 5635	90 1769	3
59	8 3368	4 0867	95 7499	9 8231	4 7917	94.9686	2
60	4 2501	4.2501	100 0000	5 0314	5.0314	100 0000	1
	Present value.	Current depreciation during year		Present value.	Current deprectation during year.	Accrued depreciation	Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	
60 YE	ar Life				`	60 Үел	r Life
		6 per cent.			7 per cent		
1 2 3 4 5 6 7 8 9 10	\$100 0000 99.8124 99 6136 99 4028 99.1794 98 9426 98.6916 98 4255 98 1435 97 8446	\$0 1876 0 1988 0 2108 0 2234 0 2368 0 2510 0 2661 0 2820 0 2989 0 3169	\$ 0 1876 0 3864 0 5972 0 8206 I 0574 I 3084 I 5745 I 8565 2 1554 2 4723	\$100 0000 99 8771 99 7455 99 6048 99 4542 99 2931 99 1207 98 9362 98 7388 98 5276	\$0 1229 0 1316 0 1407 0 1506 0 1611 0 1724 0 1845 0 1974 0 2112 0 2260	\$ 0 1229 0 2545 0 3952 0 5458 0 7069 0 8793 1 0638 1 2612 1 4724 1 6984	60 59 58 57 56 55 54 53 52 51
11 12 13 14 15	97 5277 97 1917 96 8357 96 4582 96.0582	o 3360 o 3560 o 3775 o 4000 o 4241	2 8083 3 1643 3 5418 3 9418 4 3659	98 3016 98 0598 97 8011 97 5243 97 2280	o 2418 o 2587 o 2768 o 2963 o 3169	1 9402 2 1989 2 4757 2 7720 3 0889	50 49 48 47 46
16 17 18 19 20	95 6341 95 1846 94 7081 94.2030 93 6671	 4495 4765 5051 5354 5675 	4 8154 5 2919 5.7970 6 3324 6.8999	96 9111 96 5719 96 2090 95 8208 95 4053	0 3392 0 3629 0 3882 0 4154 0 4446	3 4281 3 7910 4 1792 4 5947 5 0393	45 44 43 42 41
21 22 23 24 25	93 1001 92 4985 91 8608 91 1849 90 4684	0 6016 0.6377 0 6759 0 7165 0.7594	7 5015 8.1392 8 8151 9 5316 10 2910	94 9607 94 4851 93 9761 93 4315 92.8488	0 4756 0 5089 0 5446 0 5827 0 6235	5 5149 6 0239 6 5685 7 1512 7 7747	39 38 37 36
26 27 28 29 30	89 7090 88 9039 88 0506 87 1461 86 1872	o 8051 o 8533 o 9045 o 9589 i o163	11 0961 11 9494 12 8539 13 8128 14 8291	92 2253 91 5581 90 8443 90 0804 89 2631	o 6672 o 7138 o 7638 o 8173 o 8745	8 4419 9 1557 9 9196 10 7369 11 6114	35 34 33 32 31
31 32 33 34 35	85 1709 84 0936 82 9516 81 7412 80 4581	1 0773 1 1420 1 2104 1 2831 1 3601	15 9064 17 0484 18 2588 19 5419 20 9020	88 3886 87 4529 86 4517 85 3804 84 2341	0 9357 1 0012 1 0713 1 1463 1 2265	12 5471 13 5483 14 6196 15 7659 16 9924	29 28 27 26
36 37	79 0980 77 6563	I 4417 I 5282	22 3437 23 8719	83 0076 81 6952	I 3124 I 4043	18 3048 19 7091	25 24
,	Present value	Current depreciation during year		Present value.	Current depreciation during year.		Expect- ancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

	(Continued)							
Year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year.		
60 YE	AR LIFE					60 Үел	R LIFE	
		6 per cent.			7 per cent			
38 39 40	\$76 1281 74 5082 72 7911	\$1 6199 1 7171 1 8201	\$25 4918 27.2089 29 0290	\$80 2909 78 7884 77 1806	\$1 5025 1 6078 1 7203	\$21.2116 22 8194 24.5397	23 22 21	
41 42 43 44 45	70 9710 69 0417 66 9965 64 8288 62.5310	1 9293 2 0452 2 1677 2 2978 2 4357	30 9583 33 0035 35.1712 37 4690 39 9047	75 4603 73 6196 71 6501 69.5427 67 2877	1 8407 1 9695 2 1074 2 2550 2.4127	26 3804 28 3499 30 4573 32.7123 35.1250	20 19 18 17 16	
46 47 48 49 50	60 0953 57 5134 54 7767 51 8757 48 8007	2 5819 2 7367 2.9010 3 0750 3 2596	42 4866 45 2233 48.1243 51 1993 54 4589	64 8750 62 2933 59 5309 56 5752 53 4125	2 5817 2 7624 2 9557 3 1627 3 3841	37 7067 40 4691 43 4248 46 5875 49 9716	15 14 13 12 11	
51 52 53 54 55	45 5411 42.0860 38 4236 34 5415 30 4264	3 4551 3 6624 3 8821 4 1151 4 3620	57 9140 61.5764 65 4585 69 5736 73 9356	50 0284 46 4075 42 5331 38 3875 33 9517	3 6209 3 8744 4 1456 4 4358 4 7463	53 5925 57 4669 61 6125 66 0483 70 7946	9 8 7 6	
56 57 58 59 60	26 0644 21 4407 16 5395 11 3443 5 8374	4 6237 4 9012 5 1952 5 5069 5 8374	78 5593 83 4605 88 6557 94.1626 100 0000	29 2054 24 1268 18 6928 12 8784 6 6569	5 0786 5 4340 5 8144 6 2215 6 6569	75 8732 81.3072 87.1216 93 3431 100 0000	5 4 3 2 1	
75 YE	ar Life					75 YEA	r Life	
		4 per cent.			5 per cent.			
1 2 3 4 5 6 7 8	\$100.0000 99 7771 99 5453 99 3042 99 0535 98.7927 98 5215 98.2395	\$0.2229 0 2318 0 2411 0 2507 0 2608 0.2712 0 2820 0 2934	\$0.2229 0.4547 0.6958 0.9465 1.2073 1.4785 1.7605 2.0539	\$100 0000 99 8678 99 7291 99 5834 99 4304 99 2697 99 1011 98 9239	\$0 1322 0.1387 0.1457 0 1530 0.1607 0.1686 0.1772 0.1859	\$0.1322 0.2709 0.4166 0.5696 0.7303 0.8989 1.0761 1.2620	75 . 74 . 73 . 72 . 71 . 70 . 69 . 68	
.10 ———	97.9461 97.6411	0 3050 0.3172	2 3589 2 6761	98 7380 98 5427	0.1039	1.4573 1.6623	67 66	
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expect- ancy. Years.	

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year	Remaining investment beginning of year.	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year.	Total amortiza- tion end of year.	
75 YE	AR LIFE					75 YEA	R LIFE
		4 per cent		5 per cent			
11	\$97 3239	\$0.3299	\$3 0060	\$98 3377	\$0 2153	\$1 8776	65
12	96 9940	0.3432	3 3492	98 1224	0 2260	2 1036	64
13	96 6508	0.3568	3 7060	97 8964	0 2374	2 3410	63
14	96 2940	0.3712	4 0772	97 6590	0 2492	2 5902	62
15	95 9228	0.3860	4 4632	97 4098	0 2616	2 8518	61
16	95 5368	0 4014	4 8646	97 1482	0 2749	3 1267	60 59 58 57 56
17	95 1354	0 4175	5 2821	96 8733	0 2884	3 4151	
18	94 7179	0 4342	5 7163	96 5849	0 3029	3 7180	
19	94 2837	0 4515	6 1678	96 2820	0 3181	4 0361	
20	93 8322	0.4696	6 6374	95 9639	0 3339	4 3700	
21	93 3626	0 4884	7 1258	95 6300	o 3507	4 7207	55
22	92 8742	0 5080	7 6338	95 2793	o 3682	5 0889	54
23	92 3662	0 5282	8 1620	94 9111	o 3866	5 4755	53
24	91 8380	0 5494	8 7114	94 5245	o 4059	5 8814	52
25	91 2886	0 5714	9 2828	94 1186	o 4262	6.3076	51
26	90 7172	0 5942	9 8770	93 6924	0 4476	6 7552	50
27	90 1230	0 6180	10 4950	93 2448	0 4699	7 2251	49
28	89 5050	0 6427	11 1377	92.7749	0 4934	7 7185	48
29	88 8623	0 6684	11 8061	92 2815	0 5181	8.2366	47
30	88 1939	0 6952	12 5013	91 7634	0 5440	8 7806	46
31	87 4987	0 7229	13.2242	91,2194	0 5712	9 3518	45
32	86 7758	0 7519	13 9761	90 6482	0 5997	9 9515	44
33	86 0239	0 7819	14 7580	90 0485	0 6298	10 5813	43
34	85 2420	0 8132	15 5712	89 4187	0 6612	11 2425	42
35	84 4288	0 8458	16 4170	88 7575	0 6943	11 9368	41
36 37 38 39 40	83 5830 82 7034 81 7886 80 8373 79 8479	o 8796 o 9148 o 9513 o 9894 I 0290	17 2966 18 2114 19 1627 20 1521 21.1811	88 0632 87 3342 86 5688 85 7651 84 9211	o 7290 o 7654 o 8037 o 8440 o 8861	12 6658 13 4312 14 2349 15 0789 15 9650	39 38 37 36
41	78.8189	1 0702	22 2513	84 0350	0 9304	16 8954	35
42	77 7487	1 1129	23 3642	83 1046	0.9769	17 8723	34
43	76 6358	1 1575	24.5217	82 1277	1 0258	18 8981	33
44	75.4783	1 2038	25.7255	81.1019	1.0771	19 9752	32
45	74 2745	1 2519	26 9774	80 0248	1.1309	21 1061	31
46		Current depreciation during year	28 2794	78 8939 Present value.	Current depreciation during year	22.2936	Expect- ancy Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	
75 YE	ar Life	<u> </u>				75 Yea	r Life
		4 per cent			5 per cent		
47	\$71 7206	\$1 3541	\$29 6335	\$77 7064	\$1 2468	\$23 5404	29
48	70 3665	1 4082	31.0417	76 4596	1 3092	24 8496	28
49	68 9583	1 4646	32 5063	75 1504	1 3746	26 2242	27
50	67 4937	1 5232	34 0295	73 7758	1 4434	27 6676	26
51	65 9705	1 5840	35 6135	72 3324	1 5155	29 1831	25
52	64 3865	1 6475	37 2610	70 8169	1 5913	30 7744	24
53	62 7390	1 7133	38 9743	69 2256	1 6709	32 4453	23
54	61 0257	1 7819	40 7562	67 5547	1 7545	34 1998	22
55	59 2438	1 8532	42 6094	65 8002	1 8421	36 0419	21
56	57 3906	1 9272	44 5366	63 9581	1 9343	37 9762	20
57	55 4634	2 0044	46 5410	62 0238	2 0309	40 0071	19
58	53 4590	2 0845	48 6255	59 9929	2 1326	42.1397	18
59	51 3745	2 1680	50 7935	57 8603	2 2391	44 3788	17
60	49 2065	2 2546	53 0481	55 0212	2 3511	46 7299	16
61	46 9519	2 3448	55-3929	53 2701	2 4687	49 1986	15
62	44 6071	2 4387	57 8316	50 8014	2 5921	51 7907	14
63	42 1684	2 5361	60 3687	48 2093	2 7217	54 5124	13
64	39 6323	2 6376	63 0053	45 4876	2 8577	57 3701	12
65	36 9947	2 7432	65 7485	42 6299	3 0007	60 3708	11
66	34 2515	2 8528	68 6013	39 6292	3 1507	63 5215	10
67	31 3987	2 9670	71 5683	36 4785	3 3082	66 8297	9
68	28 4317	3 0856	74 6539	33 1703	3 4737	70 3034	8
69	25 3461	3 2091	77 8630	29 6966	3 6473	73 9507	7
70	22 1370	3 3374	81 2004	26 0493	3 8297	77 7804	6
71	18 7996	3 4709	84 6713	22 2196	4 0212	81 8016	5
72	15 3287	3 6097	88 2810	18 1984	4 2222	86 0238	4
73	11 7190	3 7542	92 0352	13 9762	4 4334	90 4572	3
74	7 9648	3 9043	95 9395	9 5428	4 6550	95 1122	2
75	4 0605	4 0605	100 0000	4 8878	4 8878	100 0000	1
		6 per cent.			7 per cent		
1	\$100 0000	\$0 0769	\$0 0769	\$100 0000	\$0 0441	\$0 0441	75
2	99 9231	0 0814	0.1583	99 9559	0 0471	0.0912	74
3	99 8417	0 0864	0 2447	99 9088	0 0504	0 1416	73
4	99 7553	0 0916	0 3363	99.8584	0 0540	0 1956	72
5	99 6637	0 0970	0 4333	99 8044	0 0578	0 2534	71
	Present value	Current depreciation during year		Present value.	Current depreciation during year		Expect- ancy. Years.

TABLE 27 AMORTIZATION AND DEPRECIATION TABLES (Continued)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortiza- tion end of year	-
75 YE.	ar Life					75 YEAR	R LIFE
		6 per cent				-	
6	\$99 5667	\$0 1029	\$0 5362	\$99 7466	\$0 0618	\$0 3152	70 69 68 67 66
7	99 4638	0 1090	0 6452	99 6848	0 0661	0 3813	
8	99 3548	0 1156	0 7608	99 6187	0 0707	0 4520	
9	99 2392	0 1225	0 8833	99 5480	0 0757	0 5277	
10	99 1167	0 1299	1 0132	99 4723	0 0810	0 6087	
11	98 9868	0 1376	1 1508	99 3913	o o867	o 6954	65
12	98 8492	0 1459	1 2967	99 3046	o o927	o 7881	64
13	98 7933	0 1547	1 4514	99 2119	o o992	o 8873	63
14	98 5486	0 1640	1 6154	99 1127	o 1062	o 9935	62
15	98 3846	0 1738	1 7892	99 0065	o 1136	i io7i	61
16	98 2108	0 1842	1 9734	98 8929	o 1216	1.2287	60
17	98 0266	0 1952	2 1686	98 7713	o 1301	1.3588	59
18	97 8314	0 2070	2 3756	98 6412	o 1392	1 4980	58
19	97 6244	0 2194	2 5950	98 5020	o 1489	1 6469	57
20	97 4050	0 2326	2 8276	98 3531	o 1593	1 8062	56
21	97 1724	o 2465	3 0741	98 1938	0 1705	1 9767	55
22	96 9259	o 2613	3 3354	98 0233	0 1824	2 1591	54
23	96 6646	o 2770	3 6124	97 8409	0 1952	2 3543	53
24	96 3876	o 2936	3 9060	97 6457	0 2089	2 5632	52
25	96 0940	o 3113	4 2173	97 4368	0 2235	2 7867	51
26	95 7827	0 3299	4 5472	97 2133	0 2391	3 0258	50
27	95 4528	0 3497	4 8969	96 9742	0 2559	3 2817	49
28	95 1031	0 3707	5 2676	96 7183	0 2738	3 5555	48
29	94 7324	0 3929	5 6605	96 4445	0 2929	3 8484	47
30	94 3395	0 4165	6 0770	96 1516	0 3134	4.1618	46
31	93 9230	0 4414	6 5184	95 8382	0.3354	4 4972	45
32	93 4816	0 4680	6 9864	95 5028	0 3589	4 8561	44
33	93 0136	0 4961	7 4825	95 1439	0 3840	5 2401	43
34	92 5175	0 5258	8 0083	94 7599	0 4109	5 6510	42
35	91 9917	0 5574	8 5657	94 3490	0 4396	6.0906	41
36 37 38 39 40	91 4343 90 8435 90 2173 89 5535 88 8498	 5908 6262 6638 7037 7459 	9 1565 9 7827 10.4465 11 1502 11 8961	93 9094 93 4390 92 9357 92 3971 91 8208	o 4704 o 5033 o 5386 o 5763 o 6166	6 5610 7.0643 7 6029 8 1792 8 7958	39 38 37 36
41	Present value.	Current deprecia- tion during year	12 6867	91 2042 Present value	Current depreciation during year.	9.4556	Expectancy Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES (Concluded)

Year.	Remaining investment beginning of year	Amortiza- tion during year	Total amortization end of year.	Remaining investment beginning of year.	Amortiza- tion during year	Total amortization end of year.	
75 YE	ar Life					75 YE	R LIFE
		6 per cent.			7 per cent		
42	\$87 3133	\$0 8381	\$13 5248	\$90 5444	\$0.7059	\$10 1615	34
43	86 4752	0.8883	14.4131	89.8385	0.7554	10 9169	33
44	85 5869	0 9417	15.3548	89.0831	0.8082	11 7251	32
45	84 6452	0 9982	16 3530	88 2749	0.8648	12 5899	31
46	83 6470	1 0580	17 4110	87 4101	0 9253	13 5152	30
47	82 5890	1 1215	18 5325	86 4848	0 9901	14 5053	29
48	81 4675	1.1888	19 7213	85 4947	1 0594	15 5647	28
49	80 2787	1 2602	20 9815	84 4353	1 1336	16 6983	27
50	79 0185	1 3357	22 3172	83 3017	1.2129	17 9112	26
51	77 6828	1 4159	23.7331	82 0888	1.2979	19 2091	25
52	76 2669	1 5009	25 2340	80 7909	1 3887	20 5978	24
53	74.7660	1 5909	26 8249	79 4022	1 4859	22 0837	23
54	73 1751	1 6864	28.5113	77 9163	1 5899	23.6736	22
55	71 4887	1 7875	30.2988	76 3264	1 7012	25.3748	21
56	69 7012	1 8948	32 1936	74 6252	1 8203	27 1951	20
57	67.8064	2 0085	34 2021	72 8049	1 9477	29 1428	19
58	65 7979	2 1290	36 3311	70 8572	2 0841	31 2269	18
59	63 6689	2 2567	38 5878	68 7731	2 2299	33 4568	17
60	61 4122	2 3921	40 9799	66 5432	2 3860	35 8428	16
61	59 0201	2 5357	43 5156	64 1572	2 5531	38 3959	15
62	56 4844	2.6878	46 2034	61 6041	2 7318	41.1277	14
63	53 7966	2 8491	49 0525	58 8723	2 9230	44.0507	13
64	50 9475	3 0200	52 0725	55 9493	3 1276	47 1783	12
65	47 9275	3 2012	55 2737	52 8217	3 3466	50 5249	11
66	44 7263	3 3933	58 6670	49 4751	3 5808	54.1057	10
67	41.3330	3 5969	62 2639	45 .8943	3 8315	57 9372	9
68	37 7361	3 8127	66 0766	42 0628	4.0997	62.0369	8
69	33 9234	4 0415	70 1181	37 9631	4.3867	66.4236	7
70	29 8819	4.2839	74.4020	33 5764	4.6937	71 1173	6
71	25 5980	4.5410	78 9430	28.8827	5.0233	76.1406	5
72	21.0570	4 8135	83 7565	23.8594	5.3738	81 5144	4
73	16.2435	5.1022	88.8587	18.4856	5.7500	87 2644	3
74	11.1413	5.4084	94 2671	12.7356	6.1525	93.4169	2
75	5.7329	5.7329	100 0000	6.5831	6.5831	100.0000	1
	Present value.	Current deprecia- tion during year.		Present value.	Current deprecia- tion during year,	Marie e e e e e e e e e e e e e e e e e e	Expect- ancy. Years.

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